Concepts 1

Equations

(a) Simple Linear Equations :

Transposition : A term can be transposed from one side of the equation to the other by changing its sign. A fraction is transposed, simply by inverting it. When simplifying the equations, always keep the terms containing the unknowns on left hand side (LHS) of the equation and all the other terms on right hand side (RHS).

1 $5x + 17 = 67$

Sol. $\therefore$ $5x = 67 - 17 = 50 \Rightarrow 5x = 50$
$\therefore$ $x = (1/5) \times 50 = 10$
$\therefore$ $x = 10$.

2 $(7x - 4)/3 = x + 4$

Sol. $\therefore$ $7x - 4 = 3(x + 4)$
$\therefore$ $7x - 4 = 3x + 12$ $\therefore$ $7x - 3x = 12 + 4 = 16$
$\therefore$ $4x = 16$
$\therefore$ $x = 4$.

3 $(3x - 5)/7 + (5x + 2)/4 = 2$

Sol. In order to remove fractions, multiply throughout by the L.C.M. of the three terms appearing in the denominators of the above equation (L.C.M. of 7, 4, 1 is 28).

$$\frac{28(3x-5)}{7}+\frac{28(5x+2)}{4}=\frac{28\times2}{1}$$

$\therefore$ $4(3x - 5) + 7(5x + 2) = 56$ $\therefore$ $12x - 20 + 35x + 14 = 56$
$\therefore$ $12x + 35x = 56 + 20 - 14 = 62$ $\therefore$ $47x = 62$
$\therefore$ $x = 62/47$.

4. $$\frac{1.2x+2}{3.5}+\frac{x-4}{0.7}=\frac{2.4x+0.8}{2.0}$$

Sol. Remove decimal points by multiplying numerator and denominator throughout by 10.

$$\frac{12x+20}{35}+\frac{10x-40}{7}=\frac{24x+8}{20}=\frac{6x+2}{5}$$

Multiplying throughout by 35 (LCM of 35, 7 and 5)

$$\frac{35(12x+20)}{35}+\frac{35(10x-40)}{7}=\frac{35(6x+2)}{5}$$

$12x + 20 + 5(10x - 40) = 7(6x + 2)$ $\therefore$ $12x + 20 + 50x - 200 = 42x + 14$
$12x + 50x - 42x = 14 - 20 + 200$ $\therefore$ $20x = 194$ or $x = 1/20 \times 194$
$\therefore$ $x = 9.7$

(b) Simultaneous Equations (Two Unknowns, Two Equations)

In order to obtain the two unknowns from the two given equations, equalize the coefficients of any one unknown of the two equations. Add the equations or subtract one from the other so that one unknown (whose coefficients were equalized) is eliminated. Solve the resulting simple equation. Substitute the value thus obtained in either of the original equations so as to obtain second unknown.

5. 3x + 4y = 13 (1)
5x + 3y = 18 (2)

Sol. To equalize the coefficients of y in the above two equations, multiply equation.(1) by 3 & equation.(2) by 4 The resulting equations are...
9x + 12y = 39 (3)
20x + 12y = 72 (4)
To eliminate one unknown, subtract equation.(3) from equation.(4)

$$\begin{array}{rl} & 20x + 12y = 72 \\ - & \underline{9x + 12y = 39} \\ & 11x \quad\quad = 33 \\ & \therefore x = 3 \end{array}$$

Substitute x = 3 in equation.(1) 3 × 3 + 4y = 13
$\therefore$ 4y = 4 $\Rightarrow$ y = 1
Thus, the resulting solution is: x = 3 ; y = 4.

6.
$$\frac{3x+2y}{4}+\frac{4x-y}{5}-4=0 \quad \cdots\cdots\cdots \quad (1)$$
$$\frac{2x+y}{7}+x+y=6 \quad \ldots\ldots\ldots \quad (2)$$

simplify the two equations by eliminating fractions. Multiply equation (1) through by 20

$$20\times\frac{3x+2y}{4}+20\times\frac{4x-y}{5}-20\times 4=0$$

$\therefore$ 15x + 10y + 16x - 4y - 80 = 0
31 x + 6 y = 80 (3)

Multiply equation.(2) throughout by 7.

$$7\times\frac{2x+y}{7}+7x+7y=42$$

$\therefore$ 2x + y + 7x + 7y = 42
9 x + 8 y = 42 (4)
To equalize the coefficients, multiply equation.(3) by 4 and equation.(4) by 3. Which results in. . .

$$\begin{array}{rll} 124x + 24y &= 320 & (5) \\ \underline{27x + 24y} &= \underline{126} & (6) \\ 97x &= 194 & \\ \therefore x &= 2 & \end{array}$$

Substituting x = 2 in equation.(4),
18 + 8y = 42
8y = 24 $\therefore$ y = 3
Solution Set is (2, 3).

* **Some other Methods of solving Simultaneous equations**
* To solve the equations of the type a x +b y = c and b x +a y = d. Add and subtract these two equations. Again add and subtract the resulting equations to get the solution.

e.g. 13x + 17y = 124 and 17x + 13y = 116.
Sol : Adding these two equations we get 30x + 30 y = 240 i.e. x+y = 8. Subtracting them we get - 4x + 4y = 8. $\therefore$ - x + y = 2. Adding these two resulting equations we get y = 5. Hence x = 3

* To solve the equations of the type a/ (αx+β) + b/ (γy+δ) = m and c/(αx+β) + d /(γy+δ) = n. Put 1/(α x+β) = X and 1/ (γ y+δ) = Y. Solve the equations for X and Y. Then solve for x and y.

* **General Method :** In General to solve the equations ax + by = c and mx + ny = d, Write the coefficient of y, x and constants as shown below

x	y	-1	
b	c	a	b
n	d	m	n

Then x/(bd-cn) = y/(cm-ad) = (-1)/(an-bm)

* **Number of Solutions :** Consider the two equations
$a_1 x + b_1 y = c_1$
$a_2 x + b_2 y = c_2$
Then depending on the coefficients the number of solutions are as follows.

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Infinite Solutions

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ No Solution (i.e. Inconsistent Equations)

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ Unique Solution

(c) Quadratic Equations

An equation of the form a x^2 + b x + c = 0, where, a, b and c are given numbers such that a is not zero and x is an unknown is called a quadratic equation. A quadratic equation cannot have more than two roots. The two roots of a quadratic equation are given by:

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$\Delta = b^2 - 4ac$ is called the discriminant of the quadratic equation. The term Δ can be negative, zero or positive. The following table illustrates the relationship between Δ and the two roots of the quadratic equation.

$\Delta = b^2 - 4ac$	***Nature of Roots***	***Form/Value***
> 0	Real and Unequal; Two Distinct Roots	$(-b \pm \sqrt{\Delta})/2a$;
	If it is a perfect square then roots are rational otherwise irrational.	
= 0	Real and Equal	-b/2a
< 0	Imaginary Roots	p + iq

If α and β are the two roots of a quadratic equation then the quadratic equation can be written as:
$(x - \alpha)(x - \beta) = 0$. Or, $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

$$\boxed{x^2 - (\text{sum of the roots})\,x + (\text{product of the roots}) = 0}$$

(d) The sign of the expression $(x - a)(x - b)$ where $(a < b)$

Case I.

$(x - a)(x - b)$ is + ve

Above is possible if either both factors are + ve or both - ve

As $a < b$; x does not lie between a and b, i.e. $x < a$ or $x > b$.

Case II.

$(x - a)(x - b)$ is - ve

Above is possible if one of the factors is + ve and the other is - ve

$\therefore$ $x < a$ and $x > b$ **or** $x > a$ and $x < b$

As $a < b$; x lies between a and b, i.e. $a < x < b$

Ex. Form a quadratic equation whose roots are 3 and 7.

Sol. A quadratic equation whose roots are known can be written in the form:

x^2 - (Sum of the Roots)x + Product of the Roots = 0

i.e. $x^2 - (3 + 7)x + (3 \times 7) = 0$ $\quad \therefore x^2 - 10x + 21 = 0$ is the required answer.

Ex. Solve the following equations for x and y

$2x^2 + y^2 = 3$ (1) and $x + y = 2$ (2)

Sol. From equation.(2), $x = 2 - y$. Substitute, (2-y) for x in equation.(1)

$2(2-y)^2 + y^2 = 3$ $\quad \therefore 2(4 - 2y + y^2) + y^2 = 3$

$8 - 8y + 2y^2 + y^2 = 3$ $\quad \therefore 3y^2 - 8y + 5 = 0$

$3y^2 - 3y - 5y + 5 = 0$ $\quad \therefore 3y(y - 1) - 5(y - 1) = 0$

$(y - 1)(3y - 5) = 0$ $\quad \therefore (y - 1) = 0$ Or, $(3y - 5) = 0$

$y = 1$ Or, $y = 5/3$

In equation.(2), when $y = 5/3$, $x = 2 - 5/3 = 1/3$ $\quad \therefore$ when $y = 1$, $x = 2 - 1 = 1$

Ex. Solve: $\dfrac{3}{x+1} - \dfrac{1}{x+3} = \dfrac{7}{4x}$

Multiply both sides of the equation by $(x + 1)(x + 3)(4x)$

$3(x + 3)(4x) - (x + 1)(4x) = 7(x + 1)(x + 3)$

$12x^2 + 36x - 4x^2 - 4x = 7(x^2 + 4x + 3)$

$8x^2 + 32x = 7x^2 + 28x + 21$, $\quad$ i.e., $x^2 - 4x - 21 = 0$

$x^2 - 3x + 7x - 21 = 0$ $\quad$ i.e., $x(x - 3) + 7(x - 3)$

$(x + 7)(x - 3) = 0$ $\quad (x + 7) = 0$ Or, $(x - 3) = 0$

$x = -7$ Or, $x = 3$.

Ex. Solve the following quadratic equations.

(I) $15y^2 - 11y + 2 = 0$ $\qquad$ (ii) $x^2 - 6x + 9 = 0$

Sol. (i) $15y^2 - 5y - 6y + 2 = 0 \Rightarrow 5y(3y - 1) - 2(3y - 1) = 0$

$\therefore$ $(5y - 2)(3y - 1) = 0 \Rightarrow (5y - 2) = 0$ Or, $(3y - 1) = 0$

$\therefore$ $y = 2/5$ Or, $y = 1/3$.

(ii) $(x - 3)^2 = 0$ $\qquad x = 3$.

Ex. For what value of m is the expression $4x^2 - 6x + m$ a perfect square ?

Sol. For Given expression to be a perfect square the last term **(L.T.)** is given by

$$\boxed{\textbf{L.T.} = (\textbf{M.T.})^2 / 4 \times \textbf{F.T.}}$$

Where M.T. = Middle term and F.T. = First term

$\therefore$ L .T. = $(-6x)^2 / 4\times(4x^2) = 9/4$.
Thus the given equation is a perfect square when, m = 9/4.

Ex. For what value of k will the equation $2y^2 + 3ky + 18 = 0$ have equal roots ?

Sol. Here, a = 2, b = 3k and c = 18. A quadratic equation has equal roots when the discriminant $b^2 - 4ac$ is zero.
If, $(3k)^2 - 4(2 \times 18) = 0$ Then, $9k^2 = 144$ and $k = \sqrt{16}$ $k = \pm 4$.

(d) Indeterminate Equations

Consider the following equation
$3x + 4y = 10$
The above equation has two unknowns viz. x and y. It is clear that by ascribing any value to x, we get a corresponding value of y.
e.g. if x = 2, y = 1
if x = -2, y = 4.
Thus the above equation can have infinite solutions. In general, if the number of unknowns is greater than the number of equations, there will be an unlimited number of solutions. The equations in that case are said to be indeterminate.

(e) Simultaneous Quadratic Equations

Ex. Solve x + y = 18, xy = 45

Sol. x + y = 18 ……………………(1)
xy = 45 ……………………(2)
Squaring (1) we get $x^2 + 2xy + y^2 = 324$
From 2 we get 4xy = 180
by subtraction $x^2 - 2xy + y^2 = 144$ or $x - y = \pm 12$
Combining this result with equation 1, we have, x + y = 18, x - y = 12 $\therefore$ x = 15, y = 3
x + y = 18, x - y = -12 $\therefore$ x = 3, y = 15

Ex. Solve $x^2 + y^2 = 160$, x + y = 16

Sol. $x^2 + y^2 = 160$ …………………(1)
x + y = 16 …………………(2)
Subtract equation 1 from the square of equation 2,
$2xy = 96 \Rightarrow xy = 48$ …………(3)
Now solving equation 2 & 3 as in the previous example we get, x = 12, y = 4 or x = 4, y = 12

Ex. Solve $x^3 - y^3 = 999$, x - y = 3

Sol. $x^3 - y^3 = 999$ … (1)
x - y = 3 ………………(2)
Dividing (1) by (2) , $x^2 + xy + y^2 = 333$... ……(3)
Squaring (2), $x^2 - 2xy + y^2 = 9$……… (4)
Subtracting, 3xy = 324 or xy = 108………(5)
From 2 & 5, proceeding as in the previous problem, x = 12, y = 9 or x = -9, y = -12

Ex. Solve 1/x - 1/y = 1/3, $1/x^2 + 1/y^2 = 5/9$

Sol. 1/x - 1/y = 1/3 ………………(1)
$1/x^2 + 1/y^2 = 5/9$ ……………(2)
Squaring 1, $1/x^2 - 2/xy + 1/y^2 = 1/9$ ……(3)
Subtracting 3 from 2, 2/xy = 4/9
Adding this to 2, we get,
$1/x^2 + 2/xy + 1/y^2 = 1$ $\therefore$ $1/x + 1/y = \pm 1$

Solving together with 1, $1/x = 2/3$ or $1/x = -1/3$, $1/y = 1/3$ or $1/y = -2/3$
$x = 3/2$ or -3 and $y = 3$ or $-3/2$

(f) Theory Of Equations

The degree of an equation is the highest power of the variable in the given equation and the roots are those values of the variable, which satisfy the given equation and maximum possible number of real roots of the equation can be equal to the degree of the equation.

Rules for finding the nature of roots of the given equation

1. If all the coefficients of the given equation are positive, the equation has no positive root; thus the equation $x^5 + x^4 + x^3 + 2x + 1 = 0$ cannot have any positive root.
2. If the coefficients of the even powers of x are all of same sign, and the coefficients of the odd powers are all of the opposite sign, the equation has no negative root;
for e.g. $X^7 + x^5 - 2x^4 + x^3 - 3x^2 + 7x - 5 = 0$ this equation has no negative root.

3. If the equation contains only even powers of x and coefficients are all of the same sign, the equation has no real roots;
Thus the equation $2x^6 + 4x^4 + 2x^2 + 9 = 0$; will have no real roots.

4. If the equation contains only odd powers of x and coefficients are all of the same sign, the equation has no real root, except $x = 0$;
$x^7 + x^5 + x^3 + x = 0$ will have no real root except $x = 0$

All the above rules can be summarized with help of the rule known as Descartes' *rule of signs*.
The rule states that f (x) = 0 cannot have more positive roots than there are changes of sign in f (x), and cannot have more negative roots than there are changes of sign in f (-x)
For example we have the equation $3x^4 + 12x^2 + 5x - 4 = 0$
Only one change of sign takes place in the given equation so it cannot have more than one positive root.
$F(-x) = 3x^4 + 12x^2 - 5x - 4 = 0$ in this also we there is only one change in sign is therefore it cannot have more than one negative roots.

As the equation can have one positive, one negative root and degree of the equation is 4 so the total no. of roots must be equal to 4. So there must be at least 2 imaginary roots.

EXERCISE 1

1. Solve for x : $5x - 17 + 3x - 5 = 6x - 7 - 8x + 115$.

2. $x - [3 + \{x - (3 + x)\}] = 5$

3. $(x + 1)^2 + 2(x + 3)^2 = 3x(x + 2) + 35$

4. $(x + 1)(x + 2)(x + 6) = x^3 + 9x^2 + 4(7x - 1)$

5. $(3a + 1)(2a - 7) = 6(a - 3)^2 + 7$

6. $5/x + 6/y = 3$, $15/x + 3/y = 4$

7. The sum of a number of two digits and of a number formed by reversing the digits is 110, and the difference in the digits is 6. Find the numbers.

8. Six pigs and seven goats can be bought for Rs. 2500, and thirteen goats and eleven pigs are bought for Rs. 4610. What is the cost of each animal ?

9. The middle digit of a three digit number is zero, the sum of its digits is 11. If the digits are reversed, the number so formed exceeds the original number by 495. What is the number ?

10. A fraction becomes 1/2 when 1 is subtracted from the numerator and 2 is added to the denominator, and becomes 1/3 when 7 is subtracted from the numerator and 2 from the denominator. What is the fraction ?

11. Solve the following quadratic equations
(a) $x^2 - 8x + 16 = 0$ (b) $12x^2 - 11x + 2 = 0$ (c) $7x^2 - 13x + 3 = 0$

12. Form the quadratic equation whose roots are...
(a) 1/3, -5/2 (b) p/q, q/p (c) $(3 \pm 5\sqrt{-1})/2$

13. If p and q are the roots of the equation $x^2 + bx + c = 0$, form an equation whose roots are $(p - q)^2$ and $(p + q)^2$

14. Find a condition for which the roots of the equation $ax^2 + bx + c = 0$ are...
(i) Equal in magnitude but opposite in sign (ii) Reciprocals

15. What is the common root in terms of a, b, c, p, q and r for the equations $ax^2 + bx + c = 0$ and $px^2 + qx + r = 0$?

16. $x^2 - 6x + m = 0$ is a quadratic equation with one root given by $3 + \sqrt{2}$. Find m and the second root.

17. What is the nature of roots of the following quadratic equations
(a) $3x^2 - 14x + 2 = 0$ (b) $3(5x^2 + 1) = 17x$

18. Find a value of h for which the roots of the equation $5x^2 - 2hx = -5$ are real and equal.

19. Solve $x^3 + y^3 = 407$, $x + y = 11$

20. Solve $1/x + 1/y = 2$, $x + y = 2$

21. Solve $1/x^2 + 1/y^2 = 61/900$, $xy = 30$

22. Solve $1/x + 1/y = 7/12$, $xy = 12$

23. Solve $x^2 + y^2 = 185$, $x - y = 3$

24. $x^3 - y^3 = 56$, $x^2 + xy + y^2 = 28$

25. $(x - y)/10 = 1$, $x^2 - 4xy + y^2 = 52$

26. Find two numbers such that their sum is 29, and the larger exceeds the smaller by 9.

27. A man walks 1 km, covers the second part of the journey on horseback, and then twice as much by car. If the entire journey is 10 km, how far does he travel on horseback ?

28. Divide Rs. 38 between A, B, and C, such that B has Rs. 5 more than A, and C has Rs. 10 more than B.

29. Find a number such that if 5, 15, and 35 are added to it, the product of the first and the third results is equal to the square of the second.

30. If seven-eighteenths, five-twelfths, and a sixteenth part of a number add up to 125, find the number.

31. Two-fifths of A's money is equal to B's. and seven-ninths of B's is equal to C's. In all they have Rs. 770. how much does each have ?

32. The width of a room is two-thirds of its length. If the width had been 3 feet more, and the length 3 feet less, the room would have been square. Find the dimensions of the room.

Solve the following equations.

33. $\frac{x}{4} + \frac{x-5}{3} = 10$ 34. $\frac{x+19}{5} = 3 + \frac{x}{5}$ 35. $\frac{x+5}{6} - \frac{x+1}{9} = \frac{x+3}{4}$

36. $\frac{4(x+2)}{3} - \frac{6(x-7)}{7} = 12$ 37. $\frac{x-8}{5} + \frac{4+x}{4} + \frac{x-1}{7} = 7 - \frac{23-x}{5}$

38. P, Q, R, S have Rs. 290 between them. P has twice as much as much as R, and Q has three times as much as S. R and S together have Rs. 50 less than P. Find the share of each.

39. One-eleventh of A's age exceeds one-seventh of B's by two years. Twice B's age is same as A's age thirteen years ago. Find their present ages.

40. A man has a number of one rupee and five-paise coins. He observes that if the rupees were turned into five-paise coins and the five-paise coins into rupees, he would have Rs. 5.70 more. If the rupees were turned into fifty-paise coins and the five-paise coins into ten-paise coins, he would have Rs. 1.95 less. How much amount does he have ?

41. In a bag containing black and white balls , half the number of white ones is equal to a third of the number of black, and twice the whole number of balls exceeds three times the number of black balls by four. How many balls does the bag contain ?

42. The wages of 10 men and 8 boys amount to Rs. 37. If 4 men together receive Rs. 1 more than 6 boys, what are the wages of man and boy ?

43. A man is five times as old as his son. The sum of the squares of their ages is 2106. Find their ages.

44. The sum of the reciprocals of two consecutive numbers is (15 / 56). Find the numbers.

45. A hall can be paved with 200 square tiles of a certain size. If each tile were 5 cm longer each way, covering the same area would take 128 tiles. Find the length of each tile.

46. A merchant bought certain metres of cloth for Rs. 50. He kept 5 m for himself and sold the rest at Re.1 per metre more than he gave, and made a profit of Rs. 10. How many metres did he buy ?

47. Two farmers A and B have 30 cows between them. They sell their cows at different prices, but each receives the same sum. If A had sold his cows at B's price he would have received Rs.3200. Had B sold his cows at A's price he would have received Rs. 2450. How many cows did each one have ?

Find the values of x and y in each of the following sets of equations.

48. $ax + by = 2$; $abxy = 1$.

49. $x - y = 10$; $x^2 - 4xy + y^2 = 52$

50. $\dfrac{1}{x} + \dfrac{1}{y} = 2\ ;\ x + y = 2$

51. What is the value of $(\alpha^3 + \beta^3)$, if α and β are the roots of the equation $px^2 + qx + r = 0$?

52. A square metallic plate of side d, increases by $1/\ell$ on heating. What is the fractional change in the area of the plate?

53. $x^4 + 4x^3 + 6x^2 + 4x + 5 = 0$ one root being $\sqrt{-1}$ discuss the nature of the other roots.

54. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$, be equal. Then find the relation between a, b and c.

55. If the roots of the equation $x^2 + a^2 = 8x + 6a$ be real then. Find the interval in which a lies.

56. Find the value of q for which the roots of the equation $3x^2 - 7x + 7q = 0$ are reciprocals of each other.

57. In $x^9 + 5x^8 - x^3 - 7x + 2 = 0$, how many minimum imaginary roots are possible?

58. Find the condition that $x^3 - px^2 + qx - r = 0$ may have two roots equal but with opposite sign.

59. Find the sum of the squares of the roots of the equation
$x^3 - px^2 + qx - r = 0$

60. What may be inferred regarding the roots of the equation $4x^6 + x^4 + 2x - 3 = 0$?

ANSWERS

1. 13	**2**. 5	**3**. 2
4. 2	**5**. 4	**6**. 5, 3
7. 28, 82	**8**. 230, 160	**9**. 308
10. (15 / 26)	**11**(a). 4	(b) 1/4, 2/3
(c) $(13 \pm \sqrt{85})/14$	**12**(a) 6x2 + 13x - 5 = 0	(b) $(pq)x^2 - (p^2 + q^2)x + pq = 0$
(c) $2x^2 - 6x + 17 = 0$	**13**. $X^2 - 2(b^2 - 2c)x + b^2(b^2 - 4c) = 0$	**14**. (i) b = 0 (ii) c = a
15. $\alpha = (cp - ra)/(aq - pb)$	**16**. $(7, 3 - \sqrt{2})$	
17.(a)Complex	(b)Real&Unequal	
18. $h = \pm 5$	**19**. (7,4), (4, 7)	**20**. (1, 1)
21. $(\pm 6, \pm 5), (\pm 5, \pm 6)$	**22**. (4, 3), (3, 4)	**23**. (11, 8), (-8, -11)
24. (4, 2), (-2, -4)	**25**. (12, 2), (-2, -12)	**26**. 10, 19
27. 3 km	**28**. Rs. 6, 11, and 21	**29**. 5
30. 144	**31**. Rs. 450, 180, and 140	**32**. 18, 12
33. 20	**34**. no solution	**35**. - (1/7)
36. 7	**37**. 8	**38**.P-140, Q-60, R-70, S-20
39. 55, 21	**40**. Rs.5.55	**41**. b = 12, w = 8
42. Rs. 2.5, 1.5	**43**. 9, 45	**44.** 7, 8
45. 20 cm	**46**. 25 m	**47**. 16, 14
48. (1/a, 1/b)	**49**. 12 & 2 or -2 & -12	**50**. 1, 1
51. $(1/p^3)(3pqr - q^3)$	**52.** $(1/\ell d)(2 + 1/\ell d)$.	
53. Two complex & two negative real roots.		
54. a = 0 or $a^3 + b^3 + c^3 = 3abc$	**55.** $-2 \le a \le 8$	**56.** 3/ 7
57. At least 6 imaginary roots	**58.** p. q = r	**59.** $p^2 - 2q$
60. One positive, one negative and 4 imaginary		

Concepts 2

Ratio, Proportion, Variation

Number 5 is one third of 15 or 15 is three times 5. In such a situation we say that *the ratio of 5 to 15 is equal to 1/3* or *the ratio of 15 to 5 is equal to 3.* The ratio of number a and number b is the quotient of a and b and is expressed as a : b or a/b.
Any ratio a : b = ka : kb k not equal to 0

If **a / b = c / d** then
b/a = d/c
a/c = b/d
d/b = c/a
(a + b)/b = (c + d)/d
(a - b)/b = (c - d)/d
(a + b)/(a - b) = (c + d)/c - d)

If $a_1/b_1 = a_2/b_2 = a_3/b_3 = ... = a_n/b_n$, then each of these fractions is

$(k_1a_1 + k_2a_2 +...+ k_na_n)/(k_1b_1 + k_2b_2 +...+ k_nb_n)$ or $(a_1 + a_2 + a_3 +...+ a_n)/(b_1 + b_2 + b_3 +...+ b_n)$

If four numbers a, b, c, d are in proportion, then a : b = c : d. The number d is called the fourth proportional to a, b and c. Numbers a and d are called the extremes and the numbers b and c are called the means.

If a : b = c : d then the *product of the means is the product of the extremes* or, mathematically, ad = bc. This is the test for proportionality.

If a : b = b : c = c : d then, the numbers a, b, c and d are said to be in continued proportion. If a, b, c are in *continued proportion* then, b is called the mean proportional between a & c and $b^2 = ac$.

Some Useful Results:

1. If a/b = 1, then (a + x) / (b + x) = a/b. if $x \neq -a$.
2. If a/b > 1, then (a + x) / (b + x) < a/b. if x > 0.
3. If a/b < 1, then (a + x) / (b + x) > a/b. if x > 0
4. If a/b = 1, then (a - x) / (b - x) = a/b. if $x \neq a$
5. If a/b > 1, then (a - x) / (b - x) > a/b. if x > 0 and x < b
6. If a/b < 1, then (a - x) / (b - x) < a/b. if x > 0 and x < b

Ex. (2a + 3b): a - b :: 2:3 find the value of 2 + (a/b)

Sol. (2a + 3b)/ (a - b) = 2/3 $\therefore$ 6a + 9b = 2a - 2b
i.e. 4a = -11b $\therefore$ (a/b) = -(11/4)

Thus, 2 + (a/b) = -(3/4)

Ex. If 4, x and 25 are in continued proportion, find x.

Sol. Since the three numbers are in continued proportion
$x^2 = 25 \times 4 = 100 \quad \therefore x = \pm 10$

Ex. Find the fourth proportional to: 12, 20, 9

Sol. Let the fourth proportional be **x**
Product of the means is equal to the product of the extremes
$\therefore$ 12 × **x** = 20 × 9 i.e. **x** = 180/12 = 15

Variation: A quantity *A* is said to vary directly as quantity *B* when, the two quantities depend upon each other in such a manner that if *B* is changed, *A* is changed in the same ratio.

$A \propto B$ (A varies as B)
Then, $A = kB$ Where, k is a constant.

A quantity *A* is said to vary inversely as quantity *B* when *A* varies directly as the reciprocal of *B*. Mathematically, this can be expressed as:

A = k/B Where, k is a constant

A quantity is said to vary jointly as number of others when it varies directly as their product.
A = k (B × C)

A is said to vary directly as *B* and inversely as *C* if A varies as *B/C*

Ex. If A = 4 when B = 24, find B when A = 7, where B varies directly as A.

Sol. A = kB $\therefore$ k = A/B i.e. k = 4/24 = 1/6
When, A = 7, 1/6 = 7/B i.e. B = 42

Ex. Square of x varies as cube of y Given that x = 4 when y = 2. Find x when y is 4.

Sol. $x^2 \propto y^3$ $\therefore x^2 = k\,y^3$ (1)
Substituting x = 4 and y = 2 in Equation. (1)
16 = k × 8 $\therefore$ k = 2
When y = 4, $x^2 = 2 \times 64$ Thus, $x = \pm 8\sqrt{2}$

Ex. Area of a triangle varies as its base when height is constant and as height when base is constant. If the base is 4 cm and height is 10 cm, the area of the triangle is 20 cm^2. Find the length of the base if the area of the triangle is 25 cm^2 and its height is 5 cm.

Sol. Area $\propto$ Base × Height
A = k × B × H
20 = k × 4 × 10
k = 1/2
25 = 1/2 × B × 5 $\therefore$ B = 10 cm

EXERCISE 2

1. What number must be added to each term of the ratio 5 : 37 to make it equal to 1 : 3 ?

2. If x : y :: 3 : 4 Find the ratio of 7x - 4y : 3x + y

3. Find the fourth proportional to 5, 3, 10

4. Find the mean proportional between (x/y) + (y/x) and $x^3/y(x^2 + y^2)$

5. If a/3 = b/5 = c/1 = (3a + 5b - 7c)/x Find x.

6. The two sides of a rectangle are in the ratio 3:2. If the area of the rectangle is 96 cm^2. Find length of each side

7. If $4a^2 + 8b^2 = 12ab$. Find a : b

8. Ages of husband and wife are in the ratio 5 : 4. After 25 years, the ratio of their ages will be 10 : 9. Find their present ages.

9. Find four proportional such that the sum of their means is 19, sum of their extremes is 21 and sum of the squares of all the four numbers is 442.

10. Volume of a right circular cylinder varies as the square of the radius of the base when height is constant, and as height when radius of base is constant. The volume of the cylinder is 88 cm^3 when the radius of the base is 21 cm and the height is 12 cm. Find the radius when the height is 7 cm and the volume is 462 cm^3.

11. If a varies directly as the square root of b and inversely as cube of c and if a = 4 when b = 16 & c = 2. Find b when a = 2 and c = 4.

12. Two identical containers contain a mixture of alcohol and water in the ratio x : 1 and y :1. Find the ratio of alcohol to water when the contents of the two containers are mixed.

13. The force of attraction due to gravity between the two bodies varies directly as their masses and inversely as the square of the distance separating the two bodies. If two bodies weighing 20×10^7 kg and 4×10^{10} kg are separated by a distance of 1000 m, the force of attraction is 48N. Find the distance separating the two bodies weighing 11×10^9 kg and 2×10^6 kg and exerting a force of 66 N.

14. The area of a circle varies as the square of its radius. If the area of the circle is 154 cm^2 when the radius is 7 cm, find the area when the radius is 10.5 cm.

15. The value of a coin varies directly as the square of its diameter, thickness remaining constant. The value also varies directly as the thickness, the diameter remaining constant. Two coins have their diameters in the ratio 4 : 3. If their values are in the ratio 4 : 1 find the ratio of their thickness'.

16. The time taken by an object to fall from a height varies as the square root of the height from which it falls. It takes 3 seconds to fall 44.10 metres. What time would it take to fall 122.5 metres?

17. The altitude of a triangle varies directly as its area and inversely as its base. A triangle has a base of 4 metres, an area of 6 square metres and an altitude of 3 metres. Find the altitude of a triangle with area 12 square metres and base 8 metres.

18. Two numbers are in the ratio 3 : 4. If 7 is subtracted from them the remainders are in the ratio 2 : 3. Find the two numbers.

19. The volume of a certain solid varies jointly as the area of its base and its height. when the area of its base is 60 square metres and the height is 14 metres, the volume is 280 cubic metres. What is the area of base if the volume is 390 cubic metres and the height is 26 metres ?

20. p varies inversely as $q^2 - 1$. If p = 24 when q = 10, find p when q = 5.

21. If $(x^2 + y^2) / (m^2 + n^2) = xy/mn$, then find $(x + y) / (x - y)$.

22. If $12x^2 + 3y^2 - 13xy = 0$, then find the ratio x/y.

23. The numerator of a fraction is smaller than the denominator by 2. Find the fraction, if it becomes 10/9 times the original, when the numerator and the denominator both are increased by 1.

24. If the volume of a sphere is given by $V = (4/3)\pi r^3$ and the surface area by $S = 4\pi r^2$. Find the volume in terms of the surface area.

25. If the volume of a sphere is given by $V = (4/3)\pi r^3$ and the surface area by $S = 4\pi r^2$. How many liters of paint is required to paint the surface of a sphere of radius 4 m, if one needs 4 liters of the same paint to paint a similar sphere of radius twice as big?

26. If u/v = w/x = y/z, then show that $\{(2u^6v - 7w^5y + 15uw^2y^5)/(2v^7 - 7x^5z + 15vx^2z^3y^2)\}^{½} = uwy/vxz$.

27. Divide 372 into five parts in continued proportion so that the ratio of the sum of the second and fourth parts to the sum of the third and the fifth parts is 1:2.

28. The ratio of the measures of $\angle A$ and $\angle B$ of ΔABC is 2:5. The ratio of $\angle A$ and $\angle C$ is 1:5½. What are the measures of the angles of ΔABC?

29. If $(x^3 + y^3) / (x^3 - y^3) = 793/665$, determine the values of $\{(3x^2 - 4y^2)/ (4y^2)\}$ and $\{(2x - 3y)/ (2x + 3y)\}$.

30. The measures of the angles of a quadrilateral are in the ratio 1:2:5:4. What are the measures of the angles and what is the type of the quadrilateral?

31. Samar obtained an aggregate of 252 marks in a total of five different subjects. Each subject had a maximum of 100 marks and required a minimum of 40% for passing in that subject. Samar's marks in Hindi are twice those obtained in English. The ratio of marks obtained in English to those obtained in Science is 7:9, whereas, the ratio of the marks obtained in Science to those obtained in Sanskrit is 3:5. In Mathematics, the marks obtained by Samar are exactly equal to half the total marks obtained by him in the above three languages. In which subject did Samar fail and by how many marks?

32. The wavelength (λ) of sound and its frequency (η) are in inverse proportion to each other. The frequency is 400 Hz when the wavelength is 80 cm. What is the wave-length of sound when its frequency is 320 Hz.

33. The distance through which a heavy body falls from rest varies directly as the square of time for which it falls. In 4 seconds a body covers a distance of 33 metres more than the distance it covers in 3 seconds. Find the distance through which the body will fall in 7 seconds.

34. In a deflection magnetometer, the deflection 'd' of a given material varies directly as the applied load 'W', cube of its length 'L' and inversely as its moment of inertia 'I'. If the load is increased by 25% and the moment of inertia of the material is decreased to $^{16}/_{25}$ times the original, what should be the percent change in the length of the material so that deflection remains the same?

35. The value of a diamond is proportional to the square root of its weight. Find the gain by cutting a diamond worth Rs. 1000 into two pieces whose weights are in the ratio 9 : 16.

ANSWERS

1. 11	**2.** 5:13	**3.** 6
4. x/y	**5.** 27	**6.** 12, 8
7. 1 or 2	**8.** 25, 20	
9. 6 : 9 : : 10 : 15 or 6 : 10 : : 9 : 15	**10.** 63 cm	**11.** 256
12. $\frac{x+y+2xy}{2+x+y}$	**13.** $20\sqrt{5}$ m	**14.** 346.5 cm^2
15. 9 : 4	**16.** 5 seconds	**17.** 3 metres
18. 21, 28	**19.** 45 sq. m	**20.** 99
21. $\pm$ (m+n)/(m-n)	**22.** 3/4, 1/3.	**23.** 3/5 or –6/-4
24. $V = (1/3).(S^3/4\pi)^{1/2}$.	**25.** 1 litre.	**27.** 12, 24, 48, 96, 192
28. 20°, 50°, 110°.	**29.** 1/5, 179/64.	
30. Cyclic; 30°, 60°, 150°, 120°	**31.** English-12, Science-4.	**32.** 100 cm.
33. 231 m.	**34.** 20% decrease.	**35.** Rs. 400.

Concepts 3

Functions

Function is a relation between independent and dependent variable such that for one value of independent variable there is only one value of dependent variable.

For example: If x and y are the two variables, let x be the independent and y be the dependent variable, so it can be written as y = f(x), meaning thereby, y depends on x.
The significance of "f" is to show the relation between independent and dependent variable. One can also write "g","h" etc.in place of "f".
Now if $y = f(x) = x^2 + 1$, here "$x^2 + 1$" gives the relation.

Terms related to functions

Domain: Set of values which independent variable can take is known as domain.

Range: Set of values which dependent variable can have corresponding to the values of independent variable is known as Range of the function.

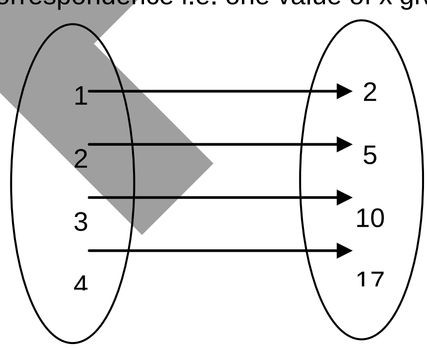
Types of correspondence: There are three types of correspondence

1. **One to one correspondence:** This means for one value of independent variable there is only one value of dependent variable.

For example: $y = f(x) = x^2 + 1$, here we know that y depends on x and let x can take four values, they are {1,2,3,4} and we have to find out the values of y, corresponding to the values of x.Now substitute the first value of x,which is 1 in the given function will give the first value of y which is equal to 2, ($y = 1^2 + 1 = 2$).Similarly we can find out other values of y and they are {2,5,10,17}

The pictorial representation of this is known as mapping. The values of y corresponding to x and the mapping for one to one correspondence is shown below.

The arrow shows the correspondence i.e. one value of x gives only one value of y.



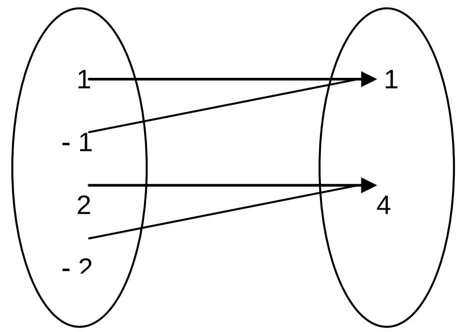
Values of independent variable , x   Corresponding values of dependant variable, y

2. **Many to one correspondence:** For many values of independent variable there is only one value of dependent variable

For example: $y = f(x) = x^2$, Let domain of x is{1,-1,2,-2},corresponding values of y i.e. range becomes {1,2}.

The mapping is shown in the next figure.

The arrow shows the correspondence i.e. many values of x give only one value of y.



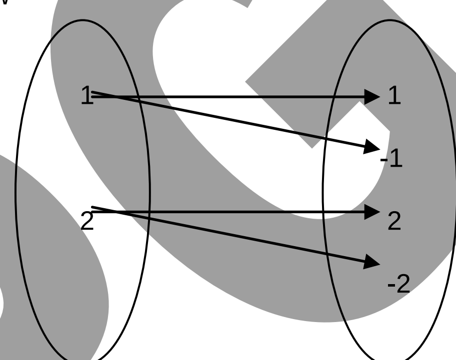
Values of independent variable, x

Corresponding values of dependant variable, y

3. **One to many correspondence:** It means, one value of x gives many values of y, since it is violation of the definition of function hence not a function.(Out side the scope)

For example: $y = f(x) = \pm \sqrt{x}$. Let x takes two values {1,4}, the corresponding values of y becomes {1, -1, 2, -2}. Here one can see that one value of x, say x = 1, gives many values of y , i.e. y = 1, or y = -1. Hence the relation is not a function.

The mapping is shown below



Values of independent variable, x

Corresponding values of dependant variable, y

Types of functions

1. Even functions: Functions which follow the rule f(-x) = f(x) are known as even functions.
   For example: $f(x) = x^2$; $f(-x) = (-x)^2 = x^2 = f(x)$

2. Odd functions: Functions which follow the rule f(-x) = -f(x) are known as odd functions.
   For example: f(x) = x; f(-x) = (-x) = -x = -f(x)

3. Neither even nor odd: There are some functions which neither follow f(-x) = f(x) nor f(-x) = -f(x) rule are known as neither even nor odd.
   For example: f(x) = x + a where a is a constant. $f(-x) = (-x) + a = -x + a \neq f(x) \neq -f(x)$

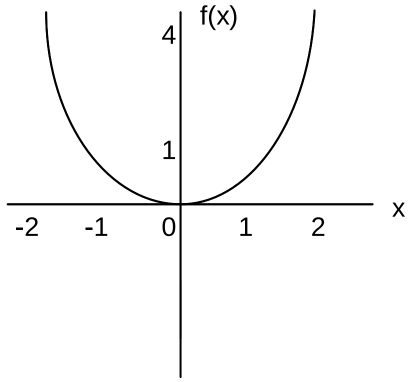
4. Composite functions: Composite functions are function of a function.
   For example: f(x) = x + 1; $g(x) = x^2 + 4$, two functions are given. Now $f_og(x)$ and $g_of(x)$ are known as composite functions can also be written as f(g(x)) and g(f(x)). Substitute g(x) in place of x in f(x) and the value of g(x) in place of x in the relation of f(x) x +1 we get $f(g(x)) = (x^2 + 4) +1$. Similarly $g(f(x)) = (x + 1)^2 + 4$

5. Constant functions: Constant functions are one which give a constant value for all the values of independent variable. f(x) = K where K is a constant.

6. Identity function: there is only one identity function i.e. $y = f(x) = x$, it's plot always passes through origin and the values of independent and dependent variables are always same.

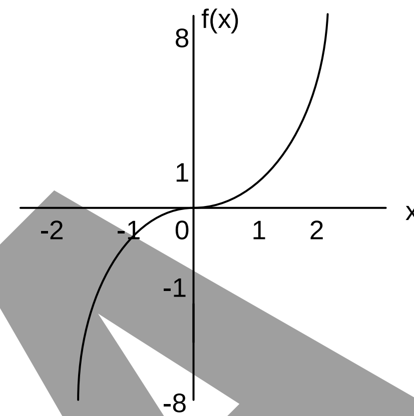
Types of curves

1. $f(x) = x^2$



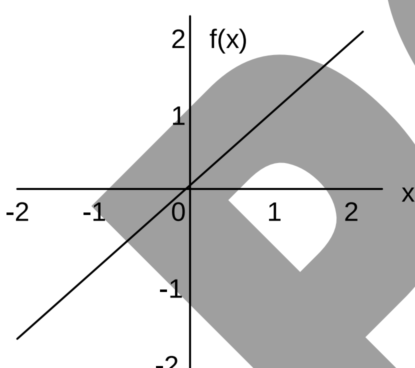
x	2	1	0	-1	-2
f(x)	4	1	0	1	4

2. $f(x) = x^3$



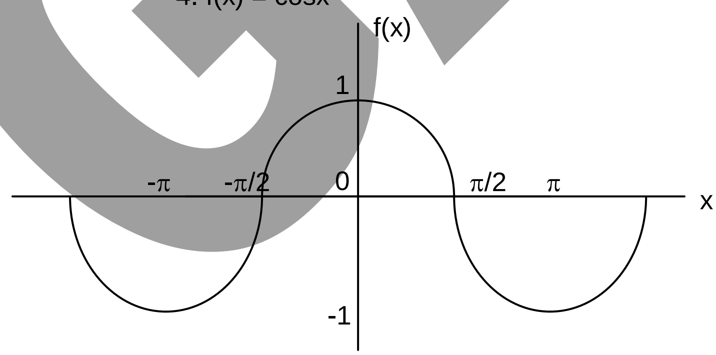
x	2	1	0	-1	-2
f(x)	8	1	0	-1	-8

3. $f(x) = x$



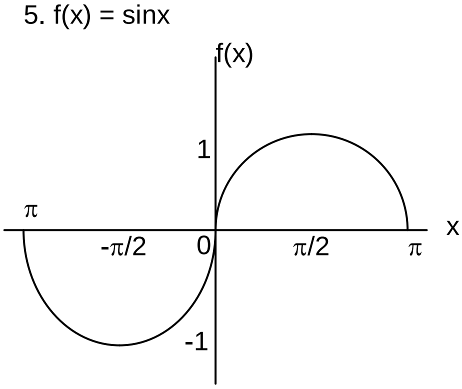
x	2	1	0	-1	-2
f(x)	2	1	0	-1	-2

4. $f(x) = \cos x$

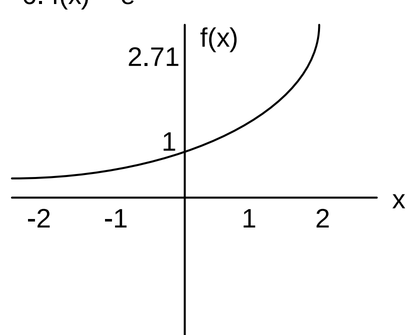


x	2π	π	0	-π	-2π
f(x)	-1	0	1	0	-1

5. $f(x) = \sin x$



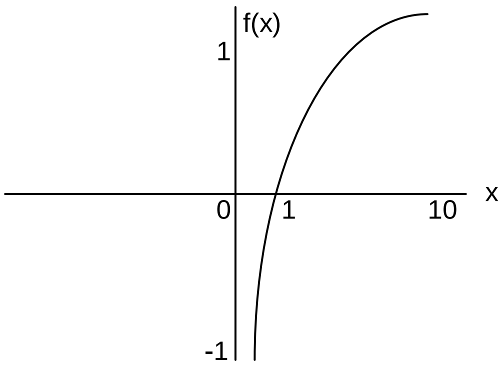
6. $f(x) = e^x$



X	π	$\pi/2$	0	$-\pi/2$	$-\pi$
f(x)	0	1	0	-1	0

x	-2	-1	0	1	2
f(x)	0.13	0.37	1	2.71	7.34

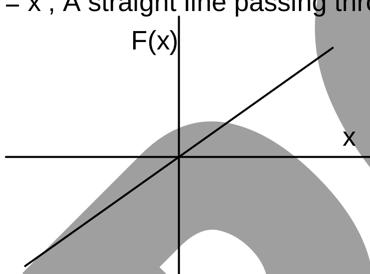
7. f(x) = log x



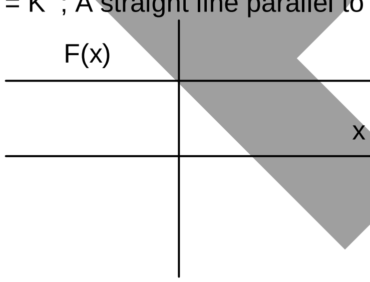
x	0.1	1	10
f(x)	-1	0	1

Some properties of straight lines

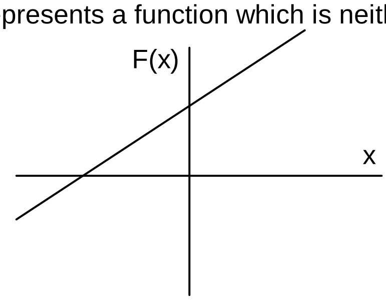
1. f(x) = x ; A straight line passing through origin always represents an odd function.



2. f(x) = K ; A straight line parallel to x axis, always represents an even function.

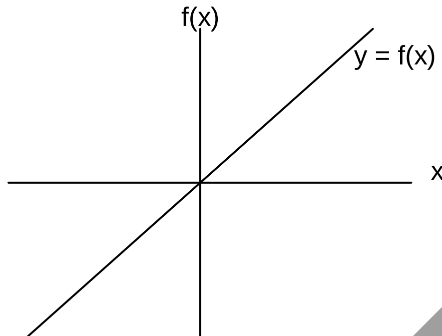


3. f(x) = mx + c ;A straight line neither parallel to x axis nor passing through origin, always represents a function which is neither even nor odd.

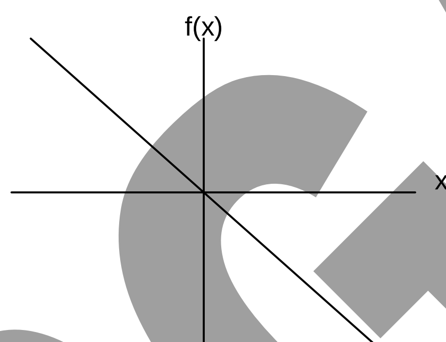


Reflection of curve about various axis

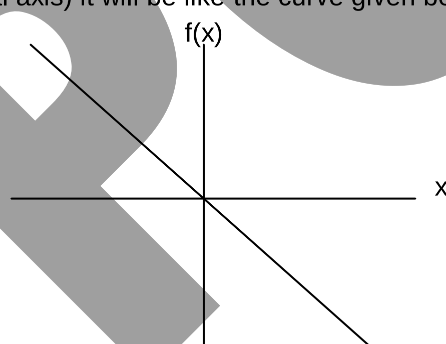
We will take an example to learn the reflection(rotation) of a curve.
Let y = f(x) = x and the curve is drawn below



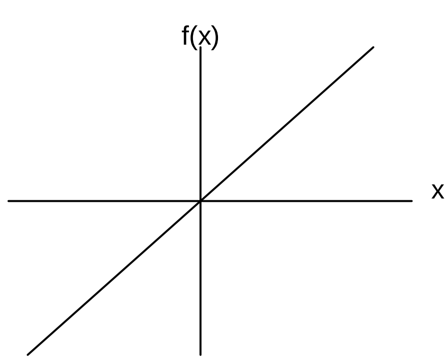
1. **Reflection of the curve about vertical axis:** Now if take the reflection of this curve about f(x) axis(i.e. vertical axis) it will be like the curve given below



2. **Reflection of the curve about horizontal axis:** Now if take the reflection of original curve about x axis(i.e. horizontal axis) it will be like the curve given below

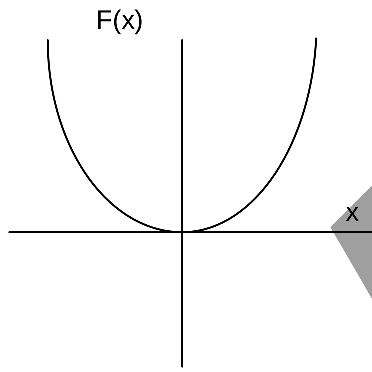


3. **Reflection of the curve about both the axis:** This can be done in both ways i.e. either the curve can be reflected first about vertical axis followed by the reflection of this about horizontal axis or vice-a-versa. The final figure is shown below

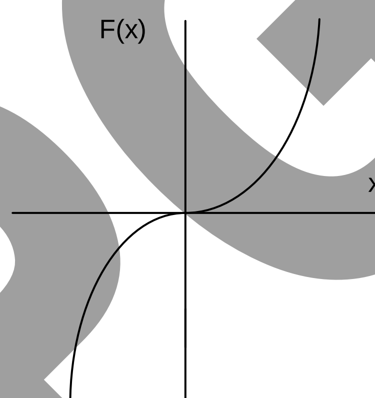


Method of reflection (Mirror image method): This method is used to know the type of function by rotating the curve about various axis.We assume that y is a function of x

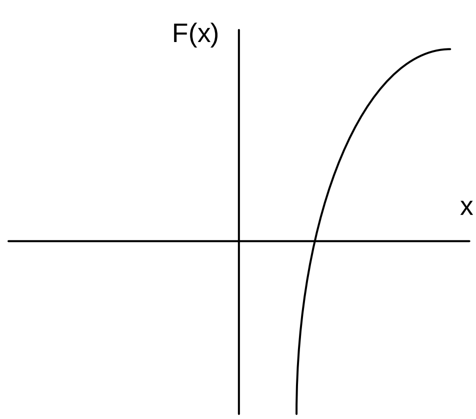
Method of reflection for even function: If the curve is symmetrical about vertical axis(i.e. f(x) axis), it represents an even function.In other words if we rotate the given curve about vertical axis and if it is same as the original one, the function (whose curve is given) is an even function.
For example: The curve given below is symmetrical about vertical axis and again we know that it is the curve of $y = f(x) = x^2$,which is an even function.



Method of reflection for odd function: If the curve is first rotated about one axis followed by the reflection of the reflected curve about the other axis gives the same curve as the original one, then the function(whose curve is given) is an odd function.
For example: The curve given below is of $y = f(x) = x^3$ which is an odd function and if the curve is rotated as per the method,it remains same as the original one.



Note: The curves that do not follow either of the two methods given above represent functions that are neither even nor odd.
Example: The given curve is of y = f(x) = log x and it is neither even nor odd



EXERCISE 3

1. If $f(x) = x^4 - 3x^3 + 6x^2 - 10x + 16$, find f(4)

2. If $f(t) = 2t^2 + 2/t^2 + 5/t + 5t$, what is f(1/t) ?

3. If $f(x, y) = 3x^2 - 2xy - y^2 + 4$, find f(1, -1)

4. If f(x) = (x + 3)/(4x - 5), and t = (3 + 5x)/(4x - 1); what is f(t) ?

5. f (x) = [x + (1/x)], g (x) = x +1. Find g { f [$(1/x)^2$]}.

6. If f (x) = x^x , find, in terms of f (x) , the value of $[f(-x)]^2$

7. If f (x) = 8 and g (x) = $y^2 + 1$, find $g_o f(x)$.

8. If f (x) = a $\log_{10} x$, g (x) = $\log_x 10$ where a is nonzero. find g (10/x) in terms of f (x).

9. If f (x,y) = $x^2 + y^2$, g (x,y) = x^2y^2, and h (x,y) = x - y, find a relation between the three functions.

10. If $f(x) = x^3 - x^2 - x + 1$ and $g(x) = x^2 - 2x + 1$, then which of the following is true ?
    a. f(x) = (x + 1) g(x) b. f(x) / g(x) = 1 c. $f(x) - g(x) = x^3 - 2x^2$ d. none of these

11. If $f(x) = \log_2 x^2$ and $g(x) = \log_x 4$, then find a relation between f and g.

12. What is f (2), if $f(\sqrt{x}) = x^3 + x^2 + x + 1$?

13. What is f (√3/2), if $f(x) = [(1 + x)^{1/2} + (1 - x)^{1/2}] / [(1 + x)^{1/2} - (1 - x)^{1/2}]$?

14. f (x) = 3x - 5 and f(g(x)) = 2x then g(x) = ?

15. A function is defined as y = f(x) = 2x + 3. Find the co – ordinates of a point such that x = f(y).

16. If f(a, b) = (ab), g(a) = $(a^3 + a)$ and a, b $\in$ R, what is the value of f{3, g(3)}?

17. A function is defined as $f(x) = 5/x^5 + 4/x^4 + 3/x^3 + 2/x^2 + 1/x + 1 + x + 2x^2 + 3x^3 + 4x^4 + 5x^5$. If f(2) = 260.78, what is the value of f(1/2)?

18. f (o, y + 2) = y + 5
    f (x + 5, y) = f (x, y-1) find the value of f (5, 6)

19. If the function is same as defined above, then how many steps will be required to have the solution for f (15, 6).

20. f (x, y) = x^{3n} when x > y
    f (x, y) = y^{3n} when y > x then f (4, 5) - f (4, 3) is always divisible by (given n is an even number)
    a. 61 b.189 c. both a and b d. none of these.

21. f (x, y) = $x^3 + y^3$, g (x, y) = $x^2 + y^2$, h (x, y) = xy what is the relationship in between the three functions?

22. f (x) = y = (3x + 4) / (5x + 3) then how is x = f (y) defined?

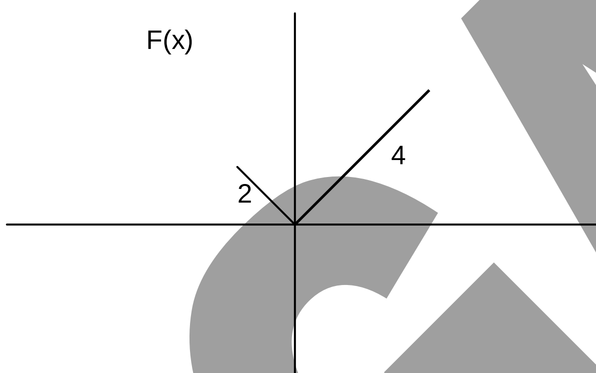
23. f (a, b) = $a^2 + b^3$, g (a, b) = a + b Then what is the value of f (3, g (3, 4))

24. $f(x, y) = 1 + 2 + 3 + \ldots\ldots + x$, if $x > y$
$= 1 + 2 + 3 + \ldots\ldots + y$, if $y > x$
Then find the value of $(f(0, 1) + f(2, 1) + f(2, 3) + f(4, 3) + \ldots\ldots\ldots\ldots f(10, 9)$

25. $f(x) = x / (x+1)$
$f^{n}(x) = (n-1) f^{n-1}(x)$ $n > 1$
find the value of $f^{4}(1)$.

26. If f(x) is a polynomial function of the second degree such that f(-3)=6,f(0)=6 & f(2)=11,then the graph of the function f(x) cuts at the ordinate x=1 at which point ?

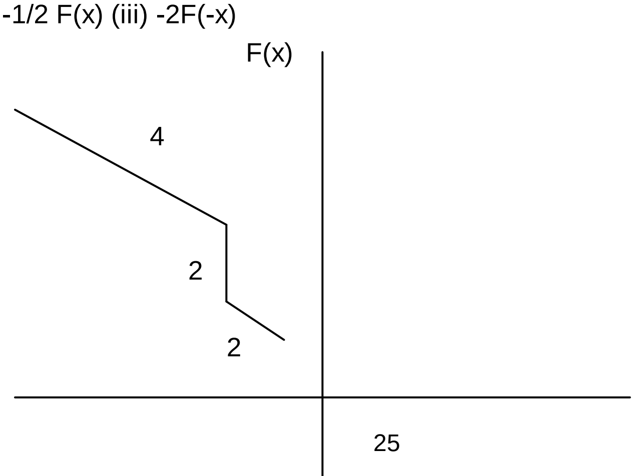
27. If following three sets of instructions are applied to the given graph, then what will be the answer Graphs? And what will be the relation among all these graphs?
(a) (i) F(-x) (ii) -F(-x) (iii) -F(x)
(b)-1/2F(-x) (ii) F(-x) (iii) -2F(x)
(c) (i)F(-x) (ii) -3F(x)(iii) -1/3F(-x)



28. Which of the following set of instructions we should give to change Graph F(x) into F1(x)?

(a) (i) -1/3 F(x) (ii) F(-x) (iii) -F(x)
(b) (i) -F(x) (ii) F(-x) (iii) -1/3F(x)
(c) (i) F(x) (ii) -1/3 F(-x) (iii) -F(x)

29. If the following instructions are applied to graph of F(x), then what will be the answer figure?
(i) F(-x) (ii) -1/2 F(x) (iii) -2F(-x)



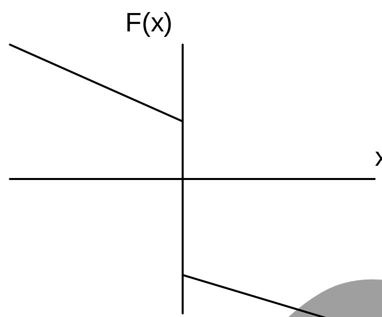
30. If $f(x) = 1/(1-x)$, $g(x)= f[f(x)]$ & $h(x) = f[f\{f(x)\}]$; $a=f(x)+g(x)+h(x)$ & $b = f(x)g(x)h(x)$ then what will be a & b?

31. Function $y = \log_a(x + \sqrt{(x^2 +1)}\ )$, is odd or even? What will be the property of graph of this function?

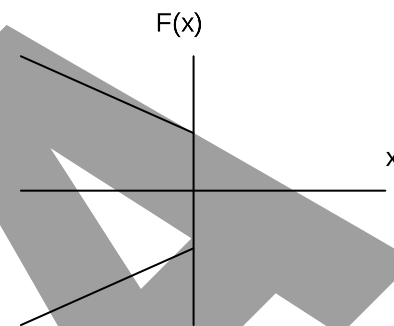
32. Find Domain and Range of following functions?
(i) $f(x) = 1/\sqrt{(x-|x|)}$ (ii) $f(x) = (x+1)/(2x – 1)$

33. Assuming y to be a function of x, state whether the functions plotted below are odd or even.

(a)



(b)



34. Find out whether the function is odd or even.
(i) $y = 2^{-x^2}$ (ii) $y = 2^{x-x^4}$

35. If $f(x) = x/(x – 1)$, then for which real values of x, $f(1/x) = f(1-x)$?

36. If $f(x, y) = \sum_{n=1}^{m} n\sqrt{x}$ and $g(x,y) = \sum_{n=1}^{m} n\sqrt{y}$;
Where n is a natural number, and m is square root of highest four digit perfect square.
Find out $f(x, y) + g(x, y)$ and $f(x, y)\ g(x,y)$.

37. If $f (x,y) = X^{\sum_{n=2}^{\infty} 1/n}$ and $g (x,y) = X^{\sum_{m=3}^{\infty} 1/m}$
Where n and m are such multiples of ‘2’& ‘3’ respectively, which are obtained when their powers are consecutive natural numbers. Then find out the value of $f(x,y).g(x,y)/2$.

38. Ordinate of a line parallel to $x+y = 4$, is 9.Find the area enclosed by two parallel lines and two co-ordinates axis.

39. US is planning to conduct a missile defense test. The path of the missile, which has to be targeted, is a second-degree polynomial function of time, in which constant term is negligible. The path of the Interceptor, which is supposed to hit the missile, is also given by the same polynomial, with constant term having considerable value. Will this test be successful?

40. If $F(x,y,z) =|x| + |y| +|z|$,for all real values of x and $G(x,y,z) = |x+y+z|$, for all real vales of x, then what will be the relation between these two functions?

ANSWERS

1. 136 **2.** f(t) **3.** 8 **4.** x **5.** $[f(x)]^2 - 1$

6. $1/[f(x)]^2$ **7.** $y^2 + 1$ **8.** a / [a - f(x)] **9.** $[h(x,y)]^2 = f(x,y) - 2\sqrt{g(x,y)}$

10. (a) **11.** f(x) = 4/g(x) **12.** 85 **13.** $\sqrt{3}$ **14.** (2x + 5) / 3

15. (-3, -3). **16.** 90. **17.** 260.78. **18.** 8 **19.** 4 steps

20. c **21.** $f(x, y) = \{g(x, y) + 2h(x, y)\}^{1/2}\{g(x, y) - h(x, y)\}$

22. x = (3y - 4) / (3 – 5y) **23.** 352 **24.** 220 **25.** 3

26. (1,8) **28.** not possible **30.** a = (2-x)/(1-x) , b = -1 **31.** odd

32.(i) not defined, (ii) $D_f = R - \{1/2\}$, $R_f = R - \{1/2\}$ **33.**(a) odd, (b) not a function

34. (i) even, (ii) NENO **35.**no real value **36.** $4950(\sqrt{x} + \sqrt{y})$, $24502500 \sqrt{x}\sqrt{y}$

37. $X^{3/2}/2$ **38.** 32.5 **39.** Not successful **40.** $F \geq G$

Concepts 4

Inequalities

A quantity a is greater than another quantity b if a - b is positive. Also b less than a if b - a is negative. Zero is regarded as greater than any negative quantity and less than any positive quantity.

Elementary properties of inequalities:

(i) For any two real numbers a and b, we have
$a > b$ or $a = b$ or $a < b$.

(ii) If $a > b$ and $b > c$, then $a > c$.

(iii) If $a > b$, then $a + m > b + m$, for any real number m.

(iv) If $a > b$, then $am > bm$ for $m > 0$ and $am < bm$ for $m < 0$, that is, when we multiply both sides of the inequality by a negative quantity, the sign of the inequality reverses.

(v) If $a \neq 0$ and $b \neq 0$ and $a > b$, then
$(1/a) < (1/b)$.

(vi) If $a_1 > b_1$, $a_2 > b_2$, ..., $a_n > b_n$, then
$a_1 + a_2 + ... + a_n > b_1 + b_2 + ... + b_n$
and $a_1 . a_2 . a_3 .... a_n > b_1 . b_2 . b_3 .... b_n$.
($a_i \geq 0$ and $b_i \geq 0$, i = 1,2, 3, ...n)

(vii) If a, b, c are positive and not all equal, then $(a + b + c)(ab + bc + ca) > 9abc$ and,
$(b + c)(c + a)(a + b) > 8abc$

(viii) $(a + c + e + \ldots\ldots)/(b + d + f + \ldots\ldots)$ is greater than the least and less than the greatest of the fractions a/b, c/d, e/f, ……

(ix) If $a > x$, $b > y$, $c > z$, …… then $a + b + c + \ldots\ldots > x + y + z + \ldots\ldots$ and $abc\ldots\ldots > xyz\ldots\ldots$ abc and xyz should be all non negative numbers.

(x) $a^2 + b^2 + c^2 \geq ab + bc + ca$

(xi) $(n!)^2 \geq n^n$

(xii) For any positive integer n , $2 \leq (1 + 1/n)^n \leq 3$

(xiii) $a^2b + b^2c + c^2a \geq 3abc$

(xiv) $a/b + b/c + c/d + d/a > 4$

(xv) $a^4 + b^4 + c^4 + d^4 \geq 4abcd$

(xvi) When a+b+c+… = Constant then $a^m b^n c^p$…… will be greatest if a/m = b/n = c/p = ……

Some Important Properties:

(i) If $x > 0$ and $a > b > 0$; then $a^x > b^x$,

(ii) If $a > 1$ and $x > y > 0$; then $a^x > a^y$,

(iii) If $0 < a < 1$ and $x > y > 0$; then $a^x < a^y$,

(iv) If $a > 1$ and $x > y$, then $\log_a x > \log_a y$.

(v) If $0 < a < 1$ and $x > y$, then $\log_a x < \log_a y$.

Some theorems on inequalities:

1. Since for all real a, $a^2 \geq 0$. $\therefore$ $(\sqrt{a} - \sqrt{b})^2 \geq 0 \Rightarrow a + b - 2\sqrt{(ab)} \geq 0$.
   $\therefore$ $(a + b)/2 \geq \sqrt{(ab)}$
   $\therefore$ The **A**rithmetic **M**ean of two positive quantities is greater than or equal to their **G**eometric **M**ean. Similarly $\sqrt{(ab)} \geq [2ab / (a+b)]$. Hence the **G**eometric **M**ean $\geq$ **H**armonic **M**ean

2. If $a_i > 0$, i = 1, 2, 3, ...n, then
   $(a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_n) / n \geq (a_1 . a_2 . a_3 . \ldots\ldots\ldots . a_n)^{1/n}$
   that is, the geometric mean of n positive quantities cannot exceed their arithmetic mean.

3. If the sum of two positive quantities is constant, then their product is greatest when they are equal; and if their product is constant, then their sum is least when they are equal.

4. If $a_i > 0$ (i = 1, 2, ... n) and $a_1 + a_2 + \ldots\ldots\ldots a_n$ = constant, then the product $a_1.a_2.....a_n$ is greatest when **$a_1 = a_2 = a_3 = \ldots\ldots\ldots = a_n$.**

5. If $a_i \geq 0$, i = 1, 2,........n, then

   (i) $(a_1^m + a_2^m + \ldots\ldots + a_n^m) / n \geq (a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_n / n)^m$
   if $m \geq 1$ or m is any negative quantity.
   (ii) $(a_1^m + a_2^m + \ldots\ldots + a_n^m) / n \leq (a_1 + a_2 + a_3 + \ldots\ldots\ldots + a_n / n)^m$
   if $0 < m < 1$.

   In Particular For a, b $\geq$ 0
   $\mathbf{(a^m + b^m) / 2 \geq [(a + b) / 2]^m}$; $m \leq 0$ OR $m \geq 1$

   $\mathbf{(a^m + b^m) / 2 \leq [(a + b) / 2]^m}$; $0 < m < 1$

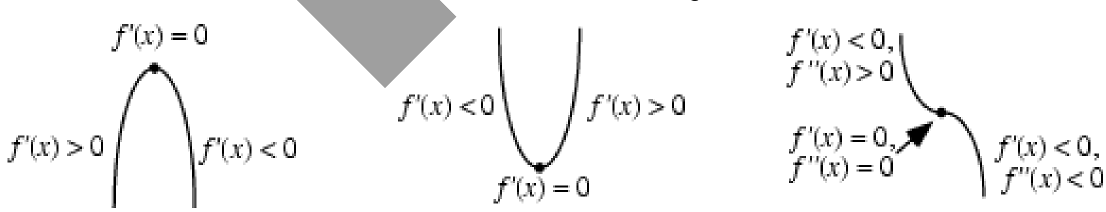
   That is, arithmetic mean of the m^{th} powers of n positive quantities is greater than the m^{th} power of their arithmetic mean in all cases except when $0 < m < 1$.

6. The product of the factorials of two numbers whose sum is constant is least when they are equal or consecutive according as their sum is even or odd.

Maxima & Minima

A continuous function may assume a maximum at a single point or may have maxima at a number of points. An absolute maximum of a function is the largest value in the entire range of the function, and a local maximum is the largest value in some local neighborhood.

A continuous function may assume a minimum at a single point or may have minima at a number of points. An abslolute minimum of a function is the smallest value in the entire range of the function, while a local minimum is the smallest value in some local neighborhood



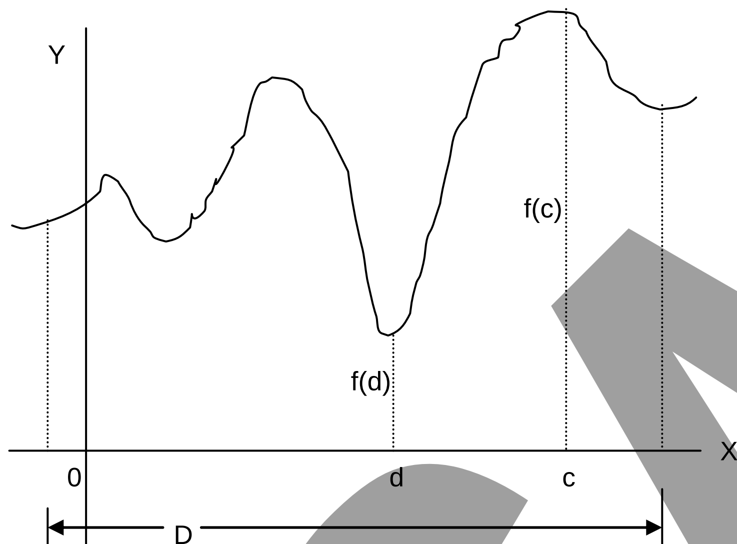
maximum *minimum* *stationary point*

Absolute Maxima and Absolute Minima

Let f be a function defined on a set D, then

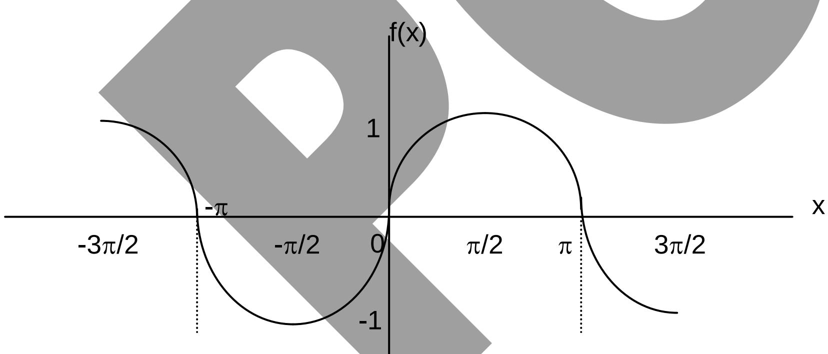
(1) f is said to be absolutely maximum at $c \in D$ if and only if $f(x) \leq f(c)$ for all $x \in D$, and c is called point of absolute maximum and f(c) is called absolute maximum value of f on D.
The absolute maximum value whenever it exists is unique. There can also be more than one points of absolute maximum.

(2) f is said to be absolutely minimum at $d \in D$ if and only if $f(x) \geq f(d)$ for all $x \in D$, and d is called point of absolute minimum and f(d) is called absolute minimum value of f on D.
The absolute minimum value whenever it exists is unique. There can also be more than one points of absolute minimum.



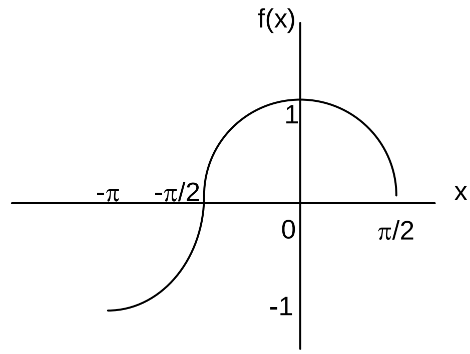
Examples

(1) Consider the function $f(x) = \sin x$, $x \in [-3\pi/2, 3\pi/2]$.its graph is shown below. The maximum (absolute) value of graph is 1 and the minimum (absolute) value is -1.



Further, we note that $\pi/2$ and $-3\pi/2$ are the points of absolute maximum as $f(\pi/2) = \sin(\pi/2) = 1$ and $f(-3\pi/2) = \sin(-3\pi/2) = -\sin(2\pi - \pi/2) = \sin(\pi/2) = 1$.
Also $-\pi/2$ and $3\pi/2$ are the points of absolute minimum as $f(-\pi/2) = \sin(-\pi/2) = -\sin(\pi/2) = -1$ and $f(3\pi/2) = \sin 3\pi/2 = \sin(2\pi - \pi/2) = -\sin(\pi/2) = -1$.

(2) Consider the function $f(x) = \cos x$, $x \in [-\pi, \pi/2]$.Its graph is shown below. The maximum (absolute) value of f(x) is 1 and the minimum (absolute) value is -1.



Further we note that 0 is the only point of absolute maximum as f(0) = 1 and $-\pi$ is the only point of absolute minimum as $f(-\pi) = \cos(-\pi) = \cos(\pi) = -1$.

Working rule for finding absolute maximum and minimum

If f is a differentiable function in [a,b] except at finitely many points,then to find absolute maximum and minimum adopt the following procedure:
(1) Find f'(x) and locate the points in (a,b) at which f'(x) fails to exist.
(2) Find the values of x in (a,b) for which f'(x) = 0.
(3) Evaluate f(x) at the points obtained in steps(1) and (2) and also find f(a) and f(b).
(4) Maximum of all the values obtained in the (3) is the absolute maximum and the the minimum of these is the absolute minimum of f(x) in [a,b].

Note: Thus, we find that if a function f(x) is derivable in its domain (except) at finitely many points then absolute maximum (or minimum) can occur only at the following points:
(1) the points at which f'(x) does not exist.
(2) the points at which f'(x) = 0.
(3) the end points a and b if the function is defined on the closed interval [a,b].

Example: Find the maximum and minimum values of the function f(x) = 3 - 2sinx
Solution: Given f(x) = 3 - 2sinx.
Clearly $D_f = R$. We know that $-1 \le \sin x \le 1$ for all $x \in R$
$\Leftrightarrow \quad 2 \ge -2\sin x \ge -2$ (Multiplying by -2)
$\Leftrightarrow \quad 5 \ge 3 - 2\sin x \ge 1$ …..for all $x \in R$
$\Leftrightarrow \quad 5 \ge f(x) \ge 1$ for all $x \in R$
$\therefore$ Minimum value of f(x) = 1 and Maximum value of f(x) is 5.
Note that $1 = f(\pi/2)$ and $5 = f(-\pi/2)$

Local Maxima and Local Minima

Let f be a function defined on a set D. Suppose there is an open interval I in the domain of f, then,
(1) f is said to be maximum at $c \in I$ if and only if $f(x) \le f(c)$ for all $x \in I$, and c is called the point of local maximum and f(c) is called local maximum value of f on D.

(2) f is said to be minimum at $d \in I$ if and only if $f(x) \ge f(d)$ for all $x \in I$, and d is called the point of local minimum and f(d) is called local minimum value of f on D.

Examples

a and b are real numbers and a < b.

Let a = -3 and b = 5. Then we can talk about the interval (3,5). This means all of the numbers between 3 and 5. If we were just talking about integers, we could write the set {3,4,5}, but there are an infinite number of real numbers betweeen 3 and 5, so they wouldn't all fit in the set brackets, so we abbreviate with interval notation. We can use interval notation because any interval on the real number line is almost completely characterized by its endpoints. This is expressed in interval notation by a bracket or a parenthesis.

Here are some examples:

(-3,5) is the set of all numbers greater than -3 and less than 5.

(-3,5] is the set of all numbers greater than -3 and less than or equal to 5

[-3,5] is the set of all numbers greater than or equal to -3 and less than or equal to 5.

We call an interval of the form (a,b) open; [a,b] closed; [a,b) or (a,b] half-open or half-closed.

Ex.1 $f(x) = 2x$

We say that the natural domain of f is (-infinity, infinity) because f will know what to do to any number that we could possibly think of. If we restrict the domain to (-3,5), then the range will be (-6,10) because if we allow f to operate on any number between -3 and 5, and we choose a number between -6 and 10, there will be a number (namely half of the number between 6 and 10) that is in (-3,5). But the natural domain of f is really (-infinity, infinity) because there is no number that we cannot multiply by 2.

Suppose that c is in the domain of the function $f = 2x$ and there is an open interval I = (-3, 5) containing c which is contained in the domain of $f = 2x$ such that, for all x in I, $f(x) \le f(c)$, then c is called a point of local maximum of $f = 2x$.

Suppose that d is in the domain of the function $f = 2x$ and there is an open interval I = (-3, 5) containing d which is contained in the domain of $f = 2x$ such that, for all x in I, $f(x) \ge f(d)$, then d is called a point of local minimum of $f = 2x$.

Ex.2 $f(x) = x^2$

The natural domain of f is (-infinity, infinity) because f will know what to do to any number that we could possibly think of. If we restrict the domain to (-3,5), then the range will be (0,25) because no square is ever negative.

We see here that though the absolute maximum value of the function is ∞, the local maximum is never greater than 25.

In this case, absolute minimum value of the function is 0, the local minimum value is also 0.

Working rule to find out local maxima and minima

Rule to find out minima:
A and B are the variables.
If $A*B$ = constant,
Then $A + B$ = minimum
When $A = B$.

Rule to find out maxima:
A and B are the variables.
If $A + B$ = constant,
Then $A*B$ = maximum
When $A = B$.

Again a, b, c are the variables
let $x = a^p b^q c^r$.
x is maximum
when $a/p = b/q = c/r$.

Example: Find the maximum volume of a right circular cone if sum of its height and radius is 9.
Solution: Let h be the height and r be the radius of the given cylinder
Given $h + r = 9$ ……. which is a constant
and volume of cone is $= \pi r^2 h/ 3$
the volume is maximum when $r/2 = h/1$
we get $h + 2h = 3h = 9$
$h = 3$ and $r = 6$
Now on substituting these values in the formula will give maxima
which is $= \pi\ 6^2 * 3 / 3 = 36\pi$.

Ex. Solve $2x + 4 < 20/3$
Sol. $2x + 4 < 20/3 \therefore 2x < 8/3 \therefore x < 4/3$

Ex. Find the minimum value of $x^2 - 4x + 7$ for real values of x
Sol. $x^2 - 4x + 7 = (x - 2)^2 + 3$
A perfect square is always positive i.e. it cannot be less than zero. $\therefore$ the given expression is the least when $(x - 2)^2 = 0$. $\therefore$ minimum value = 3

Ex. Find the value of x which satisfies $x^2 - 3x + 2 > 0$ and $x^2 - 3x - 4 \leq 0$
Sol. $x^2 - 3x + 2 > 0 \therefore (x - 2)(x - 1) > 0 \therefore x > 2, x > 1$ or $x < 2, x < 1$
$x^2 - 3x - 4 \leq 0 \therefore (x - 4)(x + 1) \leq 0 \therefore x \leq 4$, $x \geq -1$ or $x \geq 4$, $x \leq -1$, which is not possible
$\therefore$ from (I) we have $x > 2$ or $x < 1$, From (II) we have $-1 \leq x \leq 4$
$\therefore$ the required values are $-1 \leq x < 1$ & $2 < x \leq 4$

Ex. Find the least value of $3x + 4y$ if $x^2 y^3 = 6$
Sol. Let $A = 3x/2$, $B = 4y/3$
$\therefore x^2y^3 = (4A^2/9)(27B^3/64) = 6 \therefore A^2B^3 = 32$
$3x + 4y = 2A + 3B = A + A + B + B + B$
Given the product the sum will be the least when the quantities are equal.
i.e. $A = B \therefore A^5 = 32 \therefore A = B = 2$
$\therefore$ least value of $3x + 4y = 2A + 3B = 4 + 6 = 10$

Maxima and Minima by differentiation

Working Rule to find the Maxima / Minima
Note that the Maxima & Minima found with this method are not absolute Maxima & Minima, but these are the **Local** Maxima and Minima. The WORKING RULE to find Maxima/Minima for the function f(x) is as follows :

1. Find the derivative of the given function, i.e. Find $f'(x)$
2. Solve the equation $f'(x) = 0$. Let a, b, c, . . . be the solutions of this equation.
3. Find $f''(x)$ and if --
   (a) $f''(a) < 0$ ------ then f(x) is Maximum at x = a ;
   (b) $f''(a) > 0$ ------ then f(x) is Minimum at x = a ;
   (c) $f''(a) = 0$ ------ then nothing can be said, and further investigation is required.

If $f'(a) = f''(a) = f'''(a) = 0$ and $f^{(iv)} \neq 0$, then --
   (a) $f^{(iv)} < 0 \Rightarrow$ f(x) is Maximum at x = a ;
   (b) $f^{(iv)} > 0 \Rightarrow$ f(x) is Minimum at x = a ;

Ex. find the maximum and the minimum values of $f(x) = x^3 - 9x^2 + 15x + 3$
Sol. We have, $f'(x) = 3x^2 - 18x + 15 = 3.(x - 5)(x - 1)$
$\therefore f'(x) = 0$, for x = 5, or x = 1.
Also, $f''(x) = 6x - 18$.
$\therefore f''(5) = 30 - 18 = 12 > 0$ so, $f(5) = -22$.
$\therefore f''(1) = 6 - 18 = -12 < 0$ so, $f(1) = 10$.
$\therefore$ the minimum value is at x = 5 and the maximum value is at x = 1.

Some Important Results:
(1) For all rectangle of given area, the square has the smallest perimeter.
(2) The rectangle with maximum area that can be inscribed in a circle is square.
(3) An open cylinder of given surface area will have greatest volume when its height is equal to radius of its base.
(4) A right circular cone of given volume will have least surface when its height is equal to $\sqrt{2}$ times the radius of its base.
(5) The least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6r\sqrt{3}$.
(6) The cone of maximum volume and of given slant height has semi-vertical angle of $\tan^{-1}\sqrt{2}$.
(7) A right circular cone of given total surface area will have maximum volume when the semi-vertical angle is $\sin^{-1}\sqrt{2}$.

EXERCISE 4

1. For what values of x is $4x - 5 > x + 7$?

2. If x is positive what is the greatest value of $(5 - x)(x + 3)$?

3. Between what values of x is the expression $19x - 2x^2 - 35$ positive ?

4. Find the values of x which satisfy $2(x^2 - 1) > 3x$

5. Find all values of x which satisfy $x^2 + 6x - 27 > 0$ and $4 + 3x - x^2 > 0$

6. Solve the inequality $2x^2 + 9x - 35 > 0$.

7. Solve the inequality $3x^2 - 20x + 17 \leq 0$.

8. For all $x > 0$, prove that $x + 1/x \geq 2$.

9. Solve the inequality $| x/2 | < 2$.

10. Find the minimum value of $x + 1/x$, if $x > 0$.

11. Find the maximum volume of a right circular cylinder if the sum of the radius and the height is 6 m.

12. Find two numbers such that their sum is 12, and the sum of their squares is minimum.

13. A variable rectangle has given perimeter. When is its area maximum ?

14. Find the maximum value of $x^3 - 6x^2 + 9x + 2$

15. Find the minimum value of $x^2 - 12x + 27$.

16. The solution set of the inequality $x^2 - x - 240 < 0$, is

17. Find the value of the n^{th} term of the sequence $\sqrt{6}$, $\sqrt{(6 + \sqrt{6})}$, $\sqrt{[6 + \sqrt{(6 + \sqrt{6})}]}$, ---, as n tends to infinity.

18. Which of the following numbers is greater?
    a. $\{(1728)^2 (392) + (392)^2 (532) + (532)^2 (1728)\}$ and $3(1728) (392) (532)$.
    b. $(10^4 + 15^4 + 27^4 + 53^4)$ and $(4 \times 10 \times 15 \times 27 \times 53)$.
    c. $(1 \times 2 \times 3 \times \text{----} \times 20)^2$ and 20^{20}.
    d. 1000^{1000} and 1001^{999}.

19. What is the maximum value of $(7 – x)^5 (7 +x)^4$?

20. X, Y and Z are distinct integers such that X<Y<Z and $X^2 + Y^2 + Z^2 = K$. What is the smallest value of K that determines X, Y and Z uniquely?

21. Let $y = \min \{(x + 6), (4 – x)\}$. If $x \in R$, what is the maximum value of y?

22. What is the value of x if $x < 383$, $(x +y) > 326$ and $(x – y) > 436$?. Where x is an integer.

23. Find the maximum value of x^2y^3 if $x + y = 25$.

24. Find the minimum value of $3x + 4y$ if $xy = 12$.

25. The production cost per unit varies with the number of units per hour x, in the following manner:
Cost per unit = x + (100/x). Z is the profit per unit and Y is the number of units produced which is related as 4Z + 5Y = 120 – 3 Cost per unit. Find the maximum possible profit.

26. A piece of wire of length 50 cm is cut into two pieces. One piece is bent into a circular ring and the other into an equilateral triangle. What is the side of the equilateral triangle so that the combined area of the two is minimum?

27. The perimeter of the sector of a circle is 40 cm. What is the maximum area of the sector, which subtends θ radians at the center?

28. Four walls in rhombus shape enclose a scrap depot. Sum of its diagonals is 40 ft. Find its maximum area?

29. For a trapezium, the sum of parallel sides and the height is 60. And one of the parallel side is double of the other. Find the maximum area?

30. The minimum value of the function $f(x) = 2\sqrt{x} + 1/x$

31. The minimum value of the function $f(x) = e^x + e^{-x} - x^2$

32. The perimeter of triangle is 28 cm. one of the sides is 6 cm. Find the maximum area?

33. A magazine company in Pune has 600 subscribers and makes a profit of Rs. 400 per subscriber. As the number of subscribers decreases by one its profit per subscriber increases by a rupee. Find what number of subscribers will give the maximum profit?

34. Find the maximum and minimum value of the function $f(x) = \sin^6 x + \cos^6 x$

35. Find the values for x at which maximum and minimum value of the function occurs $f(x) = x^5 - 5x^4 + 5x^3 - 1$

36. Find the maximum value of the function (a) $f(x) = \sin x + \cos x$ (b) $f(x) = \sin x \cos x$

37. Show that a cylinder will have maximum volume of given surface area when height is equal to its diameter?

38. Show that the radius of right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.

39. A plane of an enemy is flying along the curve $y = x^2 + 2$. A soldier is placed at the point (3,2). What is the nearest distance between the soldier and the plane?

40. Find the maximum and minimum value of the function $f(x) = \sin^4 x + \cos^4 x$

ANSWERS

1. $x > 4$	**2**. 16	**3**. $2.5 < x < 7$	**4**. $x>2$ or $x<-1/2$
5. $3 < x < 4$	**6**. $x > 5/2$, $x < -7$	**7**. $1 \leq x \leq 17/3$	
9. $- 4 < x < 4$.	**10**. 2	**11**. 32π	**12**. 6 and 6
13. When the rectangle is a square		**14**. 6	**15**. -9
16. $-15 < x < 16$	**17.** 3	**18.** 1^{st}, 1^{st}, 1^{st}, 1000^{1000}.	**19.** $7^9 \times 2^{17} \times 5^5 / 3^{18}$.
20. 2	**21**. 5	**22.**382	**23.** 337500
24. 24	**25.** 45	**26.** $(50\sqrt{3}) / (\prod + 3\sqrt{3})$	**27.** 100
28. $200ft^2$	**29.** 450	**30.** 3	**31.** 2
32. $12\sqrt{7}$ cm^2	**33.** 500	**34.** 1, 1/4	**35.** 1,3
36.1/2	**37.**Height = Diameter	**38.** $h_{cylr} = r/2$	**39.** $\sqrt{5}$
40. 1 and 1/2			

Concepts 5

Logarithms

Before getting started with logarithms, it is absolutely essential to have a fair idea about the basic laws of indices, which are as follows:

$a^m \times a^n = a^{m+n}$ $\qquad$ $a^n b^n = (ab)^n$

$a^m/a^n = a^{m-n}$ $\qquad$ $a^n/b^n = (a/b)^n$

$(a^m)^n = a^{mn}$ $\qquad$ $a^{-n} = 1/a^n = (1/a)^n$

$\sqrt{a} = a^{(1/2)}$ $\qquad$ $\sqrt[n]{a} = a^{(1/n)}$

$a^1 = a$ $\qquad$ $a^0 = 1$, if $a \neq 0$.

We know that $8 = 2^3$ and $16 = 2^4$. If we call 2 the ***base***, then we must raise the base t.o power 3 to obtain 8 and to power 4 to obtain 16. A logarithm is defined in following way.

$3 = \log_2 8$ $\qquad$ $4 = \log_2 16$

The logarithm of any number to given base is the index of the power to which the base must be raised in order to equal the given number. Thus if $a^x = N$, x is called the logarithm of N to base a. Mathematically,

$a^x = N$ $\qquad$ $x = \log_a N$

Two common bases that are used are:

1. $e \approx 2.718$ Natural or Napierian logs
2. 10 logs to base 10

When base e is used, an alternative notation is often used. $\log_e x = \ln x$.
For base 10, the base is often left out log x means $\log_{10} x$

MANIPULATING LOGARITHMS

A $\log_n AB = \log_n A + \log_n B$ $\qquad$ A $\log_n(A/B) = \log_n A - \log_n B$

A $\log_n A^m = m \log_n A$ $\qquad$ A $\log_n A = [\log_m A]/[\log_m n]$

A log of 1 to any base x, $(x > 0)$ is zero. $\log_x 1 = 0$; As $x^0 = 1$.

A log of any number to same base is one. $\log_n n = 1$; As $n^1 = n$

A log of zero or a negative number to any base is not defined.

Ex. Find logarithm of 125 to base $5\sqrt{5}$.

Sol. Let x be the logarithm.

$$\text{Then, } (5\sqrt{5})^x = 125$$
$$= 25 \times 5$$
$$= 5^2 \times (\sqrt{5})^2$$
$$= (5\sqrt{5})^2$$
$$\therefore x = 2$$

Ex. Find x from the equation: $a^{3x} b^{-2x} = c^{3-2x}$

Sol. Taking logarithms on both the sides,

$3x \times (\log a) - 2x \times (\log b) = (3 - 2x) \times (\log c)$

$3x \times (\log a) - 2x \times (\log b) = 3 \times (\log c) - 2x \times (\log c)$

$3x \times (\log a) - 2x \times (\log b) + 2x \times (\log c) = 3(\log c)$

$x(3 \log a + 2 \log c - 2 \log b) = 3 \log c$

$$\therefore \ x = \frac{3 \log c}{3 \log a - 2 \log b + 2 \log c}$$

Ex. Express the following as a log of single number.

$2 \log x - 4 \log y + 0.5 \log z$

Sol. $2 \log x - 4 \log y + 0.5 \log z = \log x^2 - \log y^4 + \log \sqrt{z}$

$= \log (x^2/y^4) + \log \sqrt{z}$

$= \log x^2 \sqrt{z} / y^4$

Ex. Solve $(x^4 - 2x^2y^2 + y^4)^{m-1} = (x - y)^{2m} (x + y)^{-2}$

Sol. Taking logs,

$(m - 1) \log (x^4 - 2x^2y^2 + y^4) = 2m \log (x - y) - 2 \log (x + y)$

$\therefore (m - 1) \log (x^2 - y^2)^2 = 2 [m \log (x - y) - \log (x + y)]$

$\therefore 2(m - 1)\log (x^2 - y^2) = 2 [m \log (x - y) - \log (x + y)]$

$\therefore (m - 1) \log (x^2 - y^2) = m \log (x - y) - \log (x + y)$

$\therefore (m - 1) \log [(x + y) \times (x - y)] = m \log (x - y) - \log (x + y)$

$(m - 1)\log (x + y) + (m - 1)\log (x - y) = m \log (x - y) - \log (x + y)$

$m \log(x + y) - \log(x + y) + m \log(x - y) - \log(x - y) = m \log(x - y) - \log(x + y)$

$\therefore m \log (x + y) - \log (x - y) = 0 \qquad \therefore \ m = \log (x - y)/\log (x + y)$

EXERCISE 5

1. Find logarithm of 32 to base $\sqrt{2}$

2. Express the following in terms of log of a single expression.
$3/2 \log x - 5 \log y - 2 \log z$

3. Solve the following equation for x. $(a^{x+1})/(b^{x-1}) = c^{2x}$

4. Find (a) $\log_{125} (1/5)$ (b) $\log_9 27$

5. Solve. $\log_4 (x^2 - 3x + 12) = 2$

6. Express as a logarithm of a, b and c. $\log [(64 a^2)/(b^{-5}c^3)]$

7. Find x if $a^x = c\, b^x$

8. If log 2 = 0.30103 and log 3 = 0.47712, find $\log \sqrt{(36/27)}$

9. If log 2 and log 3 are known (Refer to problem 8). Find up to two decimal places, the value of x from the equation: $6^{3-4x} \times 4^{x+5} = 8$

10. Using the values given in question 8. Obtain the value of log0.8.

11. $(1/2) \log (29 + 12 \sqrt{5}) = \log (3 + 4x)$. Find x.

12. If $2x^2 + 2y^2 = 16xy$, find the value of $\log (x - y)$

13. $\log (x^2y^3) = a$, and $\log (x/y) = b$, find log x and log y.

14. If $\log (x^2 + y^2) = 2$, find $| x/y |$ if both are whole numbers

15. If 2.log a = 3.log b - 1, find the value of a in terms of b.

16. If $\log (x^3 + y^3) = 3\log x + 3\log y$, find an expression containing x and y only.

17. Find x if $p^{x-1} = q^{x-2}$.

18. If $\log x^3 + \log 6a = \log 162 + \log a$, find the value of x.

19. If $\log (x^a + y^b) = a \log x + b \log y$, express x as a function of y.

20. $6^{5x-12}.4^{5-2x} = 2.3^{2x-3}$.

21. Evaluate $\log_6(216\sqrt{6})$.

22. Evaluate $\log_6 16$, if $\log_{12} 27 = a$.

23. Evaluate $(\log_a n) / (\log_{ab} n)$

24. Simplify 7log16/15 + 5log25/24 + 3log81/80 .

25. If $a^2 + b^2 = 7ab$, prove that $\log [(a + b) / 3] = 1/2 [\log a + \log b]$.

ANSWERS

1. 10. **2.** log $[(\sqrt{x})^3/(y^5z^2)]$. **3.** (log ab) / log (bc^2/a). **4.** (a)-1/3 (b) 3/2.

5. (-1, 4). **6.** 2log 8 + 2log a + 5 log b - 3 log c.

7. log c/(log a - log b). **8.** 0.06247. **9.** 1.77.

10. - 0.09691. **11.** $\sqrt{}$ 5/2. **12.** log (6xy $)^{1/2}$

13. $\frac{a+3b}{5}, \frac{a-2b}{5}$ **14.** 3/4 or 4/3. **15.** $\sqrt{\frac{b^3}{10}}$

16. $x^3 + y^3 = x^3y^3$. **17.** log (p/q^2)/log (p/q). **18.** 3

19. $\sqrt[a]{\frac{-y^b}{1-y^b}}$ **20.** 3. **21.** 7/2.

22. 4(3 - a)/(3 + a). **23.** 1 + $\log_a b$. **24.**log2.

25. 2 log [(1/3)(a+b)] = loga + logb

Concepts 6

Set Theory

A **set** is a well-defined collection of objects. Sets are usually denoted by A, B, C, ----. The objects in the set are called 'elements' of the set. If x is an element of the set A, we say, $x \in A$ (x belongs to A).Sets are usually defined in two forms - the tabular form and the rule method.

Tabular form: In the tabular form, all elements of the set are enumerated or listed. E.g. The set of natural numbers: N = {1, 2, 3, 4, - -}, the set of integers: I = {- -, -3, -2, -1, 0, 1, 2, 3, - -}.

Rule method: Under this method, the defining property of the set is specified. If all the elements in the set have a property P, then we can define the set as A= {x: x has the property P}. E.g. N = {x: x is a natral number}, I = {y: y is an integer}.

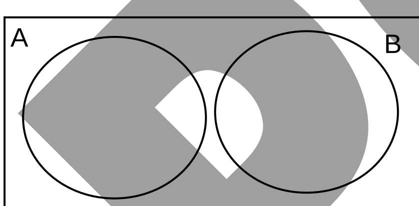
TYPES OF SETS:

Finite sets: A set with a finite number of elements is called a finite set. E.g. A = {a, e, i, o, u}.

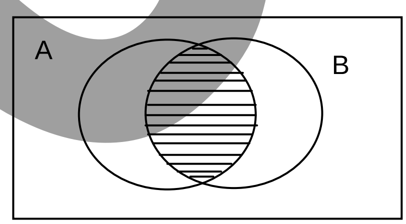
Empty set: A set which contains no elements is called an empty set or a null set. An empty set is denoted as '{ }' or ϕ.

Infinite set: A set that is neither an empty set nor a finite set is called an infinite set. Such a set will contain infinitely many elements. E.g. N = {1, 2, 3, ---}.

Disjoint sets: Two sets are said to be disjont sets if they do not have any common elements. E.g. If A = {x: x is an even number} and B = {y: y is an odd number}, then A and B will be disjoint sets.



Non – overlapping



Overlapping

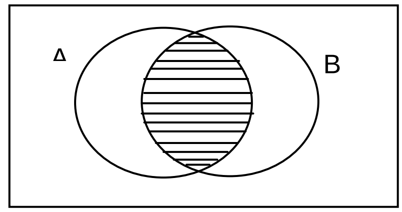
Overlapping sets: If two sets A and B have some common elements, the sets are said to be overlapping. E.g. A = { 1, 2, 3, 4, 5} and B = {3, 4, 5, 6, 7}. These sets are overlapping sets as the elements 3,4 and 5 are common to both sets. The elements common to both sets is called the intersection of the two sets and is denoted as $A \cap B$. On the other hand, the addition of two sets is called the union of two sets and is denoted by $A \cup B$.

Equal sets: Two sets are said to be equal if they contain the same elements. E.g. A = {2, 4, 6, 8} and B = {x: x is an even number between 1 and 9} are equal sets as they contain the same elements.

Universal set: A universal set is the set that contains the elements of all the sets under consideration. It is usually denoted by U, S or Ω. E.g. A = {4, 7, 8, 9}, B = {-4, -2, 0, 1, 4, 7, 10}. The set of integers, I = {--, -2, -1, 0, 1, 2, --} will be the universal set for A and B.

Complement set: Given a set A, the complement set is the set that contains elements not belonging to A and is denoted by A'. The union of the given set and its complement will give the universal set. I.e. $\Omega = A \cup A'$. e.g. A = {x: x is a Mathematics book in the CF library}. Then, A' = {y: y is not a Mathematics book in the CF library} and the universal set in this case will be U = the set of all books in the CF library.

Basic operations on sets:

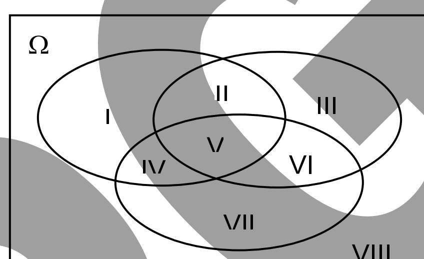


1. Intersection of two sets: The intersection of two sets is the set of elements common to both the given sets. The intersection of two sets A and B is denoted as $A \cap B$. In notation form, we can define the intersection of two sets A and B as $A \cap B = \{x: x \in A, x \in B\}$.
   If, $A \cap B = \phi$, then A and B are disjoint sets. If $A \cap B \neq \phi$, then A and B are overlapping sets.

2. Union of two sets: The union of two sets is the set containing the elements belonging to A and also the elements belonging to B. The union of these sets is denoted $A \cup B$. In notation form, we can define the union of two sets as $A \cup B = \{x: x \in A, x \in B, x \in A \cap B\}$.

3. Difference of two sets: The difference of two sets A and B is the set of elements that belong to A but do not belong to B. The difference of two sets is denoted by A - B. In notation form, we can define the difference of two sets as $A - B = \{x: x \in A, x \notin B\}$.

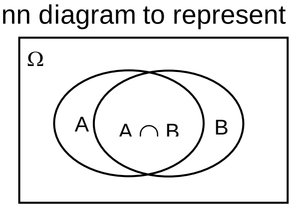
VENN DIAGRAMS: A Venn diagram is a closed figure used to denote the set of all points within the figure. Circles are used to denote the various sets under consideration.



In the above figure, Ω is the universal set. The three ovals represent three sets A, B and C. The eight regions in the figure denote different combinations of sets.

REGION I:	Contains elements in set A only - n(A).
REGION II:	Contains elements common to the sets A and B only - $n(A \cap B)$.
REGION III:	Contains elements in set B only - n(B).
REGION IV:	Contains elements common to the sets A and C only - $n(A \cap C)$.
REGION V:	Contains elements common to the sets A, B and C - $n(A \cap B \cap C)$.
REGION VI:	Contains elements common to the sets B and C only - $n(B \cap C)$.
REGION VII:	Contains elements in set C only - n(C).
REGION VIII:	Contains elements not belonging to either of the sets A, B and C - $n(A \cup B \cup C)$.

Similarly, we can also draw a Venn diagram to represent two sets A and B as follows:



If $n(A \cup B \cup C)$ is the number of elements in Ω.
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$.

Examples

Ex. If Ω is the universal set, find the following. (a) $A \cup \Omega$. (b) $A \cup A'$. (c) $A \cup \phi$. (d) $A \cup B$, if $B \subset A$.

Sol. (a) $A \cup \Omega = \Omega$. (b) $A \cup A' = \Omega$. (c) $A \cup \phi = A$. (d) $A \cup B = A$.

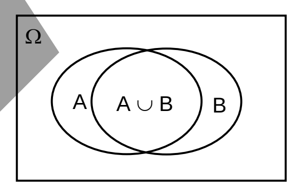
Ex. If A = {1,2,3,4}, B = {2,4,6,8}, C = {3,4,5,6,8} and Ω = {1,2,3,……,10}, verify that:
(a) $(A \cap B)' = (A' \cup B')$. (b) $(A \cup B \cup C)' = (A' \cap B' \cap C')$.

Sol. (a) We have $A \cap B$ = {2,4}. L.H.S. = $(A \cap B)'$ = {1,3,5,6,7,8,9,10}. Now we have A' = {5,6,7,8,9,10} and B' = {1,3,5,7,9,10}. Therefore R.H.S. = $A' \cup B'$ = {1,3,5,6,7,8,9,10}.

(b) $A \cup B \cup C$ = {1,2,3,4,5,6,8}. Therefore L.H.S. = $(A \cup B \cup C)'$ = {7,9,10}. We have A' = {5,6,7,8,9,10}, B' = {1,3,5,7,9,10}, C' = {1,2,7,9,10}. Now R.H.S. = $A' \cap B' \cap C'$ = {7,9,10}.

Ex. In a survey of 100 people, it was found that 60 people read the Times of India, 55 read the Indian Express and 10 read neither of the two newspapers. How many people read both newspapers?

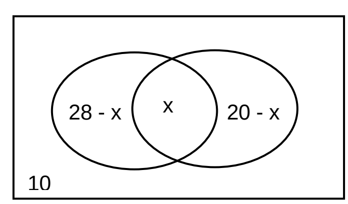
Soln. Of the 60 people reading the Times of India and the 55 reading the Indian Express, some people must be reading both newspapers. The information given in the question can be represented in the form of a Venn diagram.



A: Number of people reading the Times of India, B: Number of people reading the Indian express, $(A \cap B)$: Number of people reading both newspapers, Ω: Total number of people surveyed and $(A \cup B)'$: Number of people reading neither newspaper.
As 10 people read neither of the newspapers, the number of people reading either the Times of India or the Indian Express or both is 90. $(A \cup B) = A + B - (A \cap B)$. So, $90 = 60 + 55 - (A \cap B) \Rightarrow (A \cap B) = 25$. So, the number of people reading both newspapers is 25.

Ex. In a class of 50 students, 28 like pop music and 20 like classical music. If 10 students like neither of the two kinds of music, find the number of students who like both kinds of music.

Soln. P: students who like pop music, C: students who like classical music.
$(28 – x) + x + (20 – x) + 10 = 50 \Rightarrow x = 8$.

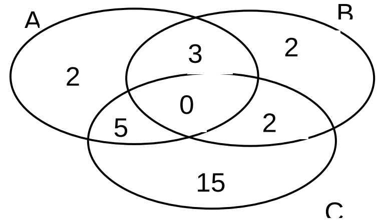


Ex. In a class of 64 students, 50% of the students have taken Sociology and 75% of the students have taken Politics. How many students have taken both the subjects?

Soln. We have n(S) = 64 x 50/100 = 32, n(P) = 64 x 75/100 = 48. Now $n(S \cup P) = n(S) + n(P) - n(S \cap P)$ = 64 = 32 + 48 - $n(S \cap P)$. Therefore, $n(S \cap P)$ = 80 - 64 = 16.

Exercises 6

1. In the Venn diagram find the value of $A \cap (B \cup C)$.



2. In a school, all 400 pupils play either Hockey or Football or both. If 264 pupils play Football and 38 students play both the games i.e Football and Hockey. Find the number of pupils who play Hockey only.

3. Two hundred students in a class were asked which T.V. programme they had watched on a particular holiday. 100 had watched programme A, 120 had watched programme B. If each student in the class had watched at least one programme from A and B, find the number of students who had watched both the programmes.

4. Students in the Delhi Public School can play any game from among Cricket, Football and Hockey. The following table gives the statistics of the students who play some or all the games in the school.

Game	Cricket	Hockey	Foot ball	Hockey & Football Only	Cricket & Hockey Only	Cricket & Football Only	All three games
No of students	70	60	60	20	20	16	6

(a) Find the percentage of students who play hockey only.
(b) Find the percentage of students who play football.
(c) Find the percentage of students who play cricket only.

5. A newspaper agent delivers morning papers to 240 families. 90 buy only the Times of India. 15 buy Times of India and The Indian Express. 100 buy The Indian Express. 30 buy The Indian Express and Hindu. 30 buy only Hindu and no family buy all three papers. Find the following:
(a) How many buy The Times of India?
(b) How many buy The Indian Express only?
(c) How many read only one paper?
(d) Which newspaper is the most popular?
(e) Which is the least popular?

6. Of the 400 members of a sport club, 144 play Squash (S), 135 play Badminton (B), 156 play Table Tennis (T) . Each member plays at least one game. The members who play badminton and table tennis both must play squash also. For every three members who play at least two games, there are two members who play all the three games. Find the number of members:
(a) who play all the three games.
(b) who play only Squash.

7. The following are the statistics of 22 students of Career Forum who appeared for CAT. 15 students got calls from the IIMA, 12 students got calls from the IIMB and 8 students got calls from the IIMC. 6 students got calls from the IIMA and the IIMB, 7 students got calls from the IIMB and the IIMC and 4 students got calls from the IIMA and the IIMC. If 4 students got calls from all three IIMs, find how many students of Career Forum got:

(a) a call from IIMA only.
(b) a call from IIMC only.
(c) a call from IIMB only.
(d) calls from exactly two IIMs.
(e) calls from more than one IIM.

8. A study of 104 students, who speak Marathi or English or Hindi, reveals 38 do not speak Marathi, 30 do not speak English and 40 do not speak Hindi. 40 speak Hindi and Marathi, 50 speak Hindi and English, 44 speak Marathi and English.
(a) How many speak all three languages?
(b) How many speak Hindi but neither Marathi nor English?
(c) How many speak exactly two languages?

9. A survey of 3000 households, T.V. set, washing machine and refrigerators were counted. Each house had at least one of these appliances. 1200 had no refrigerator, 1140 had no washing machines and 1626 had no T.V. sets. If 882 had both a washing machine and T.V. set, 831 had both a washing machine and refrigerator. 360 had both a refrigerator and T.V. set.
(a) How many had only a washing machine?
(b) How many had only a refrigerator?
(c) How many had more than one appliance?

10. In order to study the breakfast habits of the people in a particular city a survey of 400 families was conducted. Of these 400 families, 156 had cereals, 288 had eggs and 300 had toast. It was also found that 212 families had egg and toast and 104 had cereal and toast. If 128 had egg and cereal and 84 had all three, find how many had:
(a) neither toast nor cereal nor egg?
(b) egg or toast but not cereal?
(c) only two items?

DIRECTIONS: Questions 11 - 15 are based on the following data.

The following table gives information about the engineers employed by a manufacturing unit. It is known that no engineer has a degree of one discipline and diploma of the same discipline or any other discipline.

Durn. (Years)	Mechanical		Computers		Both		Total	
	Deg.	Dip.	Deg.	Dip.	Deg.	Dip.	Deg.	Dip.
0 - 2	76	24	28	22	7	3	100	50
2 - 5	32	118	35	47	15	10	52	155
> 5	67	55	8	0	3	0	78	75

11. Among the engineers with 0 – 2 years of experience, what is the percentage of employees with qualifications in Computers but not in Mechanical?

12. What is the number of employees who have neither a Mechanical nor a Computer background?

13. The organisation announces a bonus scheme for degree holders who have worked for more than 5 years as follow: An employee with only a Mechanical degree gets Rs. 9000. An employee with only a degree in Computers gets Rs.11000. An employee with degrees in both, Mechanical and Computers, gets Rs.20000. Degree holders with neither a Mechanical nor a Computers background get Rs. 6000 each. What is the additional financial burden the company has to incur on account of the bonus scheme?

14. Which category of experience has employees with qualifications in either Mechanical only, Computers only or both?

15. Among Diploma holders with upto 5 years experience, what is the ratio of employees with a Mechanical background only to those with a Computers background only?

DIRECTIONS: Questions 16 - 20 are based on the following data.
A multi - national consultancy employs people with post - graduate degrees in PMIR and Social Studies in three departments, Administration, Human Resource Development and Consultancy. The following information about the employees has been made available.

Age (Years)	Administration		HRD		Both		Total	
	PMIR	SS	PMIR	SS	PMIR	SS	PMIR	SS
20 - 25	270	220	250	180	150	120	450	300
25 - 35	300	250	240	170	120	90	550	350
> 35	220	120	180	100	120	50	400	200

16. The number of people in the age group 25 - 35 employed in Administration but not in HRD forms what percent of the total number of employees in that age group?

17. The number of Social Studies graduates employed in HRD but not in Administration forms what percent of the total number of employees?

18. 20% of the employees in the age group 25 - 35 shift to the next age group in their respective sections. The number of people in the first two age groups employed in HRD but not in Administration forms what percent of the total number of employees in HRD but not in Administration?

19. Referring to the above question, the number of employees in the age group > 35 years forms what percent of the total number of employees in the organisation?

20. With reference to the questions above, the number of employees common to Administration and HRD in the age group > 35 years forms what percent of the number of employees in Consultancy?

ANSWERS

1. 8. **2.** 136. **3.** 20. **4.** 10.61, 45.45, 21.21.

5. 125, 55, 175, Times of India, Hindu. **6.** 14, 123. **7.** 9, 1, 3, 5, 9.

8. 34, 8, 32. **9.** 186, 648, 1995. **10.** 16, 228, 192. **11.** 26.66%.

12. 36. **13.** Rs. 7,27,000. **14.** (2 - 5). **15.** 129:56.

16. 37.77%. **17.** 8.44%. **18.** 68.08%. **19.** 34.66%.

20. 53%.

Concepts 7

Base Notations

In any number normally used, the value of a digit depends upon the position the digit occupies. e.g. in a number like 2354, '2' means two thousand, 3 means three hundred, 5 means five tens (or fifty), 4 means four units. This is known as the **principle of place value.**
The Hindu-Arabic system of numbers i.e. the system of numbers we are so familiar with is the **denary or decimal system.** One system can be distinguished from the other by the number of symbols used, and this is called the base of the system.
The base of the **decimal** system is ten, **binary** system is 2, **ternary** system is 3, **quinary** system is 5, **octal** system is 8. The number denoting the base of the system is called the **radix.**

Name	Base	Symbols used	No. of symbols	Place value
Binary	**2**	**0,1**	**2**	**…2^4-2^3-2^2-2-unit**
Ternary	**3**	**0,1,2**	**3**	**…3^4-3^3-3^2-3-unit**
Quinary	**5**	**0,1,2,3,4**	**5**	**…5^4-5^3-5^2-5-unit**
Octal	**8**	**0,1,2,3,4,5,6,7**	**8**	**…8^4-8^3-8^2-8-unit**
Denary/Decimal	**10**	**0,1,2,3,4,5,6,7,8,9**	**10**	**…10^4-10^3-10^2-10-unit**
Hexadecimal	**16**	**0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F**	**16**	**…16^4-16^3-16^2-16-unit**

A suffix at the foot of the numeral is used to indicate the base.

e.g. $(47)_8$ Octal system
$(234)_{10}$ denary system

$(34)_{10}$ is read as three-four to the base 10 and its value is thirty four
$(34)_8$ is read as three-four to the base 8 and its value is $3 \times 8^1 + 4 \times 8^0 = (28)_{10}$

Converting from one base to another :

From octal to denary

$(37)_8 = 3 \times 8 + 7 \times 8^0 = (31)_{10} = 3 \times 10 + 1$

$(3456)_8 = 3 \times 8^3 + 4 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 = (1838)_{10}$

From binary to denary

$(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 4 + 1 = (13)_{10}$

$(10001)_2 = 1 \times 2^4 + 0 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (17)_{10}$

From decimal to binary

$$\begin{array}{r|l} & 0 \quad 1 \\ & 1 \quad 1 \\ & 3 \quad 0 \\ & 6 \quad 1 \\ 2 & 13 \end{array}$$

The number whose binary equivalent is to found is divided by 2 and the remainder is written on the right-hand side. Each successive quotient is similarly treated, till it is reduced to 0. The binary equivalent is then the number formed by the remainders as read from top to bottom. $\therefore (13)_{10} = (1101)_2$
* N.B.* : same method is used in case of different system, each time using the radix as divisor

EXERCISE 7

1. Convert from decimal to binary: a) 27 b) 35 c) 69

2. Convert from decimal to hexadecimal :a) 197 b) 956 c) 577

3. Convert to decimal from binary : a) 1001101 b) 11110 c) 110001

4. Convert from hexadecimal to decimal : a) 9C b) F6 c) 1A3C

5. If in a number system, 40132 corresponds to 2542 in decimal, find the base of the number system.

6. In a certain number system, 13 x 13 is written as 121 and 97 is written as 81. What is 2174 written as?

7. If in a certain number system, 35 is written as 55 , what is $(111111)_2$ written as?

8. What is $(110100101)_2$ in the number system with base 16?

9. What is the binary 11001101 equivalent to in the number system with base 7?

10. In a certain number system, 7917 - 6233 is written as 444. What is the hexadecimal FE3 - 7AA written as?

ANSWERS

1. a) 11011	b) 100011	c) 1000101
2. a) C5	b) 3BC	c) 241
3. a) 77	b) 30	c) 49
4. a) 156	b) 246	c) 6716
5. 5	**6.** 1312.	**7.** 143.
8. 1A5.	**9.** 412.	**10**. 555.

Concepts 8

Progression

Arithmetical Progression: If quantities increase or decrease by *common difference,* then they are said to be in arithmetical progression (AP).

e.g. 3, 5, 7, 9, 11, 13, 15,...
4, 8, 12, 16, 20, 24, ...

The general form of an AP is therefore:

a	a + d	a + 2d	a + 3d . . .	a + (n - 1)d
1^{st} term				n^{th} term

Thus, the n^{th} term of an AP is given by $T_n = a + (n - 1)d$

The sum of first n terms of an AP is usually denoted by S_n and is given by the following formula:

$$\boxed{\begin{aligned} S_n &= n/2\,[2a + (n-1)d] \\ &= (n/2)\,(a + l), \end{aligned}}$$

where, *a* is the first term of the AP and *d* is the common difference. Where l is the last term.

Ex. Find the sum of the series 5/2, 4, 11/2, 7, ... up to 21 terms

Sol. d = 1.5 n = 21 a = 5/2

$S_n = [21/2] \times [2 \times (5/2) + 20 \times (3/2)]$
$= [21/2] \times [5 + 30]$
$= 735/2$
$= 367.50$

Ex. Find the 74^{th} term of the series 3, 7, 11, 15, ...

Sol. a = 3 d = 11 - 7 = 4 n = 74

$T_{74} = a + (n - 1)d$
$= 3 + 73 \times 4$
$= 3 + 292$
$= 295$

When three quantities are in AP, the middle one is said to be the arithmetic mean of the other two.
If a and b are two quantities in AP and if A is the arithmetic mean then,

A = (a + b)/2

If a and b are two given quantities in A.P., and if n arithmetic means are to be inserted between a and b,
the 'm' th arithmetic mean is given by **a + { m (b - a) / (n+1) }**

Ex. Find the total earning over 6 years for a person whose salary is Rs. 30,000 p.a. and increases by 1500 p.a., every six months.

Sol. For each six months he earns half his annual salary.

$\therefore$ a = 30000/2 = 15000
To find the sum of 12 terms
d = 750
$s_{12} = 12/2\,[2 \times (15000) + 11 \times 750]$ = 6[30,000 + 8250]
= Rs. 229500

Geometric Progression: If quantities increase or decrease by *constant factor* then they are said to be in geometric progression (GP).

e.g. 4, 8, 16, 32, 64, 128, ...

2, 1, 0.5, 0.25, .125, ...

Thus, the principal characteristic of GP is the common ratio (constant factor). The general form of GP is therefore:

a	ar	ar^2	. . .	ar^{n-1}
1^{st} term				n^{th} term

The n^{th} term of GP = $\boxed{T_n = ar^{n-1}}$ Where, r is the common ratio

$$\boxed{S_n = a(1 - r^n)/(1 - r)}$$

The sum of the first n terms of a geometric progression is S_n and is given by

Ex. Find the sum of first 6 terms of the series: 3, 9, 27, 81, ...

Sol. $n = 6$ $r = 27/9 = 3;$ $a = 3$

$$S_n = 3(1 - 3^6)/(1 - 3) = 1092$$

For an infinite series, $S_\infty \rightarrow \pm\infty$ (if $|r| \geq 1$)

and S_∞ = **a/(1 - r)** (if $|r| < 1$)

Ex. Find sum of the infinite series 1, 0.5, 0.25, ...

Sol. $r = 0.5$ Since, $-1 \leq r \leq 1$ $S_\infty = a/(1 - r)$

$S_\infty = 1/(1 - 0.5) = 1/0.5 = 2$

Ex. Find the 10^{th} term of the series: 4, 16, 64, 256, 1024, ...

Sol. The given series is in geometric progression

Where, $a = 4$ $r = 4$,

To find T_{10}

$T_{10} = ar^{(10-1)}$

$= 4 \times 4^9 = 4^{10}$

When three quantities are in GP, the middle one is said to be the geometric mean of the other two If, a, b are two quantities in GP and G is their geometric mean then,

$$\boxed{G = \sqrt{(ab)}}$$

If a and b are two quantities in G. P., and if n geometric means are to be inserted between them, the m^{th} mean is given by $a(b/a)^{m/(n+1)}$

Ex. Find geometric mean between the terms 4 and 25

Sol. The geometric mean $G = (4\times25)^{1/2} = 100^{1/2} = 10$

Thus, 4, 10 and 25 are in GP.

Harmonic Progression (H. P.)

Definition : A series of quantities is said to be in harmonic progression when their reciprocals are in arithmetic progression.

e.g. 1/3 , 1/5 , 1/7 , ... and 1/a , 1/ (a + d) , 1/ (a + 2d), ... are in HP as their reciprocals 3, 5, 7...., and a, a + d, a + 2d..... are in AP.

nth term of HP.

Find the nth term of the corresponding AP and then take its reciprocal.
If the HP be as 1/a , 1/ (a + d) , 1/ (a + 2d)....
then the corresponding AP is a, a + d, a + 2d, ...
T_n of the AP is a + (n - 1)d
$\therefore$ T_n of the HP is . . . $\boxed{1/ [a + (n-1) d].}$

In order to solve a question on HP, one should form the corresponding AP.

Harmonic Mean:

The harmonic mean between two quantities a and b will be H if a, H, b be in harmonic progression and H is given by

$$\boxed{H = 2ab / (a + b) = HM}$$

A COMPARISON BETWEEN AP and GP

Description	AP	GP
Principal Characteristic	Common Difference (d)	Common Ratio (r)
n^{th} Term	$T_n = a + (n - 1)\,d$	$T_n = a\,r^{(n-1)}$
Mean	$A = (a + b)/2$	$G = (ab)^{1/2}$
Sum of First n Terms	$S_n = n/2\,[2a + (n-1)d] = n/2\,[a + l]$	$S_n = a\,(1 - r^n)/(1 - r)$
'm' th mean	$a + [m(b - a) / (n + 1)]$	$a\,(b / a)^{m/(n+1)}$

EXERCISE 8

1. The sum of n terms of the series 2, 5, 8, ... is 950 find n

2. Find the sum of the first 9 terms of the series 3/4, 2/3, 7/12...

3. Find the sum of the first 15 terms of the series whose n^{th} term is (4n + 1).

4. Find the sum of the first n natural numbers.

5. Sum of three numbers in an AP is 27, and their product is 504. Find the numbers.

6. How many terms of the series 9, 12, 15,.... must be taken so that they add up to 306.

7. What is the sum of the first 7 terms of the series 1/3, 1/2, 3/4,...

8. Sum of 3 numbers in GP is 38 and their product is 1728. Find the numbers.

9. Sum of a GP whose common ratio is 3 is 728. The last term is 486. Find the first term.

10. In a GP first term is 7 and n^{th} term is 448, and the sum of n terms is 889. Find the common ratio.

11. Find the three numbers in GP whose sum is 19 and whose product is 216.

12. If 18, a, b, c, 46 form an arithmetic sequence, find a, b, c.

13. In an AP the first term is 2, the last term is 29, sum of the terms is 155. Find the common difference.

14. The sum of 15 terms of an AP is 600, and the common difference is 5. Find the first term.

15. Find the sum of 15 terms of the series whose n^{th} term is 2n + 3.

16. A man arranges to pay off a debt of Rs. 3600 by 40 annual installments which form an A.P. When 30 of the installments repaid he dies leaving a third of the debt unpaid. What is the value of the first installment ?

17. The sum of three numbers in A.P. is 24. If the numbers be decreased by 2, 4, 4 respectively, they form a G. P. Find the numbers.

18. An A.P. has 23 terms, sum of the middle three terms is 144, the sum of the last three terms is 264. What is the 16^{th} term ?

19. In a set of four numbers first three are in G. P. and the last three are in A.P. with a difference of 6. If the first and the fourth numbers are the same, find them.

20. Find a, b, c between 2 & 18 such that (i) their sum is 25, (ii) 2, a, b are consecutive terms of an A.P. and (iii) b, c, 18 are the consecutive terms of a G. P.

21. Two sets of numbers each consist of 3 numbers in AP. The common difference of the first set is greater by 1 than that of the second set. If the sum of each set is 15, and the ratio of products of the two sets is 7 : 8, find the numbers.

22. If the sum of an AP is the same for p terms as for q terms, find the sum for (p + q) terms.

23. First and third term of an A.P. $\{A_i\}$ are $A_1 = a$, $A_3 = b$, and $a_1 = a$ and $a_5 = b$ respectively be the first and fifth terms of another A.P. $\{a_i\}$. Find the ratio of A_{n+1} and a_{2n+1} .

24. Sum of three numbers in GP is 70. If the two extreme terms are multiplied by 4, and the middle term by 5, the resultants are in AP. Find the numbers.

25. Find the first term of the infinite GP whose first two terms add up to 5, and whose each term is three times the sum of all terms that follow it.

26. If a, b and c are in AP, prove that
$(a - c)^2 = 4(b^2 - ac)$.

27. Find the sum of n terms of the series
$\log a + \log(a^2 / b) + \log(a^3 / b^2) + \log(a^4 / b^3) + \ldots\ldots\ldots\ldots$

28. Evaluate .7 + .77 + .777 + ..., in terms of a rational number.

29. If $x = 1 + a + a^2 + a^3 + \ldots\ldots\text{to } \infty$ $(|a| < 1)$,
$y = 1 + b + b^2 + b^3 + \ldots\ldots \text{ to } \infty$ $(|b| < 1)$,
Then find $1 + ab + a^2b^2 + a^3b^3 + \ldots\ldots\ldots\ldots \text{ to } \infty$

30. How many terms of the series 1, 4, 16, ..., sum up to 341?

31. If the (n+1)th term of a harmonic progression is twice the (3n+1)th term, find the ratio of the first term to (n+1)th term.

32. If the sum of the reciprocals of the first seven terms of a harmonic progression is 70, find the 4th term of the HP.

33. If the mth term of a H.P. is n and the nth term is m, what is the value of the $(m + n)^{th}$ term?

34. Prove that the first term of the series 1/n, 1/(n+m), 1/(n+2m),…… is (m+1) times the (n+1)th term.

35. If $(x^{n+1} + y^{n+1}) / (x^n + y^n)$ is the harmonic mean of x and y, find the value of n?

ANSWERS

1. 25	**2**. 15/4	**3**. 495	**4**. n(n+1)/2	**5**. 4, 9, 14
6. 12	**7**. 2059/192	**8**. 8, 12, 18	**9**. 2	**10**. 2
11. 4, 6, 9	**12**. 25, 32, 39	**13**. 3	**14**. 5	**15**. 285
16. 51	**17**. 4, 8, 12 or 10, 8, 6		**18**. 64	**19**. 8, -4, 2, 8
20. 5, 8, 12	**21**. 21, 5, -11 and 22, 5, -12 or 3, 5, 7 and 4, 5, 6			**22**. 0
23. 1	**24**. 10, 20, 40	**25**. 4	**27.** n/2 [n log(a/b) + logab]	
28. (7n / 9) - (7/81)[1 - 1/10^n].		**29**. xy / (x+y-1)	**30.** 5.	**31.** 2
32. 1/10	**33.** mn/(m + n)	**35.** -1		

Concepts 9

Series

We have gone through topics like arithmetic, geometric and harmonic progressions where we have generalized the terms and they always follow particular formula. But in case of series the terms or elements follow a definite law or they have definite relationship but it cannot be generalized, i.e reasoning has to be there while solving it which means you have to know what is the definite relationship which make the set of given terms series.

For e.g series 2, 6, 12, 20, 30 , ?
When we see the terms and the relationship that comes out is n.(n + 1), so the next term is 42
Different type of series

1. Difference or sum type of series

e.g. 1, 4, 10, 19, 31, ? as we see the difference between two consecutive terms go on increasing as multiple of 3. The answer is 31 + 15 = 46.
e.g. 1, 3, 6, 10, 15, 21, here also the difference between two consecutive terms goes on increasing but you can observe it increases with different relation. This is the reason why series usually comes under mathematical reasoning.

2. Cumulative series

e.g. 1, 3, 4, 7, 11, 18… here each term is nothing but the addition of two previous terms.

3. Power series

e.g. 0, 6, 24, 60, 120 here each term is defined as $n^3 - n$.
Here the terms are defined on the basis of powers of numbers.

4. Alternate series

e.g. 1, 5, 9, 10, 25, 15, 36,……….. here we see it is the combination of two series odd terms are the power of odd number and even terms are multiple of 5.
There are other form of series and the relationship between them must exists then only we will be able to find the term which is missing.

Exercise 9

Directions: Find the next term in the series given.

1. 0, 2, 12, 36, 80,
2. 1, 3 , 7, 13 , 21, 31, ?
3. 3, 7, 15, 31, 63,.
4. 19, 119, 1119, 11119,
5. 7, 19, 37, 61, 91,
6. 1, 3 , 7, 13, 21, 31
7. 1, 4, 27, 256, 3125,
8. 11, 13, 17, 23, 29,
9. 1! , 2!, 720!
10. 1, 6, 15, 20, 15, 6,

ANSWERS

1. 150	**2**. 43	**3**. 127	4. 111119	5. 127
6. 43	7. 46656	8. 31	9. (((4!)!)!)!	10. 1

Concepts 10

Permutations And Combinations

Tossing a coin, placing a ball in a box, rolling a dice selecting a person from a crowd are all physical processes that have a number of possible outcomes. A ball can be placed in a box in one way, the two possible outcomes of tossing a coin are "heads" and "tails", rolling a dice has six possible outcomes 1, 2, 3, 4, 5 and 6, selecting a committee of four from several hundred people has many outcomes. When considering such physical processes, it is best to follow the following rules.

Factorial Notation : The product of first n natural numbers is denoted by n! and is read as "n factorial" or "factorial n". By definition 0! = 1

Fundamental Theorems of Counting :

1. Fundamental Principle of Addition : If one thing can be done in m different ways and a second thing can be done in n different ways independent of the first, then either of them can be done in (m + n) different ways.

2. Fundamental Principle of Multiplication : If one thing can be done in m different ways and a second thing can be done in n different ways, independent of the first, then both the things can be done together in (m x n) different ways.

Ex. From a pack of 52 cards find the number of ways in which (a) a king or a queen can be drawn (b) Both a king and a queen can be drawn

Sol. A king can be drawn in 4 different ways. A queen can be drawn in 4 different ways. By fundamental principle of addition a king or a queen can be drawn in 4 + 4 = 8 ways. By fundamental principle of multiplication a king and a queen can be drawn in 4x4= 16 ways

Generalization of Fundamental Principle :

If n different things can be done in m_1, m_2,...,m_n different ways respectively, independent of each other then -

a. Any one of them can be done in $m_1 + m_2 + ... + m_n$ different ways.

b. All of them can be done in the same order in $m_1 \times m_2 \times ... \times m_n$ different ways.

Permutation :

Permutation of n objects taken r at a time is an arrangement in a straight line of r objects from the given n objects. It is denoted by ${}^nP_r$ or ${}_nP_r$ or P(n, r).

Combination :

Combination of n objects taken r at a time is the selection of r objects from the given n objects. It is denoted by ${}^nC_r$ or ${}_nC_r$ or C(n, r).

Results :

1. ${}^nP_r = n(n - 1)...(n - r + 1) = n!/(n - r)!$

2. The number of permutations of n objects taken all at a time is ${}^nP_n = n!/0! = n!$

3. The number of permutations of n objects taken all at a time when p of them are all alike of one kind, q of them are alike of a second kind and r of them are alike of a third kind is n!/p!q!r!

Ex. How many words can be made with the letters of the word MATHEMATICS ? In how many of them do the vowels occur together?

Sol. Total letters = 11, M, A, T occur twice
Total arrangements = 11!/(2!2!2!) = 4989600
Treat the four vowels A, A, E, I as one unit. This with remaining 7 letters of which two are repeated can be arranged in 8!/2!2! ways. Four vowels can be arranged among themselves in 4!/2! ways
$\therefore$ Total number of ways = (8!/2!2!) (4!/2!) = 120960

4. Number of permutations of n objects taken all at a time in a circle is (n - 1)!

***Ex.*.** Twenty persons were invited to a party. In how many ways can they and the host be seated at a circular table ? In how many of these ways will two particular persons be seated on either side of the host.

Sol. There are 20 + 1 = 21 persons to be seated at the table. Fixing the seat of one person the remaining 20 can be seated in 20! ways.
Two particular persons can be sit on either side of the host in 2! ways. Remaining 18 can arrange themselves in 18! ways.
$\therefore$ Required number of ways = 2! x 18!

5. Number of combinations of n objects taken r at a time is ${}^nC_r = n!/(n - r)!r!$

6. ${}^nC_{n-r} = {}^nC_r$

7. If ${}^nC_p = {}^nC_q$ then either p = q or p + q = n

8. ${}^nP_n = {}^nP_{n-1}$

9. ${}^nP_r = {}^{n-1}P_r + r\,{}^{n-1}P_{r-1}$

10. ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

11. **Permutations with Repetitions :** The number of permutations of n objects taken r at a time when each object may be repeated any number of times in any permutation is given by n^r. As first object can be selected in n ways, Second object can be selected in n ways, and r^{th} object can be selected in n ways
By Generalization of fundamental Principle (Multiplication):
$\therefore$ Total Number of ways = n x n x... r times = n^r

***Ex.*.** Ten different letters are given. Words with five letters are to be formed from these given letters. Find the number of words which have at least one letter repeated.

Sol When there is no repetition of letters,
total number of words with 5 letters = ${}^{10}P_5$
= 30240
When any letter is repeated any number of times, total number of words = 10^5
$\therefore$ Required number of words = 100000 - 30240
= 69760

12. **Restricted Permutation :** Permutation of n objects taken r at a time in which k particular objects are :
a. never included is ${}^{(n-k)}P_r$
b. always included is ${}^{(n-k)}C_{r-k} \times r!$

***Ex.*.** A photograph of 4 players is to be taken from 11 players of a cricket team. How many different photographs can be taken if in each photograph captain & vice captain (a) must be included (b) are never included

Sol. Two players can be chosen in ${}^{9}C_2$ ways.

a. Total number of photographs = ${}^{9}C_2 \times 4!$
$= 864.$

b. ${}^{9}P_4 = 3024$

13. Division into groups : (m + n) objects can be divided into two groups containing m & n objects respectively in (m + n)!/m! n! different ways. If m = n, then they can be divided equally in :

a. Two groups in (2m)!/2!m!m! ways

b. Two persons in (2m)!/m! m! ways

14. (a + b + c) objects can be divided into three groups of a, b and c in
(a + b + c)!/ [a!b!c!] ways.
If a = b = c then they can be equally divided in (a) 3 groups in (3a)!/3!a!a!a! ways
(b) 3 persons in (3a)!/a!a!a! ways.

Ex. In how many ways can the face cards be equally divided into a. two groups b. two persons.

Sol Total face cards are 12, can be divided into two groups in $({}^{12}C_6\ {}^{6}C_6)/2 = 462$ ways
They can be divided in two persons in 462 x 2 = 924 ways (as any group can be given to any person).

15. Combination of **n different** objects taken any number of them at a time i.e. the number of selections of some or all objects from n different objects is
${}^{n}C_1 + {}^{n}C_2 + ...+ {}^{n}C_n = 2^n - 1$
Or, it is the number of ways in which at least one object can be selected out of n distinct objects.

Ex. A person has seven friends. In how many ways can he invite one or more of them to attend a tea party ?

Sol. He can invite one or two or seven friends in
${}^{7}C_1 + {}^{7}C_2 + ...+ {}^{7}C_7 = 2^7 - 1 = 127$ ways

Ex. From 5 different green balls, four different blue balls and three different red balls, how many combinations of balls can be chosen taking at least one green and one blue ball ?

Sol. At least one green ball can be chosen from five green balls in $2^5 - 1 = 31$ ways
At least one blue ball can be chosen from four blue balls in $2^4 - 1 = 15$ ways
At least one or no red ball can be chosen in $2^3 = 8$ ways
$\therefore$ By generalization of fundamental principle, required number of ways = 31 x 15 x 8 = 3720

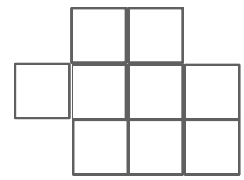
16. **Combination of objects not all different** : Total number of combinations of (p + q + r) objects where p are of one kind, q are of a second kind, r are of a third kind, taken any number of them at a time is [(p + 1)(q + 1)(r + 1) - 1]
and if p are alike of a kind, q are alike of second kind and r are different is $[(p + 1)(q + 1)2^r - 1]$

Ex. From 3 pears, 4 oranges and 5 apples, how many selections of fruits can be made by taking (i) at least one of them (ii) at least one of each kind

Sol (i) Out of 3 pears, we may take either 0 or 1 or 2 or 3. Thus pears can be chosen in 4 different ways. Similarly, oranges in 5 and apples in 6 ways.
Number of ways of selection of fruits = 4 x 5 x 6 = 120. But this also includes the case when no fruit is taken. Rejecting this case, the required number of ways = 120 - 1 = 119

(ii) At least one pear is to be chosen, we may take either 1, 2, or 3 pears. Thus pears can be chosen in 3 different ways.
Oranges can be chosen in 4 different ways
Apples can be chosen in 5 different ways
$\therefore$ Total number of ways = 3 x 4 x 5 = 60

Ex. Six crosses have to be placed in the squares of the following figure such that each row contains at least one cross. In how many ways can this be done ?



Sol. Total number of ways of filling any six squares of the given nine squares is ${}^9C_6$ = 84. If we chose first two rows six crosses can be filled in ${}^6C_6$ = 1 ways. If we chose the last two rows six crosses can be filled in 7 squares in ${}^7C_6$ = 7 ways.
$\therefore$ Number of ways which are not favorable = 1 + 7 = 8, $\therefore$ favorable cases = 84 - 8 = 76.

EXERCISE 10

1. Four persons enter a railway compartment in which there are six seats; in how many ways can they take their places ?

2. How many four digit numbers can be formed out of the digits 2, 3, 4, 5, 6, 7, if no digit is repeated in any number ? How many such numbers will be greater than 4000 ?

3. Five boys and four girls are to be seated on 9 adjacent seats in a cinema hall. It is desired that no two girls sit together. Find the number of ways in which they can be arranged.

4. There are 6 students of which 3 belong to the first year class, 2 belong to the second year class and one is in the third year. In how many ways can they stand in a line so that the students from the same class are together ?

5. In how many ways can 5 persons sit on 8 chairs in a row.

6. If ${}^nC_5 + {}^nC_4 = 3/5\ {}^7C_4$, find ${}^nC_2$.

7. If ${}^nC_{12} = {}^nC_8$ find ${}^nC_{17}$, ${}^{22}C_n$

8. From 12 books in how many ways can a selection of 5 be made, (a) when one specified book is always included (b) when one specified book is always excluded

9. Out of 14 men, in how many ways can eleven be chosen.

10. In how many ways can cricket 11 choose a captain and a vice captain from amongst themselves ?

11. How many numbers between 100 and 1000 can be formed using the digits 0, 2, 4, 6, 8, 5, if
(a) repetition of digits in a number is not allowed; (b) repetition of digits is allowed.

12. In a school, there are six classes and four teachers. If one teacher teaches only one class at a time, in how many ways can a teacher be assigned to a class ?

13. How many telephone numbers can be allotted with 6 digits from the natural numbers 1 to 9 both inclusive ?

14. A piece of paper has 8 points on it out of which 4 are collinear. How many distinct straight lines
can be drawn through these points ?

15. In how many ways can one or more cards be selected from a bunch of 5 different cards ?

16. How many selections of coins can be made if at least one coin of each type is selected from five 1 rupee, four 50 paise and three 25 paise coins ?

17. How many seven digit numbers can be formed with the digits 1, 2, 3, 4, 3, 2, 1, so that the odd digits occupy odd places ?

18. Find r if ${}^{10}P_{r+1} : {}^{11}P_r = 30 : 11$

19. In how many ways can a mixed doubles tennis match be arranged form 8 married couples if no husband and wife play in the same game ?

20. At an election a voter may vote for any number of candidates not greater than the number to be chosen. There are 7 candidates and 4 members to be chosen. In how many ways can a person vote?

21. Four-letter words are formed using 17 consonants and 5 vowels. How many will have 2 different vowels in the middle and a consonant at each end ?

22. Two out of six papers set for an examination are in mathematics. What are the number of ways in which the papers can be set so that the two mathematics papers are not together ?

23. Find n if ${}^{2n}C_3 : {}^{n}C_2 :: 44 : 3$

24. In how many ways can 45 men be allotted to 3 different districts if the districts are to be covered by 10, 15, and 20 men respectively.

25. Find r if ${}^{28}C_{2r} : {}^{24}C_{2r-4} = 225 : 11$

26. In how many ways can n things be given to p persons, when there is no restriction on the number of things each may receive ?

27. In a letter lock, each of three rings is marked with 15 letters. What is the maximum number of unsuccessful attempts that one has to make before the lock is opened ?

28. A certain code consists of 5 variables, with each variable having 4 different constant values possible. What is the total number of coded messages that can be sent with 5 constants one from each variable. ?

29. In a party of 15, each person shakes hands with every other person. What is the total number of handshakes ?

30. From 3 different soft-drinks, 4 Chinese dishes and 2 ice-creams, how many different meals are possible if at least one of each of the three items is to be included, depending upon the number of people likely to turn up ?

31. The Governing Council of an institute has fifteen members and wants to hold its annual meeting. In how many ways can the council be seated around a round table if the Chairman and the Vice-Chairman of the council are always seated together? In how many ways can the council be seated around the same table if the Chairman and the Vice-Chairman are never seated together?

32. Seven boys and five girls go out for a picnic. In how many ways can all of them be photographed if no two girls stand together?

33. Akshay is planning to give a birthday party at his place. In how many ways can he invite one or more of five friends and seat them at a circular table?

34. A cricket team of eleven is to be chosen from among 8 batsmen, 6 bowlers and 2 wicket-keepers. In how many ways can the team be chosen if there must be at least four batsmen, at least four bowlers and exactly one wicket-keeper?

35. In how many ways can five boys and five girls be photographed if at least two girls stand together?

36. A library has 11 books on Chemistry, 11 books on Mathematics and 11 books on Physics. In how many ways can these books be arranged in groups of eleven?

37. How many words can be formed from the letters in “CORRESPONDENCE” if the consonants are always written together?

38. If numbers less than 10000 are formed using the digits 3, 4, 8 and 9 how many of these will be even?

39. In how many ways can 5 boys and 3 girls be seated in a row such that no two girls sit together?

40. How many three digit numbers formed by using the digits 3, 4 and 5 are even?

41. If six persons are selected out of ten, in how many ways will a particular person be found among those six?

42. A committee of five is to be chosen from among six men and four ladies. In how many ways can this be done in order to include at least one lady?

43. A committee of four men and three women is to be formed from among six men and four women. If a particular woman, W1, refuses to be on the committee with another woman, W2, in how many ways can the committee be formed?

44. How many words can be formed from the letters of the word “SUPERANNUATION” if the vowels are always written together and all the same letters cannot be written together?

45. The manager of a football team of eleven players wants to take photographs of his team, six players at a time. The manager can choose from out of four renowned photographers to take the photographs. In how many ways can the photographs be taken if the Captain and the Vice-Captain of the team are always included in the photographs? In how many ways can the photographs be taken if the Captain and the Vice-Captain of the team are never included in the photographs?

46. What is the number of words that can be formed by using the letters A, B, C, D, E and F, taken three at a time, if each word contains at least one vowel?

47. How many five digit numbers are there containing exactly one 3.

48. A store carries four styles of pants. For each type, there are ten different possible waist sizes, six different pants lengths and four colour choices. How many different types of pants could the store have?

49. How many ways are there to pick two different cards from a deck of 52 cards such that:
a. the first card is an Ace and the second is not a Queen?
b. the first card is a spade and the second is not a Queen?

50. How many ways are there to roll two dice so that the sum of the values on the upper faces is divisible by three?

51. How many four-letter words are there if the first and the last letters are vowels?

52. How many five digit numbers are there that are the same when the order of their digits is reversed?

53. How many different outcomes are possible when a pair of dice, one red and the other white, is rolled twice?

54. How many different license plates involving three letters and three digits are there if the three letters appear together, either at the beginning or at the end of the license?

55. How many different five letter sequences can be made by using the letters A. B, C and D such that the sequence does not include the word BAD--?

56. A joint student – teacher committee of 5 members is to be formed from among 4 teachers, 3 male students and 5 female students. How many different committees can be formed if the committee must consist at least 2 teachers, 1 male student and 2 female students?

57. If ${}^{10}P_r$ = 604800 and ${}^{10}C_r$ = 120, find r.

58. There are 8 different locks, with exactly one key for each lock. All the keys have been mixed up. What is the maximum number of trials required in order to determine which key belongs to which lock?

59. There are 14 vacancies for clerks in a certain office. 20 applications are received for the vacancies. In how many ways can the clerks be appointed? How many times may a particular candidate be selected?

60. A student is allowed to select at the most X books from a collection of (2X + 1) books. If the total number of ways in which he can select a book is 63, find the value of X.

ANSWERS

1. 360	**2.** 360, 240	**3.** 43200	**4.** 72	**5.** 6720	
6. 15	**7.** 1140, 231	**8.** 330, 462	**9.** 364	**10.** 110	
11. 100, 180	**12.** 360	**13.** 9^6	**14.** 23	**15.** 31	
16. 60		**17.** 18	**18.** 5	**19.** 840	**20.** 98
21. 5780	**22.** 480	**23.** 6	**24.** (45! X 3!) / (10! X 15! X 20!)		
25. 7	**26.** pn	**27.** 3374	**28** 1024x5!	**29.** 105	
30. 315	**31.** 12 x 13!	**32.** 7! x $^8P_5$	**33.** 89	**34.** 1652	
35. 10! – (5! X 6!)	**36.** 33!/(3! X 11! X 11! X 11!)		**37.** (6! X 9!)/96	**38.** 170	
39. 14400	**40.** 9	**41.** 126	**42.** 246	**43.** 30	
44. {(8!7!)/4!} – (6! X 5!)		**45.** ($^9C_4$ x 6! X 4), ($^9P_6$ x 4)		**46.** 96	
47. 29889	**48.** 960	**49.** 188, 612	**50.** 12	**51.** 16900	
52. 900	**53.** 64	**54.** 2 x 263 x 103	**55.** 1008	**56.** 180	
57. 7	**58.** 28	**59.** $^{20}C_{14}$, $^{19}C_{13}$	**60.** 3		

Concepts 11

Binomial Theorem

The expansion of the expression $(x + a)^n$ where n is any positive integral index is given as
$x^n + {}^nc_1\, x^{(n-1)} a + {}^nc_2\, x^{(n-2)} a^2 + {}^nc_3\, x^{(n-3)} a^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ {}^nc_n\, a^n$.
the expansion written above is known as binomial expansion. In the expansion the coefficient of second term is ${}^nc_1$, the coefficient of third term is ${}^nc_2$ thus the general term of the expansion which is $(r+1)^{th}$ term is given as

$${}^nc_r\, x^{n-r} a^r \text{ or } \frac{(n)(n-1)(n-2)\ldots\ldots(n-r+1)}{\lfloor r}\, x^{n-r} a^r.$$

In applying this formula to any particular case, it should be observed that the index of a is the same as the suffix of C, and that the sum of the indices of x and a is n.
Let us take an example find the fifth term of $(a + 2x)^6$ fifth term means $(4 + 1)^{th}$ term
Therefore the fifth term is ${}^6c_4\, a^2(2x)^4 = 240a^2x^4$

The expansion of the expression $(x-a)^n$ is also same as that of $(x+a)^n$ but with alternate +ive and -ive sign i.e $x^n - {}^nc_1\, x^{(n-1)} a + {}^nc_2\, x^{(n-2)} a^2 - {}^nc_3\, x^{(n-3)} a^3 + \ldots\ldots\ldots\ldots\ldots\ldots\ {}^nc_n\, a^n$. The general term of the expansion is given as $T_{r+1} = (-1^r)\ {}^nc_r\, x^{n-r} a^r$
or

$$(-1^r)\, \frac{(n)(n-1)(n-2)\ldots\ldots(n-r+1)}{\lfloor r}\, x^{n-r} a^r.$$

To Find The Greatest Term in the Expansion Of $(x + a)^n$.

We have $(x + a)^n = x^n (1 + a/x)^n$ therefore, since x^n multiplies every term in above expansion, therefore it will be sufficient to find the greatest term in this later expansion.

Let the r^{th} and $(r + 1)^{th}$ be any two consecutive terms. the $(r + 1)^{th}$ term is obtained by multiplying the r^{th} term by $(n- r + 1)/ r .(a/x)$ that can be written $\{(n +1)/r - 1\} (a/x)$.

$T_{r+1}/ T_r = \{(n +1)/r - 1\} (a/x)$
The factor $\{(n + 1)/ r\} - 1$ decreases as r increases; hence the $(r + 1)^{th}$ term is not always greater than the r^{th} term , but only until $\{(n +1)/r -1)\} (a/x)$ becomes equal to 1, or less than 1.
Now solving the inequality $\{(n+1)/r - 1\} (a/x) >1$,

i. if we get r as an integer p then $(p + 1)^{th}$ and p^{th} terms are equal and greater than any other term
ii. if r is not an integer, denote its integral part with p and $(p + 1)^{th}$ will be the greatest term.

Example
if x = 1/3, find the greatest term in the expansion $(1 + 4x)^8$
The inequality will be given as $\{(8+1)/r - 1\} (4/3) >1$,
Solving this we will have $36 - 4r > 3r$
$36 >7r$, integer value of the r is 5 so the greatest term will be sixth term. And its value will be

${}^8c_5 \times (4/3)^5 = 57344 / 243$.

EXERCISE 11

1. Find the expansion of $\{a + (a^2 - 1)^{1/2}\}^7 + \{a - (a^2 - 1)^{1/2}\}^7$

2. $\{ x^3 + (1/x^6)\}^{36}$ which term in the expansion will be free of x.

3. $(x -3)^7 + (x + 3)^7$ what is coefficient of x^4 in the expression.

4. Find the coefficient of x^{18} in $(ax^4 - bx)^9$.

5. What is the coefficient of x^r in the expansion of $(x + 1/x)^n$.

6. find the value of $(x + \sqrt{2})^4 + (x -\sqrt{2})^4$ if x = 4.

7. Find the greatest term of the following expansion.
   i. $(x + y)^{30}$ if x =3 , y = 2
   ii. $(2a + b)^{14}$ if b = 5, a = 2
   iii. $(3 + 2x)^{15}$ if x = 3 / 2

8. What is the middle term of the expansion $(1 + x)^{2n}$.

9. Prove that ${}^nC_1 + 2.\, {}^nC_2 + 3.\, {}^nC_3 + 4\ {}^nC_4$ ……………n ${}^{ns}C_n = n (2^n - 1)$.

10. Find the value of (x , y, n) if the condition given the 2nd , 3rd, 4th terms are 240, 720, 1080 of the expansion $(x + y)^n$.

ANSWERS

2. 13th **3**. 0 **4.** ${}^9C_6\, a^3\, b^6$

5. $n! / \{ [(n - r) / 2] ! . [(n + r) / 2] ! \}$ **6.** 904 **7.** (I) 13th (II) 9th (III) 8th and 9th

8. $(2n - 1)(2n - 3)(2n - 5)(2n - 7) ..5.3.1.\ 2^n / n !$ **10**. n = 5, x = 2 and y = 3

C o n c e p t s 12

Probability

Deterministic Experiment : is the one which gives a certain definite result. e.g. acid is added to a base.

Random Experiment : is the one which gives one or more results under identical conditions. e.g. a coin is tossed.

The set of all possible outcomes of a random experiment is known as the **sample space** and every outcome is a **sample point.**

An **event** is a subset of the sample space. An event defined by a singleton set is called an **elementary event** or a **simple event.** The event defined by an empty set is called an **impossible event.** Events are said to be **equally likely** if one does not happen more often than the other. Two or more events are said to be **mutually exclusive** these events cannot occur simultaneously. Two or more events are said to be **compatible** if they can occur simultaneously. Two or more events are said to be **independent** if the happening or non happening of any event does not affect the happening of others.

Probability : If A is an event of the sample space S, then

Probability of A = P(A) = Number of cases favorable for A / Total number of cases

- $0 \le P(A) \le 1$
- $P(A) + P(A') = 1$
- If 'a' cases are favorable to A and 'b' case are not favorable to A then
  $P(A) = a/(a + b)$ $P(A') = b/(a + b)$
  We say that the **odds in favor of A are a : b i.e. a to b** and **odds against A are b : a i.e. b to a**

Thus, if an event can happen in 'a' ways and fail in 'b' ways, and each of these is equally likely, the probability or the chance of its happening is a/(a + b) and that of its failing is b/(a + b). The chance of happening of an event 'a' is also stated as "the odds are a to b in favor of the event, or b to a against the event".
If 'p' is the probability of happening of an event then '(1- p)' is the probability of its not happening.

Binomial Distribution : If n trials are performed under the same condition and probability of success in each trial is p and q = 1 - p then the probability of exactly r successes in n trials is :

$$\boxed{P(r) = {}^nC_r\, p^r\, q^{n-r}}$$

Geometric Theorem : If an experiment is performed indefinitely under the same conditions then the probability of first success in the n^{th} trial is given by :

$$\boxed{P(n) = pq^{n-1}}$$

where p = probability of success in each experiment and q = 1 - p

If p_1 and p_2 are the chances that the two independent events will happen separately then the chance that they will both happen is $p_1 \times p_2$. Also, the chance that the first event happens and the second fails is $p_1 \times (1-p_2)$ and so on.

If an event can happen in two ways which are mutually exclusive and if p_1 and p_2 are the two respective probabilities then the probability that it will happen in some one of these ways is $p_1 + p_2$.

Conditional Probability :

Consider the three events. Two dies are tossed then -

A : Sum of the scores is even.

B : Sum of the scores is less than 6

Writing the sample space it can be seen easily that the probability of event A is ½ and that of event B is 5/18. But if it is given that the event A has already occurred then the probability that sum of the scores is less than 6 is 2/9.

The event B is same but it's probability changes accordingly A has occurred or not. Thus the probability of an event B when event A has already occurred is denoted by **P(B|A)** and is defined by

$$\boxed{\mathbf{P(B|A)} = \frac{P(A \cap B)}{P(A)}}$$

Ex. What is the chance of drawing an ace from a deck of cards ?

Sol. There being 52 cards in a deck of cards, the total number of possible outcomes of drawing a card is 52. Since there are 4 aces in a pack of cards, the total number of favorable outcomes is 4.The chance or probability of drawing an ace from the given pack is the ratio of total number of *favorable* outcomes to the total number of *possible* outcomes.
Which in this case is: 4/52 = 0.077

Ex. A three digit number is to be formed by using the digits 1, 2, ..., 9. What is the probability that the digit formed is greater than 500, if (a) repetition is not allowed and (b) repetition is allowed?

Sol. (a) If the repetitions are not allowed, the total number of possibilities of forming a three digit number is ${}^9P_3 = 504$.
Number of ways of forming a number greater than 500 is $5 \times {}^8P_2 = 280$.
Probability that the three digit no. is greater than 500 is 280/504 = 0.55

(b). If the repetitions are allowed, the total number of ways in which a three digit number is formed is $9^3 = 729$.
Number of ways of forming a number greater than 500 is $5 \times 9 \times 9 = 405$.
The required probability is 405/729 = 0.55.

Ex. Harshad and Amit throw with two dice. If Harshad throws 10, what is Amit's chance of throwing a higher number ?

Sol. Total number of outcomes of throwing two dice is 6 × 6 = 36.
Amit must throw either 11 or 12. This can be done in 3 ways namely
(6 + 5, 5 + 6, 6 + 6).
Thus Amit's chance of throwing a higher number is 3/36 or 1/12.

Ex. A bag contains 4 red and 7 green balls. If three balls are drawn from the bag, replaced and once again three balls are drawn from the bag, what is the probability of obtaining 3 red balls on first drawing and 3 green balls on second drawing ?

Sol. Total no. of ways of drawing 3 balls from given 11 balls is ${}^{11}C_3 = 165$
Number of ways of drawing 3 red balls is ${}^4C_3 = 4$
Number of ways of drawing 3 green balls is ${}^7C_3 = 35$
Probability of 3 red balls on first drawing = 4/165 and the
probability of 3 green balls on second drawing = 35/165
Thus the chance that both events happen = 4/165 × 35/165 = 140/27225 = 28/5445.

Ex. In the situation explained in above problem, what is the required probability, if the balls are not replaced before second drawing ?

Sol. Total no. of ways of drawing 3 balls from given 11 balls is $^{11}C_3 = 165$
Number of ways of drawing 3 red balls is $^4C_3 = 4$
Probability of 3 red balls on first drawing = 4/165
Having drawn 3 red balls, the bag will contain 8 balls (1 red & 7 green).
Total no. of ways of drawing 3 balls from given 8 balls is $^8C_3 = 56$
Number of ways of drawing 3 green balls is $^7C_3 = 35$
Probability of 3 green balls on second drawing = 35/56

Thus the chance that both events happen = 4/165 × 35/56 = 140/9240 = 7/462.

Ex. The odds against a certain event are 9 to 7 and against a certain another event are 3 to 2. If the two events are independent of each other, find the probability of occurrence of at least one of the events.

Sol. The chance that A will not happen is 9/16
The chance that B will not happen is 3/5
Thus, the chance that both will not happen = 27/80
Chance that at least one will happen = 1 - (27/80) = 53/80

Ex. One number is selected at random from first twenty five natural numbers. What is the chance that it is a multiple of either 5 or 7 ?

Sol. The probability that the number is a multiple of 5 is 5/25
The probability that the number is a multiple of 7 is 3/25
Neither 5 nor 7 has a common multiple between 1 and 25. Thus these two events are mutually exclusive. Chance that the selected number is multiple of 5 or 7 is (5+3)/25 = 8/25

Ex. Ten horses are running in a race, the chances that A will win are 30%, that B will win are 20% and that C will win are 10%. What is the possibility that one of them will win?

Sol. We have P(A) x P(B') x P(C') + P(A') x P(B) x P(C') + P(A') x P(B') x P(C) = (0.3 x 0.8 x 0.9) + (0.7 x 0.2 x 0.9) + (0.7 x 0.8 x 0.1) = 0.398.

Ex. A bag contains 5 green apples and 7 red apples. If two apples are drawn from the bag, what is the probability that one is red and the other is green ?

Sol. The required probability is $(^5C_1 \times {^7C_1})/{^{12}C_2} = 35/66$.

Ex.. The probability of A solving a problem is 1/4 and that of B solving it is 1/5. If they try independently what is the probability that (a) Problem is solved (b) Problem is not solved

Sol. (a) Probability of A and B both solving the problem = (1/4) × (1/5) = (1/20)
The probability of either A or B solving the problem is given by P(A) . P(B') + P(B) . P(A') + P(A). P(B) = (1/4) .(4/5) + (3/4).(1/5) + (1/4).(1/5) = 8/20 = 2/5
(b) The problem is not solved if neither A nor B solves it. The probability of which is given by (3/4) × (4/5) = 3/5

EXERCISE 12

1. Find the chance of throwing more than 15 in one throw with three dice.

2. In a single throw with two dice, find the chances of throwing (a) five, (b) six

3. If four coins are tossed, what are the chances that there will be two heads and two tails ?

4. A has three shares in a lottery containing 3 prizes and 9 blanks; B has two shares in a lottery containing 2 prizes and 6 blanks. Compare their chances of success.

5. The letters forming the name CLINTON are placed at random in a row. What is the chance that the two vowels come together ?

6. A bag contains 6 different white and 9 different black balls. If three balls are drawn at random, find the probability that all of them are black.

7. Two dice are thrown, find the probability that the number obtained is a (a) perfect square (b) multiple of 3

8. Two dice are thrown. Find the probability that exactly one die shows six.

9. A bag contains 3 different white, 4 different black and 2 different red balls. Two balls are chosen at random. What is the probability that (a) one white ball and one red ball is chosen, (b) no white ball is chosen, (c) exactly one black ball is chosen.

10. From a collection of 12 bulbs of which 6 are defective 3 bulbs are chosen at random for three sockets in a room. Find the probability that the room is lighted if one bulb is sufficient to light the room.

11. A's chance of winning a single game against B is 3/4. Find the chance that in a series of five games with B, A wins exactly 3 games.

12. What is the chance that a leap year selected at random will have 53 Sundays ?

13. A university has to select an examiner from a list of 50 persons. 20 are women, 30 men, 10 know Hindi, 40 do not, 15 of them are teachers and 35 are not. What is the probability that the University selects a Hindi knowing woman teacher ?

14. The probabilities that three students, Rahul , Shashank, and Charudatta will solve a problem in arithmetic are 0.2, 0.7, and 0.6 respectively. Find the probability that the problem will not be solved.

15. A bag contains 5 blue and an unknown number of red balls. None of them are similar. When two balls are drawn at random, the probability of both being blue is 5 / 14. Find the number of red balls.

16. Find the probability of getting the same digit on the top face of two dice when rolled.

17. 5 boys and 3 girls are to be seated on chairs arranged in a row. If the arrangement is made at random, find the probability that no two girls will be seated next to one another.

18. It is known that at noon the sun is hidden by clouds on an average of two days out of every three. Find the probability that at noon on at least four out of five days the sun will be shining.

19. If on an average 1 vessel in every 10 is wrecked, find the chance that out of 5 vessels expected, at least 4 will arrive safely.

20. One MBA institute offers 1 subject in marketing, and 3 in finance, the second offers 2 in marketing and 4 in finance, and the third offers 3 in marketing and 1 in finance. If a student can take admission to either marketing or finance, but not both, and at an institute selected at random, find the chance that he takes admission in the marketing stream.

21. The daily production of a textile mill consists of three colours of yarn: Red – 40%, Blue – 35% and Green – 25%. It is seen that 2%, 5% and 7% of the Red, Blue and Green yarns are defective. What is the probability that the defective yarn selected is Green in colour? What is the probability that the yarn chosen is a non-defective red yarn?

22. A basket contains 7 white, 11 blue and 9 black balls. Five balls are removed from the basket. What is the probability that out of these removed balls, there are at least two white balls?

23. Salil has 9 five rupee coins with him. What is the probability that he obtains at least five tails when he tosses each coin once?

24. Four digit odd numbers are formed by using the digits from 0 to 9, such that 3 is always followed by 5. What is the probability that the number formed is 2695, if repetitions are not allowed?

25. A score of 5 or 9 is to be obtained from a throw of two dice. The probability is greater in which case?

26. A person forgot the last three digits of a telephone number, but remembered that these three were different digits. He dialled the last three digits at random. What is the probability that he has dialled the correct telephone number?

27. A box contains 15 distinct balls. Of these balls, five are each of a different colour and the rest are all white. Five balls are drawn at random from the box. What is the probability that four balls are coloured and one is white?

28. Two students, A and B, are given a problem to solve. The chance that A solves the problem is 1/5 and that of B is 1/6. What is the probability that:
    a. the problem is solved.
    b. B solves the problem, but A does not.
    c. the problem is not solved.

29. What is the probability that a card drawn at random from a fair pack of 52 cards is either a King or a Spade?

30. Two fair dice are thrown. What is the probability that the number of dots on the first die exceeds 3 and that on the second exceeds 4?

31. A box contains 6 white and 3 black balls. Another box contains 5 white and 4 black balls. A ball is drawn from the first box and placed in the second box. A ball is now drawn at random from the first box. What is the probability that the ball drawn is white?

32. A bag contains 7 white and 9 blue balls. If two balls are drawn at random, what is the chance that one ball is white and the other is blue?

33. A bag contains 5 red and 7 white balls. Another bag contains 3 red and 12 white balls. What is the probability that a ball drawn at random from one or the other bag is red?

34. A player rolls a biased die and receives the same number of rupees as the number of dots on the face that turns up. Die is such that it is twice likely shown an even number than an odd number. What should the player pay for each roll if he wants to make a profit of Rs. 2/3 per throw of the die in the long run?

35. Seventy college teachers are surveyed as to their possession of colour T.V.,VCR and tape recorder. Of them twenty own only colour T.V., ten own only VCR and fifteen own only tape recorder. Five teachers own all the three. Find the probability that a college teacher selected at random possess only two items.

36. A bag contains seven white and fourteen black balls. What is the probability that two balls selected at random are of different colours?

37. A person was dialling a telephone. He forgot the last three digits of the six digit telephone number but remembered that the number formed by the last three digits in the same order was a perfect square. What is the probability that he dialled a right number?

38. The probabilities of the three students A, B and C solving a certain problem are 0.5, 0.2 and 0.3 respectively. Find the probability that the problem is not solved.

39. The probability of winning a lottery is 0.25. If a person buys four lottery tickets, what are the chances that he will win on at least one of the four tickets?

40. Four persons entered the lift cabin on the ground floor of a seven-storied house. Suppose that each of them independently and with equal probability can leave the cabin at any floor beginning with the first. Find out the probability of all four persons leaving at different floors.

ANSWERS

1. 7/216	**2**. 1/9, 5/36	**3**. 3/8	**4**. 952 : 715
5. 2/7	**6**. 12/65	**7**. 7/36, 1/3	**8**. 5/18
9. 1/6, 5/12, 5/9	**10**.10/11	**11**. 135/512	**12**. 2/7
13. 3/125	**14.** 0.096	**15.** 3.	**16.** 1/6
17. 5/14	**18.** 11/243	**19.** 45927/50000	**20.** 4/9.
21. 35/86, 49/125.	**22.** $\{({}^7C_2 \times {}^{20}C_3) + ({}^7C_3 \times {}^{20}C_2) + ({}^7C_4 \times {}^{20}C_1) + ({}^7C_5)\}/{}^{27}C_5$.		
23. ½.	**24.** 1/1264.	**25.** Same.	**26.** 1/720.
27. 50/3003.	**28.** 1/3, 2/15, 2/3.	**29.** 4/13.	**30.** 1/6.
31. 2/3.	**32.** 21/40.	**33.** 37/120.	**34.** Rs. 3.
35. 2/7.	**36.** 7/15.	**37.** 1/32.	**38.** 0.28.
39. 175/256.	**40.** ${}^6P_4 / 6^4$.		

Solutions

Concepts 1: EQUATIONS.

1. $5x - 17 + 3x - 5 = 6x - 7 - 8x + 115$
   $\therefore 8x - 22 = 108 - 2x$
   $\therefore 10x = 130.$
   $\therefore x = 13.$

2. $x - [3 + \{x-(3 + x)\}] = 5$
   $\therefore x - [3 + \{x - 3 - x\}] = 5$
   $\therefore x - (3 - 3) = 5$
   $\therefore x = 5$

3. Expanding, we get, $x^2 + 2x + 1 + 2(x^2 + 6x + 9) = 3x^2 + 6x + 35$
   $\therefore x^2 + 2x + 1 + 2x^2 + 12x + 18 = 3x^2 + 6x + 35$
   $\therefore 8x = 16 \therefore x = 2$

4. $(x^2 + 3x + 2)(x+6) = x^3 + 9x^2 + 28x - 4$
   $\therefore x^3 + 3x^2 + 2x + 6x^2 + 18x + 12 = x^3 + 9x^2 + 28x - 4$
   $\therefore 16 = 8x \therefore x = 2.$

5. $(3a + 1)(2a - 7) = 6(a - 3)^2 + 7$
   $\therefore (6a^2 - 21a + 2a - 7) = 6(a^2 - 6a + 9) + 7$
   $\therefore 6a^2 - 19a - 7 = 6a^2 - 36a + 54 + 7$
   $\therefore 17a = 68 \therefore a = 4.$

6. Multiplying the first equation by 3, we get, $15/x + 18/y = 9$
   Subtracting the second equation from the above, we get,
   $15/y = 5 . \therefore y = 3$
   Substituting $y = 3$ in the first equation, we get,
   $5/x + 6/y = 3 \quad \therefore 5/x + 2 = 3 \quad \therefore 5/x = 1$
   $\therefore x = 5$
   Hence, $x = 5$ and $y = 3$.

7. If x is the number in the unit's place and y in the ten's place, the original number is $10y + x$ and the number obtained on reversing the digits is $10x + y$.
   The sum of numbers is $(10x + y) + (10y + x) = 11x + 11y = 11(x+y)$ which is stated to be 110.
   Thus, $11(x + y) = 110.$
   $\therefore x + y = 10.$
   Also, difference of digits $= (y - x) = 6.$
   Adding the above two equations, we get, $2y = 16 \therefore y = 8$
   $x = 2.$
   Hence the number is 28, or 82.

8. If p is the cost of a pig and g is the cost of a goat, we have the following relations ,
   $6p + 7g = 2500$.... (1) and $11p + 13g = 4610$....(2)
   Multiplying (1) by 11 and (2) by 6, we have,
   $66p + 77g = 27500$ ...(3) and $66p + 78g = 27660$..(4)
   Subtracting (3) from (4), we get, $g = 160.$
   Substituting $g = 160$ in (1), we get, $p = 230.$
   Thus a pig costs Rs. 230, and a goat costs Rs. 160.

9. If x is the unit's place digit, and y is the hundred's place digit, the number is y0x.
$\therefore y + 0 + x = 11$...(1)
The new number on reversing the digits is x0y
We have, difference of numbers $= (100x + 0 + y) - (100y + 0 + x) = 495$.
$99x - 99y = 495$. $\therefore x - y = 5$....(2)
adding (1) and (2), we have, $2x = 16$, i.e., $x = 8$
Substituting, $x = 8$, in (1) we get, $y = 3$.
Hence the number is 308.

10. Let the numerator be x and the denominator y.
$(x - 1)/(y+2) = 1/2$, $\therefore 2x - 2 = y + 2$ $\therefore 2x - y = 4$...(1)
also, $(x - 7)/(y - 2) = 1/3$ $\therefore 3x - 21 = y - 2$, $\therefore 3x - y = 19$...(2)
Subtracting (2) from (1), we get, $x = 15$.
Substituting in (2), we get, $y = 26$.
Hence, the fraction is 15/26

11 (a) $x^2 - 8x + 16 = 0$. We need to find two numbers whose product is 16 and sum is -8.
The factors are -4 and -4.
$\therefore x^2 - 4x - 4x + 16 = 0$.
$\therefore x(x - 4) - 4(x - 4) = 0$
$\therefore (x - 4) = 0$.
$\therefore x = 4$.

11 (b) $12x^2 - 11x + 2 = 0$.
Since 12x2 = 24, we find two factors such that their sum is -11 and product is 24.
The factors are -3 and -8.
$\therefore 12x^2 - 3x - 8x + 2 = 0$.
$\therefore 3x(4x - 1) - 2(4x - 1) = 0$.
$\therefore (3x - 2)(4x - 1) = 0$.
$\therefore 3x - 2 = 0$ or $4x - 1 = 0$.
$\therefore x = 2/3$ or $x = 1/4$.

11 (c). $7x - 13x + 3 = 0$.

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4.3.7}}{2.7}$$

$$x = \frac{13 \pm \sqrt{85}}{14}$$

12a. We have, $x^2 - (1/3 - 5/2)x + (1/3)(-5/2) = 0$
$\therefore 6x^2 + 13x - 5 = 0$.

12b. $x^2 - (p/q + q/p)x + (p/q)(q/p) = 0$.
Simplifying, $pqx^2 - (p^2 + q^2)x + pq = 0$.

12c. We have Sum of the roots = $(3 + 5\sqrt{-1})/2 + (3 - 5\sqrt{-1})/2 = 3$. and
Product of the roots $(3 + 5\sqrt{-1})/2)\,(3 - 5\sqrt{-1})/2) = 17/2$
Hence the required equation is $x^2 - 3x + 17/2 = 0$. i.e. $2x^2 - 6x + 17 = 0$.

13. $p + q = -b$; $pq = c$. $\therefore (p + q)^2 = b^2$
$(p - q)^2 = (p + q)^2 - 4pq = b^2 - 4c$
$\therefore$ Sum of the roots of the required equation $= (p+q)^2 + (p-q)^2 = 2b^2 - 4c$
and Product of the roots of the required equation $= (p+q)^2\ (p-q)^2 = b^2(b^2 - 4c)$
$\therefore$Required equation $= x^2 - 2(b^2 - 2c)x + b^2(b^2 - 4c) = 0$.

14 (1).Sum of the roots = - b/a = $\alpha + (-\alpha) = 0 \Rightarrow b = 0$
Hence the roots are equal in magnitude and opposite in sign when b = 0.
(2). Product of the roots = c/a = $\alpha \times (1/\alpha) = 1 \Rightarrow a = c$.

15. Let α be the common root. $\therefore$ α satisfies both equations .
$\therefore\ a\alpha^2 + b\alpha + c = 0$...(1)
and $p\alpha^2 + q\alpha + r = 0$....(2)
Multiplying (1) by p, and (2) by a, and subtracting (2) from (1), we get,
$(b p - a q)\alpha + (c p - a r) = 0$.
$\therefore$ The common root = $\alpha = (c p - a r) / (a q - b p)$

16. $x = 3 + \sqrt{2} \Rightarrow (x - 3) = \sqrt{2}$.
Squaring both sides, we get $x^2 - 6x + 9 = 2 \Rightarrow x^2 - 6x + 7 = 0$
$\therefore$ m = 7. and the second root = $3 - \sqrt{2}$, as irrational roots always occur in conjugate pairs.

17 (a)$b^2 - 4ac = (-14)^2 - 4(3)(20) = -44$.
Since $b^2 - 4ac < 0$, the roots are complex and complimentary.

(b)Simplifying, we have, $15x^2 - 17x + 3 = 0$.
$\therefore b^2 - 4ac = (-17)^2 - 4(3)(15) = 109$.
Since $b^2 - 4ac$ is +ve, but not a perfect square, the roots are irrational and unequal.

18. For the roots to be real and equal, $b^2 - 4ac = 0$.
i.e. $(-2h)^2 - 4(5)(5) = 0$.
$\therefore h = \pm\sqrt{25} = \pm 5$.

19. $x + y = 11,\ x^3 + y^3 = 407$.
$x^3 + y^3 = (x + y)^3 - 3xy(x + y)$
$\therefore 407 = (11)^3 - 3xy(11)$
$\therefore xy = 28$.
From the given data, $y = 11 - x$.
$\therefore x(11 - x) = 28$.
Solving the above quadratic, we get, $x = 4, 7$ and $y = 7, 4$.

20. $1/x + 1/y = 2 \quad (x + y)/xy = 2$
Substituting $x + y = 2$, in the above equation, we get,
$xy = 1$. i.e. $x = 1/y$
Substituting $x = 1/y$ in the first equation and solving the quadratic equation, we get,
$x = 1, y = 1$.

21. From the first equation, we have, $(x^2 + y^2) / x^2y^2 = 61/900$
$\therefore x^2 + y^2 = 61$, since $xy = 30$.
Substituting $x = 30/y$ in the first equation and solving the quadratic for y^2, we get,
$y = \pm 5$, or $y = \pm 6$.
Accordingly, $x = \pm 6$, $x = \pm 5$.

22. Solving as in problem no. 21, we get x =3 or 4 and y = 4 or 3.

23. Substituting $x = y + 3$ in the first equation, and solving the quadratic obtained in y, we get,
$y = -11$, or $y = 8$.
Accordingly, $x = -8$ or $x = 11$.

24. $x^3 - y^3 = (x - y)^3 + 3xy(x - y)$
$= (x - y)[(x - y)^2 + 3xy]$
$= (x - y) (x^2 + xy + y^2)$
$\therefore 56 = (x - y) (28)$
$\therefore x - y = 2$
$\therefore x = y + 2$.

Substituting the above in the first equation, we get,
$(y + 2)^3 - y^3 = 56$.
Solve the above cubic equation to get $y = -4$ or $y = 2$.
Accordingly, $x = -2$ or $x = 4$.

25. We get $x = 10 + y$
Substituting in the second equation and solving the quadratic in y, we get,
$y = -12$ or $y = 2$.
Accordingly, $x = -2$ or $x = 12$

26. Let x be the smaller number. $\therefore$ The larger number is $x + 9$.
The sum $= x + x + 9 = 29$, i.e. $2x + 9 = 29$. $\therefore$ $2x = 20$. $\therefore$ $x = 10$.
$\therefore$ The required numbers are 10 and $10 + 9 = 19$.

27. Let the man travel x km on horseback. $\therefore$ He travels 2x km by car.
$\therefore$ Total distance traveled $= 1 + x + 2x = 10$
$\therefore$ $3x + 1 = 10$. i.e. $3x = 9$. Hence $x = 3$.
$\therefore$ The man travels 3 km on horseback.

28. Let A have Rs. x. $\therefore$ B has Rs. $x + 5$, and C has $x + 5 + 10$, i.e. Rs. $x + 15$.
Together they have $x + x + 5 + x + 15 = 38$
$\therefore$ $3x + 20 = 38$. $\therefore$ $x = 6$. B has Rs. 11, and C has Rs. 21.

29. Let the required number be x. According to the condition,
$(x + 5) (x + 35) = (x + 15)^2$
$\therefore$ $x^2 + 40x + 175 = x^2 + 30x + 225$.
$\therefore$ $10x = 50$. $x = 5$.

30. If x is the required number, we have,
$7x/18 + 5x /12 + x/16 = 125$ $\therefore$ $(56x + 60x + 9x)/144 = 125$ $\therefore$ $x = 125 (144)/ 125 = 144$.
The required number is 144.

31. Let A have Rs. x. $\therefore$ B has Rs.$(2/5)x$, and C has $(7/9) (2/5)x$, i.e. $(14/45)x$.
A, B, C together have : $x + 2x/5 + 14x / 45 = 770$. $\therefore$ $(45x + 18x + 14x)/45 = 770$
$\therefore$ $x = 770 (45) /77 = 450$ $\therefore$ B has Rs. 180, and C has Rs. 140.

32. Let w be the width and l be the length.
$\therefore$ From the first condition, $w = (2/3)l$ ... (1)
The room will be square after change in dimensions. $\therefore$ the width and the length will become equal.
$\therefore$ $w + 3 = l - 3$ i.e. $w = l - 6$...(2)
From (1) and (2), we get, $l - 6 = (2/3) l$
$\therefore$ $l = 18$. and $w = 12$.

33. Equalising the denominator, we get, $3x + 4 (x - 5)/ 12 = 10$. $\therefore$ $3x + 4x - 20 = 12 (10) = 120$.
$\therefore$ $7x = 120 + 20 = 140$. $\therefore$ $x = 20$.

34. $(x + 19)/5 - x/5 = 3$. $\therefore$ $x + 19 - x = 3 (5)$.
$\therefore$ $19 = 15$, which is absurd.
Thus the above equation has no solution.

35. $\therefore$ $(x + 5)/6 - (x + 1)/9 - (x + 3)/4 = 0$
$\therefore$ Equalising denominators, we have, $6 (x + 5) -4 (x + 1) - 9 (x + 3) =0$
$\therefore$ $6x + 30 - 4x - 4 - 9x -27 = 0$. $\therefore$ $- 7x = 1$ $\therefore$ $x = - (1/7)$

36. Equalising denominators, $28(x+2) - 18(x-7) = 12(21)$
$\therefore 28x + 56 - 18x + 126 = 252$
$\therefore 10x = 252 - 126 - 56 = 70 \therefore x = 7.$

37. $\therefore (x - 8 + 23 - x)/5 + [7(4+x) + 4(x-1)]/28 = 7.$
$\therefore (24 + 11x)/28 = 7 - 3 \quad \therefore 24 + 11x = 4(28)$

$\therefore x = (112 - 24)/11 = 8.$

38. Let the shares be p, q, r, and s respectively. From the conditions, we have the following equations,
$p = 2r$, $q = 3s$, $r + s = p - 50$, and $p + q + r + s = 290$.
We have, $r + s = 2r - 50 \therefore s = r - 50$, since $p = 2r$
Also $q = 3s = 3(r - 50) = 3r - 150$.
Substituting all values in terms of r, in the total equation, we have,
$2r + 3r - 150 + r + r - 50 = 290.$
$\therefore 7r = 490. \therefore r = 70.$
$p = 140$, $q = 60$, $s = 20$.

39. If a and b are the respective present ages of A and B,
$\frac{1}{11}(a) - 2 = \frac{1}{7}(b)$ and $2b = a - 13$, i.e. $a = 2b + 13$.
$\therefore [7(2b + 13) - 154 - 11b]/77 = 0 \quad \therefore 3b - 63 = 0 \quad \therefore b = 21.$
$a = 2 \times 21 + 13 = 55.$

40. Let a be the number of one-rupee coins and b be the number of five-paise coins.
Amount = $(100a + 5b)$ Ps.
Amount after first change : $5a + 100b = 100a + 5b + 570$
$\therefore -95a + 95b = 570. \therefore -a + b = 6$.... (1)
Amount after second change : $50a + 10b = 100a + 5b - 195$
$\therefore -50a + 5b = -195 \quad \therefore 10a - b = 39$... (2)
Adding (1) and (2), we get, $9a = 45 \quad \therefore a = 5. \quad \therefore b = 6 + a = 11.$
$\therefore$ Amount : $100(5) + 11(5) = 5.55$ rupees.

41. Let the number of white and black balls be w and b respectively. $\therefore$ Total number = $w + b$
$(w/2) = (b/3)$ and $2(w + b) - 4 = 3b$.
$2w = (4/3)b$ and $2w = b + 4$
$\therefore b + 4 = (4/3)b$
$\therefore b = 12$. and $w = 8$.

42. Let m Rs. and b Rs. be the wages of a man and a boy respectively. From the given conditions,
$10m + 8b = 37$and $4m - 1 = 6b$ i.e. $4m - 6b = 1$
Multiplying the first equation by 3 and the second by 4, we get,
$30m + 24b = 111$ and $16m - 24b = 4$.
Adding the above, we get, : $46m = 115. \therefore m = 2.5$
Substituting in any of the equations, we get $b = 1.5$

43. Let the son be x years old. $\therefore$ The father is 5x years old.
From the given condition, $x^2 + (5x)^2 = 2106.$
$\therefore 26x^2 = 2106. \quad x^2 = 2106/26 = 81.$
$\therefore x = \pm 9.$
Since age cannot be negative, the son is 9 years old and the father is 45.

44. Let the numbers be x and x + 1. From the given condition,
$1/x + 1/(x+1) = 15/56 \quad [x + 1 + x]/[x(x+1)] = 15/56$
$\therefore 112x + 56 = 15x^2 + 15x \quad \therefore 15x^2 - 97x - 56 = 0.$
$\therefore 15x^2 - 105x + 8x - 56 = 0. \quad \therefore (15x + 8)(x - 7) = 0.$
$\therefore x = -(8/15)$, or $x = 7$.

Since the numbers are consecutive, x must be an integer. If x = 7, the next number is 8

45. The area of each tile becomes (200 / 128) i.e. 25/16 times due to an increase of 5 cm. which means that the dimension of each tile becomes 5/4 times x, i.e. increased by 25 %.
Thus, 5cm $\equiv$ 25%.
$\therefore$ The original length of the tile = 20 cm.
(* **N. B** *) : The above problem has been solved in an unusual manner. But the method is fast and ideal for typical problems which are to be solved under time pressure. You need to be very fast with percentages for such problems.

46. Let the man buy x m of cloth at y Rs. per m. From the conditions of the problem, we have,
$x.y = 50$ and $(x - 5)(y + 1) = 60$, since his profit is Rs. 10.
$\therefore xy - 5y + x - 5 = 60$ $\therefore 50 - 5y + x - 5 = 60$.
$\therefore x = 5y + 15$.
$\therefore (5y + 15)y = 50$, substituting for x.
$\therefore y^2 + 3y - 10 = 0$
$\therefore y = -5$ or $y = 2$.
Discarding the negative value, we get, y = 2, and x = 25.
Thus the man originally bought 25 m of cloth.

47. Let A and B have a and b cows respectively, and let x be the sum at which each sold his cows.
$\therefore$ The price each cow fetched is (x/a) in case of A and (x/ b) in case of B.
From the conditions of the problem,
$a.\ x/b = 3200$ ...(1). and $b.\ x/a = 2450$....(2)
Dividing (1) by (2) , we get,
$a^2/b^2 = 3200/2450 \therefore a/b = 8/7$, since the ratio of prices cannot be negative.
Also since they have 30 cows together, $a + b = 30$. $\therefore 8b/7 + b = 30$
Solving the above, we get, b = 14, and a = 16.

48. Multiplying the first equation by (by), we get, $ax(by) + b^2y^2 = 2by$.
Subtracting the second from the first, we have, $b^2y^2 = 2by - 1$.
$\therefore b^2y^2 - 2by + 1 = 0$. $\therefore (by - 1)^2 = 0$
$\therefore y = (1/b)$ and $x = (1/a)$.

49. From the second equation, $x^2 - 2xy + y^2 - 2xy = 52$ $\therefore (x - y)^2 - 2xy = 52$.
Substituting the value of x - y from the first equation,
$(10)^2 - 2xy = 52$. $\therefore xy = 24$.
We simply need to find two numbers whose difference is 10, and whose product is 24.
The numbers are 12, and 2 or -2 and -12.
(* **N. B.** *) : At this stage, you should start doing such problems orally, which is possible by practice. There should be no need to write lengthy equations and perform cumbersome calculations.

50. $\therefore x + y / xy = 2$, hence, $2/xy = 2$ $\therefore xy = 1$.
Again it is very obvious that both x and y must be equal to 1.

51. $\alpha + \beta = -q/p$ and $\alpha\beta = r/p$, $\alpha^3 + \beta^3 = (\alpha + \beta).(\alpha^2 + \beta^2 - \alpha\beta) = (-q/p).[(\alpha + \beta)^2 - 3\alpha\beta] = (1/p^3)(3pqr - q^3)$.

52. $[(d + 1/\ell)^2 - d^2] / d^2 = (1 + 1/\ell d)^2 - 1 = (1/\ell^2 d^2 + 2/\ell d) = (1/\ell d)(2 + 1/\ell d)$.

53. As the degree of the given equation is 4, there will be, in all, 4 roots possible. According to descartes' rule, there will be zero positive real rules, as there are no sign changes in f(x) and there will be maximum 4 negative real roots, as there are 4 sign changes in f(-x). As one of the roots is $\sqrt{-1}$, which is irrational and complex, its conjugate will be another root. Hence, there are two complex roots definitely. The other two roots can be negative real roots or complex in nature.

54. If the roots of the equation are to be equal, the discriminant B^2 - 4AC = 0, hence $4(a^2 - bc)^2 = 4(c^2 - ab)(b^2 - ac)$, which simplifies to $[\, a\,(a^3 + b^3 + c^3 - 3abc)\,] = 0$, hence the result.

55. $x^2 + a^2 = 8x + 6a$, $x^2 - 8x + (a^2 - 6a) = 0$, If the roots of the equation are to be real, the discriminant $B^2 - 4AC \geq 0$, $64 - 4(a^2 - 6a) \geq 0$. $\Rightarrow a^2 - 6a - 16 \leq 0 \Rightarrow (a - 8)\,(a+2) \leq 0 \therefore -2 \leq a \leq 8$.

56. $3x^2 - 7x + 7q = 0$, As the roots are reciprocal to each other, their product = 1.
$\therefore$ the product of the roots = 7q/3 = 1, q = 3/7.

57. Applying Descartes' rule we see that two times change of sign takes place so there will be maximum 2 +ive roots and by changing the sign of the variable we see that only one time the change of sign takes place so there will be maximum 1 negative root. As the degree of the equation is 9 so there should be 9 roots therefore we can say there will be atleast 6 imaginary roots.

58. As the roots are equal and of opposite sign let it be a, -a and the third root be b.
Then we have $(x - a)\,(x + a)\,(x - b) = x^3 - bx^2 - a^2 x + a^2 b$ thus we can say that product of the coefficient of x with power 2 and coefficient of x is equal to constant term with in the expression so condition for the two roots to be equal and of different sign for the given expression is pq = r.

59. Let the three roots be a, b, c then we $(x - a)\,(x - b)\,(x - c)$
We get $x^3 - (a + b + c)\,x^2 + (bc + ab + ac)\,x - abc$
Comparing this with the given equation we $(a + b + c) = p$, $(bc + ab + ac) = q$
Sum of square of roots $a^2 + b^2 + c^2 = (a + b + c)^2 - 2\,(bc + ab + ac) = p^2 - 2q$ and this is the answer.

60. Every equation which is of an even degree and has its last term negative has atleast two real roots, one positive and one negative, and four imaginary roots.

Concepts 2: RATIO, PROPORTION, VARIATION.

1. Let the number to be added be x.
$\therefore (5 + x)/(37 + x) = 1/3$
$\therefore 3(5 + x) = 37 + x$
$\therefore 15 + 3x = 37 + x$
$\therefore x = 11$.

2. We have, $(7x - 4y) : (3x + y) = [7(x/y) - 4] : [3(x/y) + 1]$ (dividing throughout by y)
substituting the value of x/y as 3/4,
$(7x - 4y) : (3x + y) = [21/4 - 4] : [9/4 + 1]$
$= 5/4 : 13/4$
$= 5 : 13$

3. Let the fourth proportional be x.
5 /3 = 10 /x. x = 6.

4. If a is the mean proportional, we have $(x/y + y/x)\,[x^3 / y(x^2 + y^2)] = a^2$
$\therefore [\,(x^2 + y^2)\, x^3\,] / [xy^2\,(x^2 + y^2)\,] = a^2$
$\therefore x^2 / y^2 = a^2$
$\therefore a = x/y$.

5. $a/3 = b/5 = c/1 = (3a + 5b - 7c)/x$
$\therefore (3a + 5b - 7c) / (9 + 25 - 7) = (3a + 5b - 7c)/x$
$\Rightarrow x = 27$.

6. Let the sides of the rectangle be 3x & 2x
$\therefore$ $(3x)(2x) = 6x^2 = 96$
$\therefore$ $x = 4$.
$\therefore$ The sides of the rectangle are 12 cm and 8 cm.

7. Dividing throughout by $4b^2$; we get
$(a/b)^2 + 2 = 3\,(a/b) \Rightarrow (a/b)^2 - 3\,(a/b) + 2 = 0$
$(a/b - 2)\,(a/b - 1) = 0 \Rightarrow a/b = 1$ or 2

8. Let the ages be 5x & 4x.
After 25 years $(5x + 25) / (4x + 25) = 10 / 9$.
Solving for x, we get, $x = 5$.
$\therefore$ The present ages are 25, and 20 respectively.

9. Let the 4 numbers be a, b, c, d $\therefore$ $a/b = c/d \Rightarrow ad = bc$.
$a + d = 21$, $b + c = 19$, $a^2 + b^2 + c^2 + d^2 = 442$.
$\therefore (a+d)^2 + (b+c)^2 = 442 + 4ad$
$\therefore$ $441 + 361 = 442 + 4ad \Rightarrow ad = bc = 90$.
$\therefore$ $ad = 90$ and $a + d = 21 \Rightarrow$ numbers are 15 and 6. and
$bc = 90$ and $b+c = 19 \Rightarrow$ numbers are 10 and 9
Hence the numbers are 6 : 9 : : 10 : 15 or 6 : 10 : : 9 : 15.

10. $V = kr^2h$, V = volume, r = radius, and h = height.
$V_1 = kr_1^2 h_1$ and $V_2 = kr_2^2 h_2$
$\therefore$ $88 = k.\ 21^2 .12$. $\therefore$ $k = 22 / (21^2 \times 3)$
$\therefore$ $V_2 = 462 = (22 \times r_2^2 \times 7) / [21 \times 21 \times 3]$
$\therefore$ $r_2 = 63$ cm.

11. $a = k\sqrt{b} / c^3$. $\therefore$ $4 = (k\sqrt{16}) / 8$ $\therefore$ $k = \pm 8$.
In the second case, $2 = [(\pm 8)\sqrt{b}] / 4^3$ $\therefore$ $b = 256$.

12. The ratio of alcohol to the volume of the container is $x / (x + 1)$ and $y / (y + 1)$in the two cases which is the same as $x(y + 1) / [(x + 1)(y + 1)]$ and $y(x + 1) / [(y + 1)(x + 1)]$
$\therefore$ Total volume of mixture $= 2(x + 1)(y + 1) = 2xy + 2x + 2y + 2$, and
total volume of alcohol in mixture $= x(y + 1) + y(x + 1) = 2xy + x + y$.
$\therefore$ Volume of water = volume of mixture - volume of alcohol.
$= x + y + 2$.
$\therefore$ Ratio of alcohol to water in mixture $= (2xy + x + y)/(x + y + 2)$

13. We have, $F = k.m_1m_2 / d^2$
In the first case, $48 = k\,(20 \times 10^7)(4 \times 10^{10}) / (1000)^2$ $\therefore$ $k = 6 \times 10^{-12}$.
In the second case, $66 = (6 \times 10^{-12})(11 \times 10^9)(2 \times 10^6) / d^2$
$d = (2 \times 10^3)^{1/2} = (400 \times 5)^{1/2} = 20\sqrt{5}$.

14. Area $= kr^2$
$\therefore$ $154 = k\,(7^2)$ $\therefore$ $k = 22 / 7$
$\therefore$ Area $= (22 / 7)(10.5)^2$
$= 346.5\ cm^2$

15. Let $V = kTD^2$ where V = value, T = thickness, D = diameter, k = constant
We have, $V_1 = k\,T_1D_1^2$ and $V_2 = kT_2D_2^2$
$\therefore$ $(V_1 / V_2) = (T_1/T_2)(D_1/D_2)^2$
Substituting $V_1/V_2 = 4/1$, $D_1/D_2 = 4/3$, we get,
$4 = (T_1 / T_2)\,(16 / 9)$. $\therefore$ $(T_1 / T_2) = 9/4$
$\therefore$ Ratio of thickness is 9 : 4.

16. $T_1/T_2 = \sqrt{(H_1/H_2)} \Rightarrow 3/T = \sqrt{(44.1/122.5)} = \sqrt{(441/1225)} = 21/35 \therefore T = 5$ sec.

17. Let H = kA/B where H = altitude, A = area, B = base, k = constant
$\therefore 3 = k(6)/(4) \quad k = 2$
$\therefore$ Required altitude H = 2x 12/8 = 3.

18. Let the numbers be 3x & 4x.
$\therefore (3x - 7)/(4x - 7) = 2/3$
Solving for x, we get, x = 7.
Hence the required numbers are 21 and 28.

19. Let V = kAH where V = Volume, A = area of base, H = altitude
From the first condition, 280 = k (60)(14)
$\therefore k = 1/3$.
$\therefore 390 = (1/3) .A .(26)$
$\therefore A = 45$ sq.m.

20. We have $p = k/(q^2 - 1)$
From the first condition, $24 = k/(10^2 - 1)$
$\therefore k = 24 \times 99$.
When q = 5, $p = (24)(99)/(5^2 - 1) \therefore p = 99$.

21. $(x^2 + y^2)/(m^2 + n^2) = xy/mn \therefore (x^2 + y^2)/2xy = (m^2 + n^2)/2mn$
by componendo and dividendo $x^2 + y^2 + 2xy/ x^2 + y^2 - 2xy = m^2 + n^2 + 2mn/ m^2 + n^2 - 2mn$
$\Rightarrow (x + y)^2/(x - y)^2 = (m + n)^2/(m - n)^2$
Hence, $(x + y)/(x - y) = \pm (m+n / m-n)$

22. $12x^2 + 3y^2 - 13xy = 0$, $12(x/y)^2 + 3 - 13(x/y) = 0$, Solving we get x/y = ¾ or 1/3.

23. $(N + 1)/(N + 3) = (10/9). N/(N + 2)$, $\therefore N^2 + 3N - 18 = 0 \Rightarrow N = 3$ or $- 6$, hence the fraction is 3/5 or –6/-4

24. $r = (S/4\pi)^{1/2} = 1/2\,(S/\pi)^{1/2}$, $V = (4/3)\,\pi r^3 = (S/6)\,(S/\pi)^{1/2}$.

25. $S_1/S_2 = 4\pi r_1^2 / 4\pi r_2^2$; $S_2 = 4$ lit; $r_2 = 2r_1 \Rightarrow S_1/4 = 1/4 \therefore S_1 = 1$ lit.

26. Let u/v = w/x = y/z = k. Therefore u = kv, w = kx and y = kz. Substituting these values L.H.S. $= \{[2v(kv)^6 - 7(kx)^5 (kz) + 15 (kv)(kx)^2 (kz)^5] / [2v^7 - 7x^5z + 15vx^2 z^3 (kz)^2]\}^{1/2} = \{k^6 [2v^7 - 7x^5 z + 15vx^2z^5k^2] / [2v^7 - 7x^5 z + 15vx^2z^5 k^2]\}^{1/2} = k^3$. But R.H.S. = (uwy) / (vxz) = (kv) (kx) (kz)/(vxz) $= k^3$. Therefore L.H.S. = R.H.S.

27. let the numbers be k/a^2, k/a, k, ka, ka^2. Therefore we have $(k/a + ka)/(k + ka^2) = ½$. On solving we get a = 2. But $k/2^2 + k/2 + k + 2k + 2^2k = 372$. Therefore k = 48. Therefore the numbers are 12, 24, 48, 96, 192.

28. $m\angle A = 2\theta$. Then $m\angle B = 5\theta$ and $m\angle C = 11\theta$. Therefore $2\theta + 5\theta + 11\theta = 180$. On solving $m\angle A = 20°$, $m\angle B = 50°$ and $m\angle C = 110°$.

29. We have $(x^3 + y^3)/(x^3 - y^3) = 793/665$. On solving we get $x^3/y^3 = 729/64$.
(1) Therefore x/y = 9/4.Now (2x - 3y) / (2x + 3y) = (2x9 - 3x4) / (2x9 + 3x4) = 1/ 5.
(2) x/y = 9/4. Therefore $x^2/y^2 = 81/16$. Now $(3x^2 - 4y^2)/(4y^2)$ = (3x81 - 4x16)/(4x16) = 179/64.

30. The angles of the quadrilateral be θ, 2θ, 5θ, 4θ. Therefore $\theta + 2\theta + 5\theta + 4\theta = 360°$. On solving we get the angles as 30°, 60°, 150°, 120°. Since 30° + 150° = 60°+ 120° = 180°. Therefore it is a cyclic quadrilateral.

31. Suppose Samar got 7x marks in English. Therefore marks in Hindi, Science and Sanskrit are 14x and 9x and 15x respectively. Total marks obtained in languages = 7x + 14x + 15x = 36x. Therefore marks obtained in Mathematics is 18x. Therefore 36x + 9x + 18x = 63x = 252. So x = 4. Samar obtained 28 marks in English, 56 marks in Hindi, 36 marks in Science, 60 marks in Sanskrit and 72 marks in Mathematics. Since passing marks are 40, Samar failed by 12 marks in English and by 4 marks in Science.

32. We have $\lambda = k/\eta$, where k is a constant. Now 80 = k/400, i.e. k = 32000. Therefore the equation becomes $\lambda = 32000/\eta$. Therefore $\lambda = 32000/320 = 100$ cm.

33. Let the distance and time are denoted by d_t and t respectively. We have $d = kt^2$, where k is a constant. Therefore $d_4 / d_3 = (4)^2 / (3)^2$. So $(d_3 + 33) / d_3 = 16/9$. On solving we get $d_3 = 9x33/7$. Putting in above formula 9x33/7 = 9k. Therefore k = 33/7. Therefore $d_7 = 33/7x7^2 = 231$m.

34. $D = KW_1L_1^3 / I_1 = K\ (5/4)W_1\ L_2^3 / (16I_1/25)$. On solving we get $L_1 / L_2 = 5/4$. Therefore the length should be reduced by 20%.

35. We have $V = K\sqrt{W}$, where V is the value of the diamond, W is the weight of the diamond and K is a constant. Initialy, $1000 = K\sqrt{25}$. So, k = 200.
$V_1 = 200\sqrt{9} = 200x3 = 600$
$V_2 = 200\sqrt{16} = 200x4 = 800$
So, Total value = 600+800 = 1400.
So, Net gain = 1400-1000 = 400

Concepts 3: FUNCTIONS.

1. Substitute 4 in place of x to get $f(4) = 4^4 - 3(4)^3 + 6(4)^2 - 10(4) + 16 = 136$.

2. $f\left(\frac{1}{t}\right) = 2\left(\frac{1}{t}\right)^2 + 2 / \left(\frac{1}{t}\right)^2 + 5 / \left(\frac{1}{t}\right) + 5\left(\frac{1}{t}\right) = 2/t^2 + 2t^2 + 5t + 5/t = f(t)$

3. Substituting x = 1, and y = -1 in f(x,y), we get,
$f(1, -1) = 3(1)^2 - 2.(1)(-1) - (-1)^2 + 4 = 8$.

4. Let $f(x) = y = (x + 3)/ (4x - 5) \Rightarrow 4xy - 5y = x + 3$.
$\therefore (4y - 1) x = (5y + 3) \Rightarrow x = (5y + 3) / (4y - 1)$
$\therefore f^{-1}(x) = (5x + 3) / (4x - 1) = t$.
$\therefore f^{-1}(x) = t \Rightarrow f(t) = x$.

5. $f\left(1/x^2\right) = \left(1/x^2\right) + x^2$, $g\left[f\left(1/x^2\right)\right] = \left(1/x^2\right) + x^2 + 1 = (1/x + x)^2 - 1 = [f(x)]^2 - 1$

6. $[f(-x)]^2 = [-x^{-x}]^2 = 1/\{(-1)^{2x}\ (x)^{2x}\} = 1/(x^x)^2 = 1/[f(x)]^2$

7. f (x) = 8, $g.f(x) = g(8) = y^2 + 1$.
N.B. : Here, both the functions are **constant functions.** The dependent variable always remains constant, irrespective of the independent variable.

8. $g(10/x) = \log_{10/x} 10 = \log_{10} 10 / \log_{10}(10/x) = 1/ \log_{10}10 - \log_{10} x = 1/ 1 - \log_{10} x$
$= 1/ 1 – [f(x)/a] = a / [a - f(x)]$

9. Consider the following identity : $(x - y)^2 = x^2 + y^2 - 2xy$
Substituting the functions in the above identity, we get,
$[h(x,y)]^2 = f(x,y) - 2\sqrt{[g(x,y)]}$.
N.B. : Here we consider only the positive value of $\sqrt{[g(x,y)]}$

10. $f(x) = x^3 - x^2 - x + 1 = (x + 1)(x - 1)^2$ and $g(x) = x^2 - 2x + 1 = (x - 1)^2$, hence f(x) = (x +1).g(x).

11. $f(x) = \log_2 x^2 = 2\log_2 x$ and $g(x) = \log_x 4 = 2/ \log_2 x$, hence f(x) = 4 / g(x).

12. $f(2) = 2^6 + 2^4 + 2^2 + 1 = 85$.

13. Given that $f(x) = [(1 + x)^{1/2} + (1 - x)^{1/2}] / [(1 + x)^{1/2} - (1 - x)^{1/2}]$
Rationalizing the denominator we get $f(x) = [1+ \sqrt{(1 - x^2)}] / x$
at $x = (\sqrt{3}) / 2$; we get $f(x) = \sqrt{3}$

14. Given that $f(x) = 3x - 5$ and $f (g(x)) = 2(x)$
$\therefore$ $3. g(x) - 5 = 2x$
$\therefore$ $g(x) = (2x + 5) / 3$.

15. Let (x_1, y_1) be the required point. Therefore $y_1 = f(x_1) = 2x_1 + 3$. But $x_1 = f(y_1) = 2y_1 + 3$. On solving we get, $y_1 = 2x_1 + 3 = (x_1 - 3) / 2$. Therefore $x_1 = -3$ and $y_1 = -3$.

16. We have $g(3) = (3)^3 +3 = 30$. Therefore $f[3,30] = 3 \times 30 = 90$.

17. From the definition of f(x) it is clear that $f(x) = f(1/x)$. Therefore $f(1/2) = f(2) = 260.78$.

18. $f (o, y + 2) = y + 5$
$f (x + 5, y) = f (x, y-1)$ $\quad$ $f (5, 6) = f (0 + 5, 6)$
$= f (0, 5)$
$= f (0, 3 + 2)$
$= 3 + 5$
$= 8$ ans.

19. f (15, 6) In order to have the value of the function , the first step reduces value of variable x to 10, the second step reduces to value 5 and third step reduces to value 0 and then we can have value of the function . So in total only three steps are required.
Step 1: $f(10+5, 6) = f(10,5)$
Step 2: $f(5+5,5) = f(5,4)$
Step 3: $f(0+5,4) = f(0,3)$
Step 4: $f(0,1+2) = 1+5 = 6$

20. f (4, 5) - f (4, 3) according to the function defined this can be written as
$5^{3n} - 4^{3n}$ which can be written $(5^3)^n - (4^3)^n$
Whatever be the value of n it will be always divisible by (125 – 64) = 61
Now if n is an even integer then it will be always divisible by (125 + 64) = 189

21. $f (x, y) = x^3 + y^3$ $\quad$ $g (x, y) = x^2 + y^2$ $\quad$ $h (x, y) = xy$ now
$= x^3 + y^3 = (x + y) (x^2 + y^2 - xy)$ where $x + y = (x^2 + y^2 + 2xy)^{1/2} = \{g (x, y) + 2 h (x, y)\}^{1/2}$
So $f (x, y) = \{g (x, y) + 2 h (x, y)\}^{1/2} \{g (x, y) - h (x, y)\}$

22. $y = (3x + 4) / (5x + 3)$ $\quad$ $x = f (y)$
on solving this the value of x in terms of y
$x = (3y - 4) / (3 – 5y)$

23. $f (a, b) = a^2 + b^3$, $g (a, b) = a + b$, $f (3, g (3, 4))$
$g (3, 4) = 7$
$f (3,7) = 3^2 + 7^3 = 352$

24. $f (x, y) = 1 + 2 + 3 +\ldots\ldots.+ x$, if $x > y$
$= 1 + 2 + 3 +\ldots\ldots.+ y$, if $y > x$
$(f (0, 1) + f (1, 2) + f (2, 3) + f(3, 4) +\ldots\ldots\ldots\ldots.. f (10, 9)$
$(f (0, 1) = 1$
$(f (0, 2) = 1 + 2$
$(f (2, 3) = 1 + 2 + 3$
$(f (3, 4) = 1 + 2 + 3 + 4$
Like this it will go on and which is actually a series
T_n the general term of the series will be $n (n + 1) / 2$
$T_{n =} (n^2 + n) / 2$ and the sum of the whole series will be given as

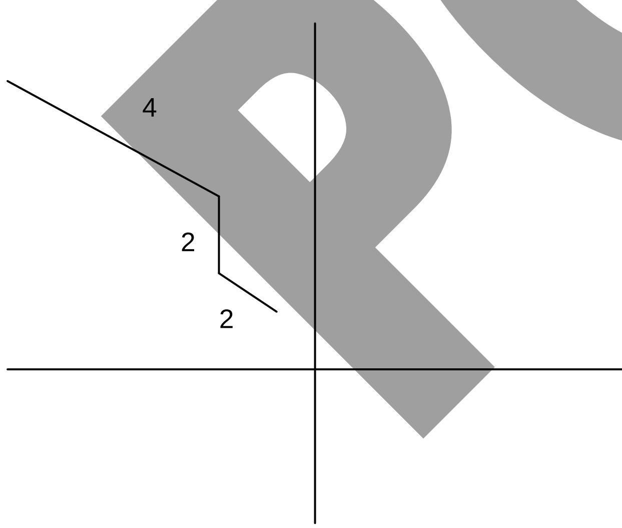
$\sum T_n = 1/2\,(\sum n^2 + \sum n)$ The first part in the parenthesis is summation of squares of natural numbers and the second part is summation of natural number and when we substitute the formulae and n = 10
½ (385 + 55) = 220 is the answers.

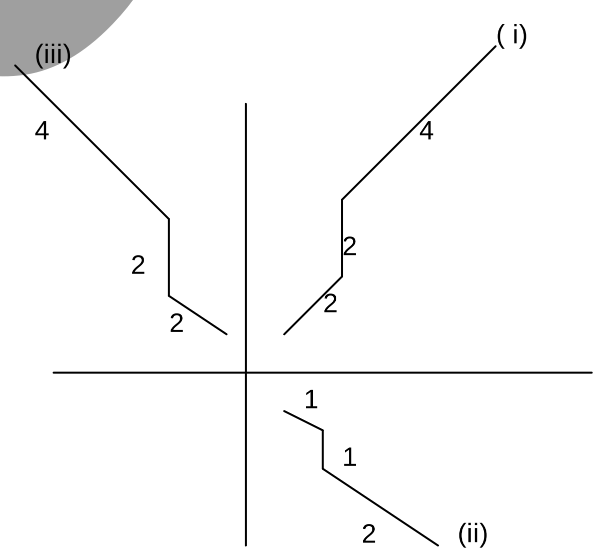
25. f (x) = x / (x+ 1)
$f^n(x) = (n-1)\, f^{n-1}(x)$ for n > 1
$f^4(1)$.
f (1). = 1 / 2
$f^2(1)$.= 1. 1 /2
$f^3(1)$.= 1. 2. 1/2 = 1
$f^4(1)$. = 3 .1 = 3 ans.

26. Let $f(x) = ax^2 + bx + c$
f(-3) = 9a – 3b +c = 6
f(0) = 0+0+ c = 6
and f(2) = 4a +2b + c =11
f(x) =4a +2b +c =11
a =1/2, b= 3/2, c=6
$f(x) = ½\, x^2 + 3/2x + 6$
f(x) = ½ + 3/2 + 6 = 8; (1,8)

27. If we apply the given three sets of instruction to F(x), we will get the same graphs, as given into question.

28. It is not possible to get graph F1 (x), from F (x), whatever may be the instruction. If we apply any instructions to F (x), all segments of this graph will change proportionally e.g. if the side with 3 units measurement becomes 1/3 rd i.e. 1,the side with 2unit measurement will become 2/3.





29. After applying the given three instruction we will get (I), (ii) & (iii) graphs respectively. Final graph i.e.(iii) graph, will be similar to the graph given in question .

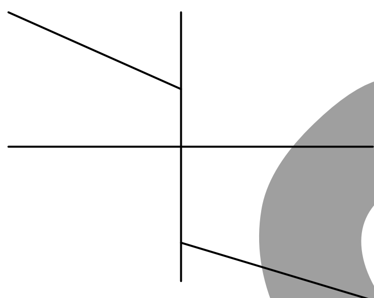
30. g(x) = f{f(x)} = f (1/1-x) = 1/(1 – 1/(1-x)) = -(1-x)/x
and h(x) = f[f{f(x)}]= f{g(x)} =1/(1- g(x)) =1/(1 +(1-x)/x) = x
f(x) g(x) h(x) = 1/(1-x).{- (1-x)/x}.x = -1=b, similarly calculate value of a. a = (2-x)/(1-x)

31. $f(-x)= \log_a(-x +\sqrt{(x^2 +1)}) = \log_a(-x +\sqrt{(x^2 +1)}) (x +\sqrt{(x^2 +1)}/ (x +\sqrt{(x^2 +1)}$
$= \log_a(x^2 +1-x^2)/ (x +\sqrt{(x^2 +1)}) = - \log_a(x +\sqrt{(x^2 +1)}) = - f(x)$
So, the function is an Odd function.

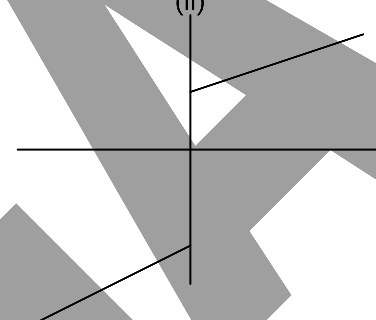
32. (i) $f(x) = 1/\sqrt{(x-|x|)}$, For real value of the function $x-|x| > 0$ i.e. $|x|< x$, which is not true for any real x, hence Domain of function ,$D_f = \{ \}$,i.e. f(x) is not defined for any real x.
(ii) For Domain of function, f(x) must be a real number, $(x+1)/(2x – 1)$ must be a real number. and for that, $2x – 1\neq 0$; $x \neq ½$; therefore Domain of given function will be set of all real number except ½ .
For Range, Let $y = (x+1)/(2x – 1)$; $2xy – y = x+1$; $x(2y – 1) = 1+y$
$x =(1+y)/(2y –1)$,since x is a real number, therefore $(1+y)/(2y –1)$,must be real; $y\neq 1/2$
Therefore range of function will be all real number. except ½.

33. (a)First take the reflection of given graph (i) about y-axis, then we will get graph (ii), now take the reflection of (ii) graph about x-axis, then we will get (iii) graph, which is similar to (i) graph.

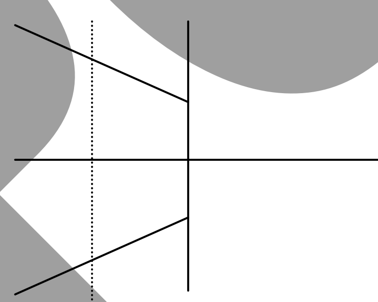
(i) & (iii)



(ii)



Therefore the given graph, is symmetrical about Origin, so the function will be an odd function.
(b)This graph does not represent any function as for one value of x, there are two values of y.



34. (i) $f(x) =y = 2^{-x^2}$; $f(-x) = 2^{-x^2} = f(x)$; Therefore function is an even function.

(ii)$f(x) = y = 2^{x- x^4}$; $f(-x) =2^{-x- x^4}$: $\neq f(-x) \neq -f(-x)$, so the given function is neither even nor odd.

35. $f(x) = x/(x –1)$, Therefore $f(1/x) = (1/x)/(1/x –1) =1/(1 – x)$
and $f(1 –x) = (1 – x)/(1-x –1) =(x-1)/x$;
$f(1/x) = f(1-x)$; $1/(1 – x) = (x-1)/x$;
On simplifying, $x^2 – x+1 = 0$;
$b^2 –4ac = (-1)^2 – 4.1.1= -3<0$, therefore roots of this equation will be complex, not real .

36. $f (x, y) = \sum_{n=1}^{m} n\sqrt{x} = 1\sqrt{x} + 2\sqrt{x} +3\sqrt{x}$ ………………..$+m\sqrt{x} =\sqrt{x}(1+2+3$ …….$+m)$
$=\sqrt{x}\, m(m+1)/2$,
value of m is 99,therefore $f(x,y) = \sqrt{x}\ 99.100/2 =4950\sqrt{x}$,
similarly, $g(x,y) =4950\sqrt{y}$

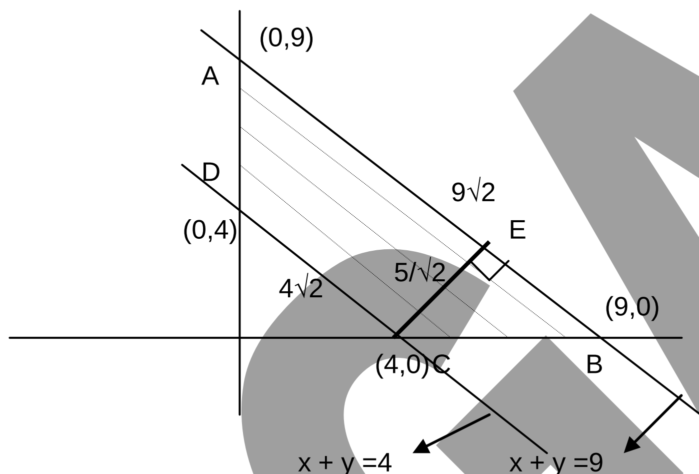
Therefore, f(x, y) + g(x, y) = $4950(\sqrt{x} + \sqrt{y})$
and f(x, y) g(x,y) = $24502500\ \sqrt{x}\sqrt{y}$

37. $f(x, y) = x^{\sum_{n=2}^{\infty} 1/n} = X^{(1/2 + 1/4 + 1/8 \ldots\ldots)} = X^{(1/2/(1-1/2))} = X$

$g(x, y) = x^{\sum_{m=3}^{\infty} 1/m} = X^{(1/3 + 1/9 + \ldots\ldots)} = X^{1/2}$

therefore $f(x,y).g(x,y)/2 = X.X^{1/2}/2 = X^{3/2}/2$

38.



Line parallel to x+y = 4, and which cuts y- axis at 9, will be x + y =9.
Area enclosed by these two lines & two axis x = 0 & y = 0,will be a trapezium ABCD (ref. to fig.)
Area of trapezium ABCD = 1/2 (AB + CD) CE
CE, is distance between two parallel lines, therefore CE = $|(C1 - C2)/\sqrt{(a^2 + b^2)}|$ =
CE= $|(9 - 4)/\sqrt{(1^2 + 1^2)}| = 5/\sqrt{2}$;
Area = $1/2\ (9\sqrt{2} + 4\sqrt{2})\ 5/\sqrt{2} = 32.5$

39. Path of missile can be given by, $f(t) = at^2 + bt + c$, where t is time, and a,b,c are constants.
Since it is given that constant term is negligible we can take c = 0; therefore Path will be
$f(t) = at^2 + bt$;
According to given information, Path of interceptor will be given by
$g(t) = at^2 + bt + c_1$, where $c_1 > 0$
For the success of test, interceptor should hit the missile, for this there path should clash, therefore f(t) = g(t): $at^2 + bt = at^2 + bt + c_1 \rightarrow c_1 = 0$, which is not true, so this test will be a failure.

40. If all values x, y & z are +ve or –ve, then F & G both will be equal. But when one or two value(s) are negative and remaining +ve, then value of F will be greater than value of G.This can be verified by taking random values.

Concepts 4: INEQUALITIES, MAXIMA & MINIMA.

1. $4x - 5 > x + 7 \quad \therefore 3x > 12 \quad \therefore x > 4.$

2. $(5 - x)(x + 3) = 15 + 2x - x^2 = -(x^2 - 2x + 1) + 16 = -(x - 1)^2 + 16$
$\therefore$ The above expression will have the greatest value when the number to be subtracted from 16 is minimum i.e. zero. $\therefore (x - 1) = 0$.
$\therefore$ The maximum value of the given expression = 16.

3. The requirement is that $19x - 2x^2 - 35 > 0$.
i.e. $2x^2 - 19x + 35 < 0$.
$\therefore (2x - 5)(x - 7) < 0$
For L.H.S. to be negative, either $2x - 5 < 0$ and $x - 7 > 0$ or $x - 7 < 0$ and $2x - 5 > 0$.
$\therefore$ $x < 5/2$ and $x > 7$ or $x > 5/2$ and $x < 7$, but $x < 5/2$ and $x > 7$ is not possible.
Hence, $5/2 < x < 7$.

4. Expanding, we get, $2x^2 - 3x - 2 > 0$.
$\therefore (2x + 1)(x - 2) > 0$.
With similar reasoning as in problem no.3, we get, $x > 2$ or $x < -1/2$.

5. $\therefore (x + 9)(x - 3) > 0$ and also $(4 - x)(x + 1) > 0$.
For the first expression to be true, both terms should be either positive or negative.
i.e. either $x < -9$ or $x > 3$.
With similar reasoning, for the second expression, $-1 < x < 4$.
The expressions will be true only for $3 < x < 4$.

6. $2x^2 + 9x - 35$ factorises to $(2x - 5)(x + 7) > 0$, hence $x < -7$ or $x > 5/2$.

7. $3x^2 - 20x + 17 = (3x - 17)(x - 1) \leq 0$, $\therefore 1 \leq x \leq 17/3$

8. $x + 1/x - 2 = x^2 + 1 - 2x = (x - 1)^2$, which is always greater than or equal to zero for any real x.

9. $|x/2| < 2$, $\Rightarrow -2 < x/2 < 2$ i.e. $-4 < x < 4$.

10. $x + 1/x = (\sqrt{x} - 1/\sqrt{x})^2 + 2$. Since square of a number is always non negative it's minimum value is 2.

11. Let x and $(6 - x)$ be the radius and the height .
$\therefore$ volume $= \pi . x^2 (6 - x) = \pi . (6x^2 - x^3) = f(x)$, say.
$\therefore f'(x) = \pi . (12x - 3x^2) = 3.\pi . x.(4 - x)$
For max. value of $f(x)$, $f'(x) = 0$, i.e. either $x = 0$ or $x = 4$.
Value 0 for radius is inadmissible. $\therefore$ the volume is maximum when radius is 4 m
$\therefore$ max. volume $= \pi . (6(4)^2 - 4^3) = 32\pi \ m^3$.
* **N.B.*** : usually it is not necessary to check whether the second derivative is negative.

12. Let the numbers be x and 12 - x.
$\therefore$ Sum of squares $= x^2 + (12 - x)^2 = f(x)$,say.
$\therefore f'(x) = 2x + 2(12 - x)(-1) = 4(x - 6)$
$\therefore f'(x) = 0$, only when $x = 6$.
The numbers are 6 and 6.

13. Let l and b be the length and breadth of the rectangle.
$\therefore$ perimeter $P = 2.(l + b)$, which is constant. $\therefore$ $l + b$ is a constant.
Area $= l \times b$. Since for the given sum of two positive quantities, their product will be maximum if and only if they are equal. i.e. $l = b$.
$\therefore$ the area is maximum when the rectangle is a square.

14. $f'(x) = 3x^2 - 12x + 9 = 0$ for max. $f(x)$.
$\therefore 3(x - 3)(x - 1) = 0$
$\therefore$ The max. value is at $x = 3$, or $x = 1$.
$\therefore f(1) = 6$, $f(3) = 2$.
Hence the maximum value is 6.

15. $f(x) = x^2 - 12x + 27 = x^2 - 12x + 36 - 9$
$= (x - 6)^2 -- 9$. Since $(x - 6)^2 \geq 0$, the minimum value of the expression is -9.

16. $x^2 - x - 240 = (x - 16)(x + 15) < 0$, hence $-15 < x < 16$.

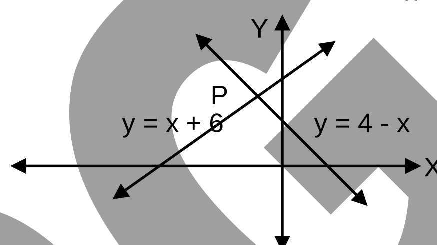
17. Let the n^{th} term = $T_n = x$. $T_{n+1} = \sqrt{(6 + x)}$. But, as n tends to infinity, $T_n = T_{n+1}$. Therefore, $x = \sqrt{(x + 6)}$. Squaring both the sides, we get $x^2 = 6 + x$. Therefore $x^2 - x - 6 = 0$. Therefore x = 3 or x= -2. Only possible answer is x = 3.

18. (a) From the property that if a, b, c, d all positive and different, then $a^2 b + b^2 c + c^2 a > 3abc$, first number is greater.
(b) From the property that if a, b, c, d all positive distinct numbers, then $a^4 + b^4 + c^4 + d^4 > 4abcd$, first number is greater.
(c) From the property $(n!)^2 > n^n$, Therefore the first number is greater.
(d) Consider $1001^{999} / 1000^{1000} = (1001 / 1000)^{1000} \times 1/1001 = (1+1 / 1000)^{1000} \times 1 / 1001 < 3 \times 1 / 1001 < 1$. Because for any positive integer $2 \le (1 + 1 / n)^n < 3$. Therefore 1000^{1000} is greater than 1001^{999}.

19. We know the property that if the product $a^m b^n c^p \ldots$ is maximum then a/m = b/n = c/p = …. Therefore (7-x)/5 = (7+x)/4. On solving x = -7/9. Therefore the maximum value of the given expression is $(7 + 7/9)^5 (7 - 7/9)^4 = 7^9 \times 2^{17} \times 5^5 / 3^{18}$.

20. For 'k' to be smallest, by estimation x = -1, y = 0 and z = 1. Therefore $k=(-1)^2 + (0)^2 + (1)^2 = 2$.

21. Consider the graph of y = x+6 and y = 4-x as shown below. At point P i.e. the point of intersection we get the maximum value of Y i.e Min of {(-1+ 6), (4 + 1)}.i.e. 5



22. We have x + y > 326 and x - y = 436. Adding these two inequalities we get 2x > 762, i.e. x > 381. But x is less than 383. Since x is an integer the only possible value of x is 382.

23. If a + b is constant then $a^n \cdot b^m$ is maximum for a/n = b/m. So x/2 = y/3 and x + y = 25. This gives x = 10 and y = 15, which gives the maximum value as 337500.

24. If ab is constant then a + b is minimum when a = b. So for minimum value of 3x + 4y, we can write the given condition as 3x . 4y = 144, which gives 3x = 12 and 4y = 12 as per the condition stated above. This gives the minimum value as 24.

25. The cost per unit will be minimum for x = 10 as per the rule stated in Q. no. 24. So the cost per unit is 20. Now the profit is no. of units x profit per unit = Z . Y. The given relation makes 4Z + 5Y = 120 - 3 x 20 = 60. Solving this in the same manner as previous question, the maximum profit comes out to be 45.

26. Let the perimeter of the triangle is x , so the perimeter of the circle is (50 – x). Say the side of the triangle is a and the radius of the ring is r.

For the triangle	For the ring
$3a = x$	$2\prod r = 50 - x$
$\Rightarrow a = x/3$	$\Rightarrow r = (50 - x) / 2\pi$
$A(T)= (\sqrt{3}/4)a^2 = x^2/ 12\sqrt{3}$	$A(R) = \pi r^2 = (50 - x)^2 / 4\pi$

Put values of a and r from above.

Say $F(x) = A(T) + A(R) = \dfrac{x^2}{12\sqrt{3}} + \dfrac{(50-x)^2}{4\pi}$

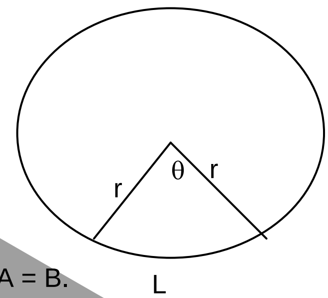
According to the question F(X) should be minimum, therefore F'(X) should be zero.

$$F'(X) = \frac{2x}{12\sqrt{3}} - \frac{2(50-x)}{4\pi} = 0$$

After solving we get $x = \dfrac{150\sqrt{3}}{(\pi + 3\sqrt{3})}$

Therefore side of the triangle $= \dfrac{50\sqrt{3}}{(\pi + 3\sqrt{3})}$

27. Given that $2r + L = 40$, where L = length of arc.



Area of the sector, $A = (1/2)\, r^2\, \theta$
$\theta = L/r$
From above two we have
$A = (1/2)\, r \times L$
But $L = 40 - 2r$.
Therefore $A = r(20-r)$
Using the rule, if A + B = constant then AB is maximum when A = B.
Therefore $r = 20 - r$. or $r = 10$.
Hence the area of the sector will be 100.

28. $d_1 + d_2 = 40$.
Area of the rhombus $= (1/2)d_1d_2$
Using the rule, if A + B = constant then AB is maximum when A = B.
Therefore, $d_1 = d_2$,so that diagonal of the given rhombus will 20.
$\text{Area}_{MAX}=(1/2)20.20=200\text{ft}^2$.

29. Let one of the parallel side is x , so the other parallel side is 2x. And let h be the height of the trapezium.
Area = (1/2) (sum of the parallel sides)(height) = $(1/2)\,(x+2x)(h) = 3xh/2$
Its given $x+2x+h = 60$ or $3x + h = 60$.
Using the rule, if A + B = constant then AB is maximum when A = B.
Therefore, $3x = h$ so that $x = 10$, $h = 3x = 30$.
Area = 450.

30. $f(x) = 2\sqrt{x} + 1/x$
$f'(x) = (1/\sqrt{x}) - 1/x^2 = 0$. Hence $(1/\sqrt{x}) = 1/x^2$
Therefore , $x = 1$.
Put $x = 1$ in $f''(x)$, that will give a positive value, so minima at $x = 1$.
So the minimum value of the function comes out to be 3.

31. $f(x) = e^x + e^{-x} - x^2$
$f'(x) = e^x - e^{-x} - 2x = 0$.
$\dfrac{e^x - e^{-x}}{2x} = 1$, But $\cos x = \dfrac{e^x - e^{-x}}{2x}$
$\cos x = 1$, so $x = 0$
$f''(x) = e^x + e^{-x} - 2$, put $x = 0$ in $f''(x)$,that will give a positive value so minima at $x = 0$.
And the minimum value of the function is 2.

32. Say the other two sides are a and b.
So $a + b + 6 = 28$. and the semi - perimeter of the triangle is $S = 28/2 = 14$ cm.
Using the Hero's formula for the area of the triangle
$\Delta^2 = 14\,(14 - a)(14 - b)(14 - 6)$
$\Delta^2 = 112(14 - a)(14 - b)$
Put $a = 22 - b$
$\Delta^2 = 112(b - 8)(14 - b)$
Using the rule, if A + B = constant then AB is maximum when A = B.
$b - 8 = 14 - b$
So $b = 11$ and therefore $a = 11$.
hence the area of the triangle $= 12\sqrt{7}\ \text{cm}^2$

33. Let x number of subscribers are there below 600.
Therefore, Profit = (600-x)(400+x)
Using the rule, if A + B = constant then AB is maximum when A = B.
Hence, 600 - x = 400 + x or x = 100.So number of subscribers that will give the maximum profit will be 500.

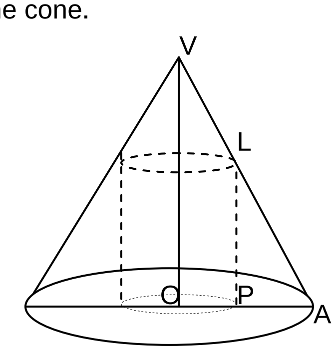
34. $f(x) = \sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3 + 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x) - 3\sin^2 x\cos^2 x(\sin^2 x + \cos^2 x)$
$= (\sin^2 x + \cos^2 x)^3 - 3\sin^2 x\cos^2 x = 1 - (12/4)\sin^2 x\cos^2 x = 1 - (3/4)\sin^2 2x$
Now the maximum and minimum value of the given function depends $(\sin^2 2x)/4$ which lies between 0 (minimum) and 1/4(maximum). Therefore, the maximum value of f (x) = 1and minimum value of f (x) = 1/4

35. $f(x) = x^5 - 5x^4 + 5x^3 - 1$
For max. or min. differentiate the given function
$f'(x) = 5x^4 - 20x^3 + 15x^2$. Now f'(x) = 0 for max or min. So we get x = 0, x = 3 and x =1.
Find the second derivative of f(x)
$f''(x) = 20x^3 - 60x^2 + 30x$.
put the values of x from the first derivative
when x = 3, f''(x) = 90 or positive so minimum value at x = 3
when x = 1, f''(x) = -10 or negative so maximum value at x = 1.

36. (a) $f(x) = \sin x + \cos x = \sqrt{2}\,\dfrac{(\sin x + \cos x)}{\sqrt{2}} = \sqrt{2}(\sin x\cos 45^o + \cos x\sin 45^o) = \sqrt{2}\sin(x+45^o)$
Now the maximum value of the function is when $\sin(x+45^o)$ is maximum and $\sin(x+45^o)$ maximum is 1. Therefore the maximum value of the function is $\sqrt{2}$.
(b) $f(x) = \sin x\cos x = \dfrac{2\sin x\cos x}{2} = \dfrac{\sin 2x}{2}$
Now the maximum value of the function is when sin2x is maximum and sin2x maximum is 1. Therefore the maximum value of the function is 1/2.

37. Surface Area (S)= $2\pi r(r + h)$. (where r and h are the radius and height of the cylinder respectively).
Volume of the cylinder,$V = \pi r^2 h$. Put the value of h in terms of S and r from above relation in V.
So $V = \pi r^2 \{(S/2\pi r) - r\} = \{(Sr/2) - \pi r^3\}$
For V to be maximum differentiate V w.r.t to r.
So $dV/dr = \{(S/2) - 3\pi r^2\}$, For V to be maximum dV/dr should be zero.
Therefore, $(S/2) - 3\pi r^2 = 0 \Rightarrow S = 6\pi r^2$
$d^2V/dr^2 = -6\pi r$ which is negative, so V is maximum at $S = 6\pi r^2$.
But $S = 2\pi r(r + h)$, so $6\pi r^2 = 2\pi r(r + h)$ that gives 2r = h, hence diameter = height.

38. Let r be the radius OA of the base and h be the height OV of the cone.
Let x be the radius OP of the inscribed cylinder in the cone.



$$\frac{PL}{OV} = \frac{PA}{OA}$$

$$\Rightarrow \quad \frac{PL}{h} = \frac{r - x}{r}$$

$$\Rightarrow \quad PL = h(r - x)/r$$

Let S be the curved surface area of the cylinder,
Therefore, $S = 2\pi.OP.PL = \dfrac{2\pi h(rx - x^2)}{r}$
$dS/dx = \dfrac{2\pi h(r - 2x)}{r} = 0$ for x = r/2.
Now S is 0 for x = 0 as well as for x = r and is positive for values of x lying between 0 and r.
Therefore S is greatest for x = r/2.

39. Let A (3,2) be the position of the soldier and P(x,y) be any point on the curve $y = x^2 + 2$, then
$AP = \sqrt{\{(x-3)^2 + (y-2)^2\}} = \sqrt{\{(x-3)^2 + (x^2+2-2)^2\}}$ (Using the equation of the curve)
$= \sqrt{\{x^4 + x^2 - 6x + 9\}}$
Let $f(x) = AP^2 = x^4 + x^2 - 6x + 9$ (distance can't be negative)
For f(x) to be minimum $f'(x) = 0$
$f'(x) = 4x^3 + 2x - 6 = 2(x-1)(2x^2 + 2x + 3) = 0$, so we get $x = 1$.(since $2x^2 + 2x + 3$ is never zero)
$f''(x) = 12x^2 + 2$, so $f''(1) = 12 + 2 = 14$ (positive value, so f(x) is minimum at x = 1)
And minimum value of f(x) = 5.
So the minimum value of AP = $\sqrt{5}$.

40. $f(x) = \sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x = (\sin^2 x + \cos^2 x)^2 - (4/2)\sin^2 x\cos^2 x$
$= (1 - \sin^2 2x/2)$.
Now the maximum and the minimum value of the function f(x) depends on the value of $\sin^2 2x$ and its maximum of it is 1 and minimum value is 0.So the maximum and minimum value of the function f(x) are 1and1/2 respectively.

Concepts 5: LOGARITHMS.

1. We have $32 = 2^5$ and $\sqrt{2} = 2^{1/2}$
$\therefore \log_{\sqrt{2}} 32 = \log_2 32 / \log_2 \sqrt{2} = \log_2(2)^5 / \log_2(2)^{1/2}$

$= 5\log_2 2 / [{}^1/_2 \log_2 2]$
$= 10$.

2. ${}^3/_2$ (log x) - 5(log y) - 2(log z)
$= \log(x)^{3/2} - \log(y)^5 - \log(z)^2$
$= \log[(x)^{3/2} / (y)^5 (z)^2]$
$= \log[(\sqrt{x})^3 / y^5 z^2]$

3. Taking log on both sides, we get
(x+1) log a - (x - 1) log b = 2x log c.
$\therefore$ x [log a - log b - 2 log c] + log a + log b = 0.
$\therefore$ x = (log a + log b) / [2 log c + log b - log a] = log ab / log (bc^2/a)

4a. $\log_{125}(1/5) = \log(1/5) / \log 125 = \log(5)^{-1} / [\log(5)^3]$
$= -\log 5 / 3\log 5 = -1/3$

4b. $\log_9 27 = \log 27/\log 9 = \log(3)^3 / \log(3)^2 = 3.\log(3) / 2.\log(3) = 3/2$

5. $\log_4(x^2 - 3x + 12) = 2$. $\therefore 4^2 = x^2 - 3x + 12$
$\therefore x^2 - 3x - 4 = 0$. $\therefore (x-4)(x+1) = 0$
Hence, x = 4, or x = -1.

6. $\log(64a^2/b^{-5}c^3) = \log 64 + \log a^2 - \log b^{-5} - \log c^3$
= 2 log 8 + 2.log a + 5.log b - 3.log c.

7. $a^x = c.b^x$
$\therefore$ x. log a = log c + x. log b
$\therefore$ x(log a - log b) = log c
$\therefore$ x. log $({}^a/_b)$ = log c
$\therefore$ x = log c / (log a - log b)

8. $\log\sqrt{({}^{36}/_{27})} = \log\sqrt{({}^4/_3)} = 1/2[\log 2^2 - \log 3]$
log 2 - (1/2) log 3 = 0.06247.

9. $6^{3-4x} . 4^{x+5} = 8$. considering log of both sides,
$(3 - 4x)\log 6 + (x+5)\log 4 = \log 8$
$(3 - 4x)(\log 3 + \log 2) + (2x+10)\log 2 = 3\log 2$
$(3 - 4x)(0.77815) + (2x+10)(0.30103) = 3 (0.30103)$
Solving the above, we get, $x =$ (approx.) 1.77

10. $\log 0.8 = \log_{10}8 - \log_{10}10 = 3\log_{10}2 - 1 = 0.90309 - 1 = 0.09691$.

11. L.H.S. $= (1/2) \log (9 + 2.3.2\sqrt{5} + 20) = \log [3^2 + 2.3.2\sqrt{5} + (2\sqrt{5})^2]^{1/2} = \log [(3 + 2\sqrt{5})^2]^{1/2}$
$= \log (3 + 2\sqrt{5})$
Comparing with R.H.S., we get, $4x = 2\sqrt{5}$. $\therefore x = (\sqrt{5})/ 2$

12. $\therefore 2x^2 + 2y^2 - 4xy = 16xy - 4xy = 12xy$
$\therefore 2 (x - y)^2 = 12xy$. $\quad \therefore 2 (x - y)^2 = 12xy$.
Taking logs of both sides, we have, $\log (x - y)^2 = \log (6xy)$
$\therefore 2 \log (x - y) = \log x + \log y + \log 6$
$\therefore \log (x - y) = 1/2 [\log x + \log y + \log 6] = (1/2) \log 6xy = \log (6xy)^{1/2}$

13. $\therefore 2 \log x + 3 \log y = a$, and $\log x - \log y = b$
multiplying the second equation by 3 and adding it to the first equation, we get, $5 \log x = a + 3b$
$\therefore \log x = (a + 3b) / 5$.
Substituting this value of log x in the first equation, we get, $\log y = (a - 2b) / 5$

14. We have , from the definition of the logarithm, $x^2 + y^2 = 10^2 = 100$.
This is possible for whole numbers x and y only if $x = \pm 6$, and $y = \pm 8$ or $x = \pm 8$ and $y = \pm 6$.
Accordingly $| x/y | = 3/4$ or $4/3$.

15. $\therefore \log a^2 = \log b^3 - \log 10$, since $1 = \log 10$.

$\therefore \log a^2 = \log (b^3/10)$ $\quad \therefore a^2 = b^3/10 \quad \therefore a = \sqrt{\dfrac{b^3}{10}}$

16. $\therefore \log (x^3 + y^3) = \log x^3 + \log y^3 = \log x^3y^3$
$\therefore x^3 + y^3 = x^3y^3$.

17. Taking logs of both sides,
$(x - 1) \log p = (x - 2) \log q$. Rearranging, we get,
$x.(\log p - \log q) = \log p - 2.\log q$
$\therefore x = (\log p - \log q^2) / (\log p - \log q) = [\log (p / q^2)] / [\log (p/q)]$

18. $\therefore \log x^3 = \log 162 + \log a - \log 6a$
$= \log [162 (a) / 6a] = \log 27$.
$\therefore x^3 = 27 \quad \therefore x = 3$.

19. $\therefore \log (x^a + y^b) = \log x^a.y^b \quad \therefore x^a + y^b = x^a.y^b \quad \therefore x^a.(1 - y^b) = -y^b$
$\therefore x = \sqrt[a]{\dfrac{-y^b}{1-y^b}}$

20. Taking logs, we have, $(5x - 12)\log 6 + (10 - 4x)\log 2 = \log 2 + (2x -3)\log 3$
Also rearranging after expressing log 6 as log 2 + log 3,
$(5x -12 + 10 -4x -1) \log 2 = (2x - 3 - 5x + 12) \log 3$.
$\therefore \log 2^{x-3} = \log 3^{9-3x}$ $\quad \therefore 2^x / 2^3 = 3^9 / 3^{3x} = 27^3 / 27^x$
$\therefore 2^x.27^x = 2^3.27^3$
$\therefore x = 3$.

21. $\log_6(216\sqrt{6}) = \log_6(6^{7/2}) = 7/2$.

22. $\log_6 16 = 4\log_6 2 = 4 / \log_2 6 = 4 / (\log_2 2 + \log_2 3)$.
$a = \log_{12} 27 = \log_{12} 3^3 = 3\log_{12} 3 = 3/\log_3 12 = 3/(\log_3 3+\log_3 4) = 3/(1+ \log_3 4) = 3/(1 + 2\log_3 2)$
$a + 2a \log_3 2 = 3$ 0r $\log_3 2 = (3 - a) / 2a$
$\log_2 3 = 2a / (3 - a)$,
$\log_6 16 = 4 / [(1 + 2a / (3 - a)] = 4(3 - a) / (3 + a)$.

23. $\log_a n / \log_{ab} n = \log_n ab / \log_n a = (\log_n a + \log_n b) / \log_n a = 1 + \log_n b / \log_n a = 1 + \log_a b$.

24. Given expression $= 7 \log(2^4 / 3x5) + 5\log(5^2 / 3x2^3) + 3\log(3^4 / 5x2^4)$
$= 7[4\log 2 - \log 3 - \log 5] + 5 [2\log 5 - \log 3 - 3\log 2] + 3[4\log 3 - \log 5 - 4\log 2]$
$= \log 2$.

25. $a^2 + b^2 = 7ab$, $(a + b)^2 = 9ab$, $[(a + b) / 3]^2 = ab$,
Taking logs on both sides, $2 \log[(1/3)(a + b)] = \log a + \log b$.

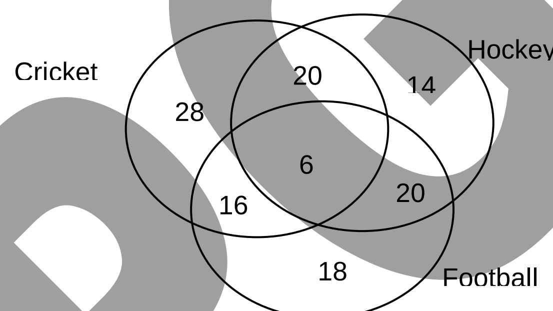
Concepts 9: SET THEORY.

1. $A \cap (B \cup C) = 5 + 3 = 8$.

2. We have $n(H \cup F) = n(H) + n(F) - n(H \cap F)$. Therefore $400 = n(H) +264 - 38$. On solving $n(H) = 136$.

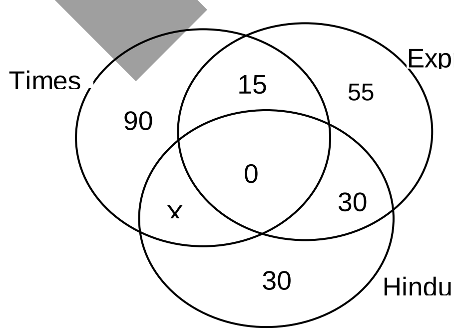
3. have $n(A \cup B) = n(A) + n(B) - n(A \cap B)$. Therefore $200 = 100 + 120 - n(A \cap B)$. ON solving we get $n(A \cap B) = 20$.

4.



We have the total no of students = 28 + 20 + 6 + 16 +14 +20 + 18 = 122. From the Venn diagram, the % of students who play Hockey only = (14/132) 100 = 10.61, % of the students who play football is (60/132)100 = 45.45, % of the students who play cricket only = (28/132)100 = 21.21.

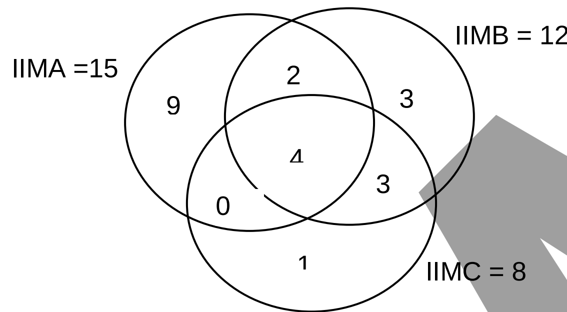
5.



We have 90 + 15 + x + 55 + 30 + 30 = 240. Therefore x = 20
(1) 125 families buy The Times of India.
(2) 55 families buy The Indian Express only.
(3) 175 families read only one paper.
(4) The Times of India is the most popular as 120 families read it.
(5) Hindu is the least popular as only 80 families read it.

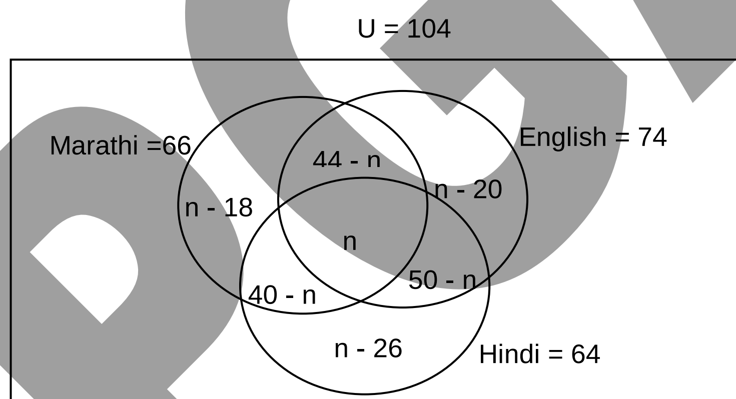
6. $S \cup B \cup T = 400$; $S \cup B \cup T = S+B+T-(SB_{only} + ST_{only} + BT_{only}) - 2.SBT$
i.e. $SB_{only}+ST_{only}+BT_{only} + 2.SBT = 144 + 135 + 156 - 400 = 35$
The number of members who play at least two games is ‘$SB_{only}+ST_{only}+BT_{only} + SBT$ ‘
Let this number be 3k. Then SBT will be 2k. i.e. $SB_{only}+SB_{only}+SB_{only} = k$
hence $SB_{only}+ST_{only}+BT_{only} + 2.SBT = k+2.(2k)=5k=35$ i.e. k = 7
(a)The number of members who play all the three games is SBT = 2k =14
(b) Also $BT_{only} = 0$ $SB_{only}+ST_{only}+BT_{only} = 7$ Therefore $SB_{only}+ST_{only} = 7$
$S = S_{only}+SB_{only}+ST_{only} + SBT$; $S_{only} = 144 - 7 - 14 = 123$

7.



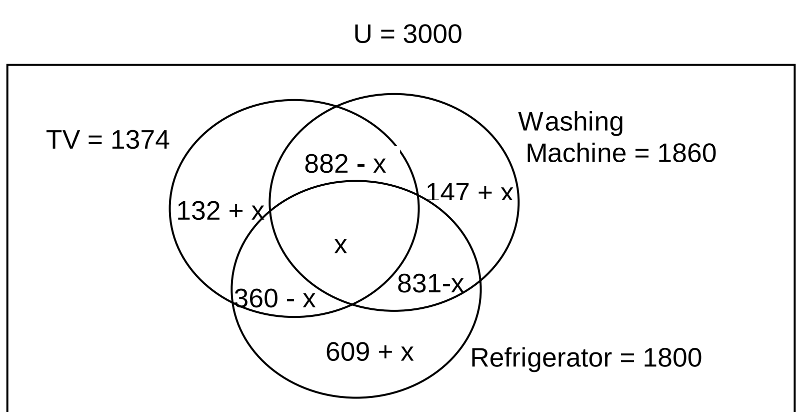
(a) 9 students get calls from IIMA only.
(b) 1 student gets a call from IIMC only.
(c) 3 students get calls from IIMB only.
(d) 5 students get calls from exactly two IIMs.
(e) 9 students get calls from more than one IIM.

8.



We have 104 - 38 = 66 speak Marathi, 104 – 30 = 74 speak English and 104 – 40 = 64 speak Hindi. Let n students speak all three languages. Therefore, from the Venn diagram we get, 66 + n – 20 + 50 – n + n – 26 = 104. On solving we get n = 34.
(a) 34 students speak all three languages.
(b)8 students speak Hindi only.
(c) 44 + 40 + 50 – (3 x 34) = 32 speak exactly two languages.

9.



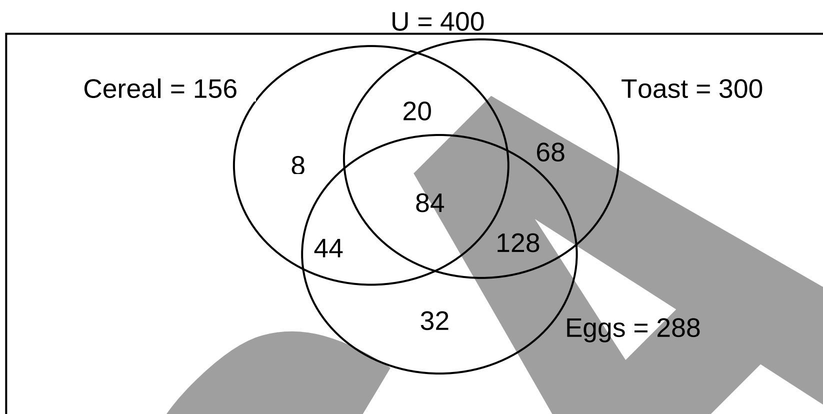
We have 3000 – 1200 = 1800 had refrigerators, 300 – 1140 = 1860 had washing machines and 3000 – 1626 = 1374 had T.V. sets. Let x households have all three appliances. Then, we have, 188 + (132 + x) + (882 – x) + (147 + x) = 3000. On solving the equation, we get, x = 39.

(a) 186 had washing machines only.

(b) 648 had refrigerators only.

(c) 882 + 360 + 831 – 2(39) = 1995 had more than one appliance.

10.



(a) 400 – (8 +20 + 84 + 44 + 68 + 128 + 32) = 16 families had neither toast nor cereal nor eggs for breakfast.

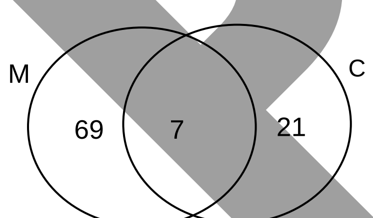
(b) 32 + 128 + 68 = 228 families had eggs and toast but not cereal for breakfast.

(c) 20 + 44 + 128 = 192 families had only two foods for breakfast.

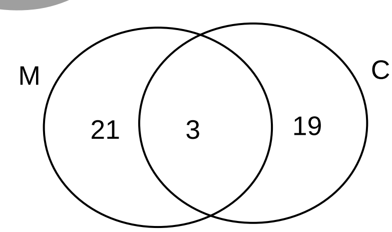
QUESTIONS 11 - 15:

(0 - 2) YEARS:

DEGREE (97) + (3 - not M, not C)

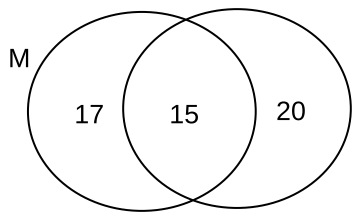


DIPLOMA (43) + (7 - not M, not C)

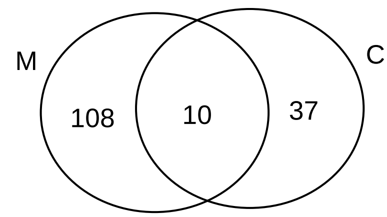


(2 - 5) YEARS:

DEGREE (52) + (0 - not M, not C)

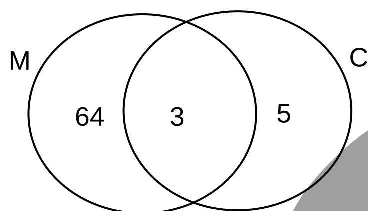


DIPLOMA (155) + (0 - not M, not C)

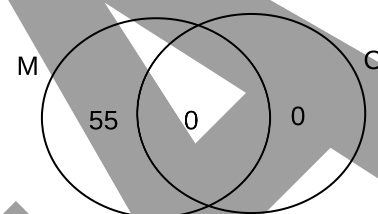


> 5 YEARS:

DEGREE (72) + (6 - not M, not C)



DIPLOMA (55) + (20 - not M, not C)



11. Out of the 150 employees in the (0 - 2) group, 40 employees have qualifications in Computers only. This forms 26.66% of the total number of employees in the (0 - 2) group.

12. From the Venn diagram, it is clear that 36 employees have neither a Mechanical background nor a Computers background.

13. The number of employees with a Mechanical degree only is 64. The additional expenses with regard to these employees are Rs. 5,76,000. The number of employees with a Computers degree only is 5 and the additional expenses associated with them are Rs. 55,000. The number of employees with both degrees is 3 and that of those with neither degree is 6. The additional expenses with regard to these employees are Rs. 60,000 and Rs. 36,000 respectively. Therefore, the total additional expenses associated with the bonus scheme are Rs. 7,27,000.

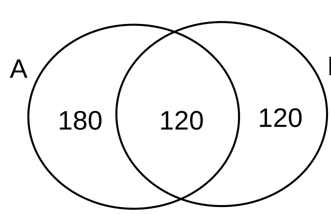
14. It is clear from the Venn diagram that the (2 - 5) group does not have employees with qualifications other than Mechanical or Computers or both.

15. In the (> 5) group, the number of employees with a Mechanical diploma only is 129 and that of employees with a Computers diploma only is 56. Therefore, the required ratio is 129:56.

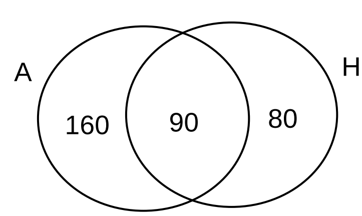
QUESTIONS 16 - 20:

(25- 35) YEARS:

PMIR (420) + (130 - not A, not H)

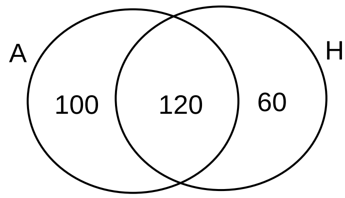


SS (330) + (20 - not A, not H)

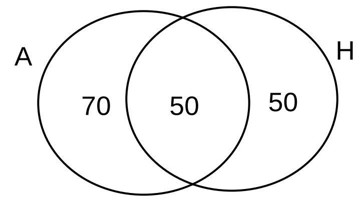


> 35 YEARS:

PMIR (280) + (120 - not A, not H)



SS (170) + (30 - not A, not H)



16. The number of people in the age group 25 - 35, employed in Administration but not in HRD is 340 and the total number of employees in that age group is 900. Therefore, the required percentage is 37.77%.

17. The number of Social Studies graduates in HRD but not in Administration is 190 and the total number of employees in the organisation is 2250. Therefore, the required percentage is 8.44%.

18. The total number of employees in HRD but not in Administration is 470. In the age group 25 - 35, 200 people are employed in HRD but not in Administration. 20% of these, i.e. 40 people shift to the >35 age group. The number of people in the first two age groups employed in HRD but not in Administration is now 320. So, the required percentage is 68.08%.

19. Out of the 900 people in the 25 - 35 age group, 180 shift to the > 35 age group. The number of employees in the > 35 age group is now 780, which forms 34.66% of the total number of employees in the organisation.

20. In the 25 - 35 age group, 210 people are employed in both, Administration and HRD. Of these, 42 people shift to the > 35 age group. In the > 35 age group, the number of people employed in both, Administration and HRD is 212. From the Venn diagram, it is clear that the number of people employed in Consultancy is 400. Therefore, the required percentage is 53%.

Concepts 7: BASE NOTATIONS.

1. (a)

$$\begin{array}{r|l} & \underline{0} \quad 1 \\ & \underline{1} \quad 1 \\ & \underline{3} \quad 0 \\ & \underline{6} \quad 1 \\ & \underline{13} \quad 1 \\ 2 & 27 \end{array} \downarrow$$

Hence, $(27)_{10} = (11011)_2$

(b)

$$\begin{array}{r|l} & \underline{0} \quad 1 \\ & \underline{1} \quad 0 \\ & \underline{2} \quad 0 \\ & \underline{4} \quad 0 \\ & \underline{8} \quad 1 \\ & \underline{17} \quad 1 \\ 2 & 35 \end{array} \downarrow$$

$\therefore (35)_{10} = (100011)_2$

(c)

$$\begin{array}{r|l} & \underline{0} \quad 1 \\ & \underline{1} \quad 0 \\ & \underline{2} \quad 0 \\ & \underline{4} \quad 0 \\ & \underline{8} \quad 1 \\ & \underline{17} \quad 0 \\ & \underline{34} \quad 1 \\ 2 & 69 \end{array} \downarrow$$

$\therefore (69)_{10} = (1000101)_2$

2.(a)

$$\begin{array}{r|l} & \underline{0} \quad C \\ & \underline{12} \quad 5 \\ 16 & 197 \end{array} \downarrow$$

$\therefore (197)_{10} = (C5)_{16}$

(b)

$$\begin{array}{r|l} & \underline{0} \quad 3 \\ & \underline{3} \quad B \\ & \underline{59} \quad C \\ 16 & 956 \end{array} \downarrow$$

$\therefore (956)_{10} = (3BC)_{16}$

(c)

$$\begin{array}{r|l} 16 & 577 \\ \hline & 36 \quad 1 \\ & 2 \quad 4 \\ & 0 \quad 2 \end{array} \quad \therefore (577)_{10} = (241)_{16}$$

3. (a) $(1001101)_2 = 1\text{x}2^6 + 0\text{x}2^5 + 0\text{x}2^4 + 1\text{x}2^3 + 1\text{x}2^2 + 0\text{x}2^1 + 1\text{x}2^0$
$= (77)_{10}$
   (b) $(11110)_2 = 1\text{x}2^4 + 1\text{x}2^3 + 1\text{x}2^2 + 1\text{x}2^1 + 0\text{x}2^0$
$= (30)_{10}$
   (c) $(110001)_2 = 1\text{x}2^5 + 1\text{x}2^4 + 0\text{x}2^3 + 0\text{x}2^2 + 0\text{x}2^1 + 1\text{x}2^0$
$= (49)_{10}$.

4. (a) $(9C)_{16} = 9\text{x}16^1 + 12\text{x}16^0$, since C in hexadecimal is 12.
$= (156)_{10}$
   (b) $(F6)_{16} = 15\text{x}16^1 + 6\text{x}16^0$
$= (246)_{10}$
   (c) $(1A3C)_{16} = 1\text{x}16^3 + 10\text{x}16^2 + 3\text{x}16^1 + 12\text{x}16^0$
$= (6716)_{10}$

5. Since, four is one of the characters in the given number, the base for that number system must be ≥ 5. Assume that the base = 5. Then, $(4 \times 5^4) + (0 \times 5^3) + (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^0) = 2542$. So, by estimation, we can say that the base of the number system is 5.

6. Let the base of the number system be N. So, $13 \times 13 = 169 = 1N^2 + 2N^1 + 1N^1$. The quadratic equation $N^2 + 2N - 168 = 0$ gives N = 12. In this number system, 2174 is written as 1312.

7. Let the base of the number system be N. So, $35 = 5N^1 + 5N^0$, giving N = 6. $(111111)_2 = (1 \times 2^5) + (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 63$. In the number system with base 6, 63 will be written as 143.

8. $(110100101)_2 = (1 \times 2^8) + (1 \times 2^7) + (1 \times 2^5) + (1 \times 2^2) + (1 \times 2^0) = 421$. In the hexadecimal system, 421 will be written as 1A5.

9. $(11001101)_2 = (1 \times 2^7) + (1 \times 2^6) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^0) = 205$. In the number system with base 7, 205 will be written as 412.

10. Let the base of the number system be N. $(7917 - 6233) = 1684 = (4 \times N^2) + (4 \times N^1) + (4 \times N^0)$, giving N = 20. The hexadecimal $(FE3 - 7AA) = (4067 - 1962)_{10} = 2105$. In the number system with base 20, 2105 will be written as 555.

Concepts 8: PROGRESSIONS.

1. It is an A.P. with a = 2, d = 3
Using the formula $S_n = n/2[\,2a + (n-1)d\,] = 950$
i.e. $n/2[\,4 + (n-1)\text{x}3\,] = 950$.
$3n^2 + n = 1900. \Rightarrow (3n+76)(n-25) = 0. \Rightarrow n = 25$.

2. It is an A.P. with a = 3/4, d = -1/12
Using the formula for S_n, we get,
$S_9 = (9/2)[\,2(3/4) + (9-1)(-1/12)] = 15/4$.

3. 1st term = 4(1) + 1 = 5, 2nd term = 4(2) + 1 = 9, 3rd term = 4(3) + 1 = 13
It is an A.P. with a = 5, d = 4 $\therefore$ Using the formula for S_n,
$S_{15} = \frac{15}{2}\{2(5) + (15-1)4\}$ $\therefore S_{15} = 495$.

4. For natural numbers, First term = 1, Common difference = 1. Using the formula for S_n

$S_n = (n/2)\{ 2(1) + (n-1)(1) \}$, $\therefore S_n = [n (n + 1)] / 2$

5. Let the three numbers be a - d, a, a + d

$\therefore$ a - d + a + a + d = 27 $\Rightarrow$ a = 3 $\therefore$ (9 - d) (9) (9 + d) = 504.

Solving for d, we have, $729 - 9d^2 = 504$.

$\therefore d^2 = 25$. $\therefore d = \pm 5$.

$\therefore$ The numbers are 4, 9, 14 or 14, 9, 4

6. a = 9, d = 3, S_n = 306 Using the formula for S_n

$306 = n/2 \{ 2(9) + (n-1)(3) \}$ $\therefore 612 = 3n^2 + 15n$

$n^2 + 5n - 204 = 0$. $\therefore (n + 17)(n - 12) = 0$.

Discarding the negative value for n, we have, n = 12.

7. a = 1/3, r = 3/2. Using formula for S_n for a Geometric series,

$S_7 = \{1/3 \ [(3/2)^7 - 1] \} / (3/2 - 1) = 2059/192$

8 Let the numbers be a/r, a, ar

$\therefore$ (a/r).a.(ar) = a^3 = 1728 $\Rightarrow$ a = 12 $\therefore$ 12/r + 12 + 12r = 38.

$\therefore$ Solving the above equation as a quadratic, we get, (3r - 2) (2r - 3) = 0, i.e. r = 2/3 , or r = 3/2.

$\therefore$The numbers are 18, 12, 8.(both values of the common ratio give the same three numbers)

9. Let the first term = a, number of terms = n

$a.3^{n-1} = 486$ $\therefore 3^n/3 = 486 / a$ $\therefore 3^n = 1458/a$

Also, the sum of n terms = $S_n = a.(3^n - 1) / (3-1) = 728$

Substituting the value of (3^n) in the above equation ,

a. [1458/a- 1] = 1456 $\therefore$ a [1458 - a] = 1456.a $\therefore a^2 = 2.a$

$\therefore$ a = 2.

10. Proceeding as in the above problem, we get, $7.\ r^{n-1} = 448$, if n terms are considered.

$\therefore r^{n-1} = 64$ $\therefore r^n = 64.r$

Also, 7. $(r^n - 1) / (r-1) = 889$. $\therefore$ 7 (64r - 1) / (r-1) = 889.

$\therefore$ 442.r = 882. $\therefore$ r = 2

11. Let the three numbers be (a/r), a, a.r, with r as the common ratio.

$\therefore$ their product = (a/r).a. a.r = 216 i.e. a^3 = 216. $\therefore$ a = 6.

$\therefore$ the sum is : 6/r + 6 + 6.r = 19. $\therefore 6r^2 - 13r + 6 = 0$

$\therefore$ (3r - 2) (2r - 3) = 0

$\therefore$ r = 3/2 or r = 2/3.

The numbers are 4, 6, 9 or 9, 6, 4.

12. Since 46 is fifth term of an A.P. whose first term is 18;

46 = 18 + (5 - 1) d $\Rightarrow$ d = 7.

$\therefore$ a = 18 + 7 = 25, b = 32 and c = 39.

13. Let the number of terms be n, difference = d

$\therefore$ 155 = (n/2)[a + l] = n/2 (29 + 2) $\Rightarrow$ n = 10.

Also l = a + (n-1) d $\Rightarrow$ 29 = 2 +d (n - 1).

$\therefore$ 9 d = 27 $\Rightarrow$ d = 3.

14. If a is the first term, S_{15} = 600 = (15/2) [2.a + (15 - 1) (5)]

$\therefore$ 2.a = 10. $\therefore$ a = 5.

15. Given series becomes 5, 7, 9, 11, . . . , 33. Which is an A.P. with a = 5, and l = 33

$\therefore S_{15}$ = (n/2) (a + l) = (15/2) (5+ 33) = 285

16. If Rs.a is the first installment and the common difference is Rs. d,
sum of all installments = S40 = (40/2) [2.a + (40 - 1) d] = total debt = 3600.
$\therefore$ 2a + 39d = 180...(1)
Also, S30 = (30/2) [2.a + (30 - 1) d] = 2/3 of the total debt = 2400.
$\therefore$ 2.a + 29d = 160...(2)
Subtracting (2) from (1), we get, 10d = 20. $\therefore$ d = 2.
$\therefore$ a = Rs. 51.

17. If the numbers are a - d, a, and a + d, their sum = 3.a = 24. $\therefore$ a = 8
$\therefore$ the numbers are 8 - d, 8, 8 + d.
$\therefore$ The numbers 8 - d -2, 8 - 4, and 8 + d - 4 form a GP.
$\therefore$ The middle number is the geometric mean.
$\therefore (4)^2 = (6 - d) (4 + d)$
$\therefore d^2 - 2d - 8 = 0 \quad \therefore (d - 4) (d + 2) = 0$
$\therefore$ d = 4, or d = -2.
$\therefore$ the numbers are 4, 8, and 12, or 10, 8, and 6.

18. If a_{11}, a_{12}, a_{13} are the middle terms, and a_{21}, a_{22}, a_{23} are the last three terms,
$a_{11} + a_{12} + a_{13} = a + 10d + a + 11d + a + 12d = 144$
$\therefore$ 3a + 33d = 144 $\therefore$ a + 11d = 48.
Also, $a_{21} + a_{22} + a_{23} = a + 20d + a + 21d + a + 22d = 264$.
$\therefore$ a + 21d = 88. Subtracting the earlier equation from this, we get,
10d = 40. $\therefore$ d = 4. and a = 4.
The 16th term = 4 + (16 - 1)(4) = 64.

19. Since the first and the fourth are same, the numbers can be : a + 6, a - 6, a, a + 6.
Since the first three are in GP, $(a - 6)^2 = a (a + 6)$.
$\therefore$ 36 = 18.a $\therefore$ a = 2.
The numbers are 8, -4, 2, 8.

20. We have the following equations : a + b + c = 25...(1), 2.a = 2 + b...(2), and c^2 = 18b... (3)
Substituting (2) in (3), $c^2 = 36.(a - 1)$
Substituting (2) in (1), 3a + c = 27. $\therefore 3a + \sqrt{36 (a - 1)} = 27$.
$\therefore (27 - 3a)^2 = [\sqrt{36 (a - 1)}]^2$
$\therefore 9 (81 - 18a + a^2) = 36.(a - 1)$
$\therefore$ Rearranging, we get, : (a - 5) (a - 17) = 0
$\therefore$ a = 5, or a = 17.
Correspondingly, c = 12, or c = -24.
Correspondingly, b = 8, or b = 32.
Thus the required values can be 5, 8, 12 or 17, 32,-24.
The second triplet is inadmissible, because a, b and c must be between 2 and 18.

21. Let the two sets be $a_1 - d_1$, a_1, $a_1 + d_1$ and $a_2 - d_2$, a_2 , $a_2 + d_2$.
$\therefore a_1 - d_1 + a_1 + a_1 + d_1 = 15$ and $a_2 - d_2 + a_2 + a_2 + d_2 = 15$
$\therefore a_1 = 5 = a_2$.
Also, $d_2 + 1 = d_1$, and $(a_1 - d_1). a_1.(a_1 + d_1) / (a_2 - d_2). a_2.(a_2 + d_2) = 7/8$
$\therefore (25 - d_1^2) / (25 - d_2^2) = [25 - (d_2 + 1)^2] / (25 - d_2^2) = 7/8$
Expanding the above equation, we get : $d_2^2 + 16d_2 - 17 = 0$. $\therefore (d_2 + 17) (d_2 - 1) = 0$.
$\therefore d_2 = -17$, or $d_2 = 1$, i.e. $d_1 = -16$ or $d_1 = 2$.
Thus we have two possible cases : (1). The numbers are 21, 5, -11 and 22, 5, -12.
(2). The numbers are 3, 5, 7 and 4, 5, 6.

22. Let a and d be the first term and the common difference respectively.
$\therefore S_p = (p/2) [2a + (p - 1).d] = S_q = (q/2) [2a + (q - 1).d]$
$\therefore 2pa + p.d (p - 1) = 2qa + q.d (q - 1)$
Rearranging, we get, $2a (p - q) + (p^2 - q^2).d - (p - q).d = 0$
Assuming $p \neq q$, 2a + (p + q) .d -d = 0.
$\therefore$ 2a + (p + q - 1).d = 0.

Also, $S_{p+q} = [(p + q)/2] [2a + (p + q - 1).d]$
$= 0$, from the above equation.

23. Let D and d be the two common differences.
$\therefore A_3 = b = a + 2D = b = a_5 = a + 4d$ $\therefore D = 2d.$
$\therefore A_{n+1} = a + n.D = a + 2n.d$ and $a_{2n+1} = a + 2n.d$
$\therefore [A_{n+1}] / a_{2n+1} = 1.$

24. Let the numbers be (a/r, a, ar) . $\therefore$ (a/r) + a + ar = 70
$\therefore$ the numbers (4a/r, 5a, 4ar) are in AP. $\therefore$ 2 (5a) = 4a/r + 4ar
$\therefore 2r^2 - 5r + 2 = 0.$ $\therefore (2r - 1) (r - 2) = 0.$
$\therefore$ r = (1/2) or r = 2.
Substituting the above in the first equation, we get, a = 20.
The numbers are 10, 20, 40. (The same result is achieved by r = 1/2)

25. Let the numbers be a, ar, ar^2, ar^3....., with a as the first term and r as the common ratio.
The numbers ar, ar^2, ar^3, . . . are also infinite terms in GP with ar being the first term and r being the common ratio.
The sum of the above GP = (a.r) / (1 - r)
Each term is thrice the sum of all the following terms. Obviously, r < 1.
$\therefore$ a = 3. a.r / (1 - r) $\therefore$ r = 1/4.
Also, sum of first two terms = a + ar = 5 $\therefore$ a = 4.

26. Since a, b, c are in A.P. , b = (a+c)/2
RHS = $4(b^2 - ac) = 4\{ [(a+c)^2 /4] - ac \} = (a - c)^2$.

27. $\log a + \log(a^2 / b) + \log (a^3 / b^2) + \log (a^4 / b^3) + . . .$ n terms
= (loga + 2loga + 3loga + . . . n loga) - (logb + 2logb + 3 logb + . . . (n-1) logb)
These are A.P.'s with c.d. loga and logb respectively
= n/2(loga + n loga) - ((n-1)/2) [logb + (n -1) logb]
= n/2 [n log(a/b) + logab]

28. S = .7 + .77 + .777 + . . . n terms
= 7/9 [.9 + .99 + .999 + . . . n terms]
= 7/9 [(1 - 1/10) + (1 - 1/100) + (1 - 1/1000) + ...]
= $7/9 [(1 + 1 + 1 + . . . \text{n times}) - (1/10 + 1/10^2 + 1/10^3 + . . . +1/10^n)]$
= $7/9 [n - (1/10). (1 - (1/10^n)) / (1 - 1/10)]$
= $(7n / 9) - (7/81)[1 - 1/10^n]$.

29. $S = 1 + ab + a^2b^2 + a^3b^3 + ... \text{to } \infty = 1 / 1 - ab$
x = 1/ (1 - a), a = (x -1) / x, y = 1/(1 - b), b = (y - 1) / y,
S = 1/ [1 - (x - 1)(y - 1) / xy] = xy / (x + y - 1).

30. $1(4^n - 1) / (4 - 1) = 341,\ 4^n - 1 = 1023 \Rightarrow 4^n = 1024 = 4 \times 256 = 4^5 \therefore n = 5.$

31. Let the harmonic progression be represented by 1/a, 1/(a+d), 1/(a+2d),……We have (n+1)th term as 1/(a+nd) and (3n+1)th term as 1/(a+3nd). It is given that 1/(a+nd) = 2x1/(a+3nd). On solving we get a = nd. Therefore the (n+1)th term = 1/(a+nd) = 1/(a+a) = ½ (1/a). Therefore, the ratio of the first term and the $(n + 1)^{th}$ term is 2.

32. Let the harmonic progression be represented by 1/a, 1/(a+d), 1/(a+2d),……. Therefore we have a+(a+d)+(a+2d)+……+(a+6d) = (7/2) [(a)+(a+6d)] = 7(a+3d) = 70,i.e. a+3d = 10. Now the fourth term of the H.P. is 1/(a+3d) = 1/10.

33. Let the harmonic progression be 1/a, 1/(a+d), 1/(a+2d), ---. Now we have mth term = 1/[a+(m-1)d] = n and nth term = 1/[a+(n-1)d] = m. On solving these two equations we get a = d = 1/(mn). Therefore (m+n)th term is 1/[a+(m+n-1)d] = mn/m+n.

34. Given series is in H.P. Therefore we have (n+1) term = 1/(n+mn) = (1/n) (1+m).

35. We have $(x^{n+1} + y^{n+1})/(x^n+y^n) = 2xy/(x + y)$. Therefore $x^{n+2} + yx^{n+1} + xy^{n+1} + y^{n+2} = 2x^{n+1}y + 2xy^{n+1}$. So, $x^{n+2} + y^{n+2} = x^{n+1}y + xy^{n+1}$. On solving $x^{n+1}(x - y) - y^{n+1}(x - y) = 0$. Therefore $(x - y)(x^{n+1} - y^{n+1}) = 0$. Therefore, n = -1.

Concepts 9: SERIES.

1. 0, 2, 12, 36, 80,
The term follows this rule i.e $n (n -1)^2$ hence the next term will be 150.

2. 1, 3 , 7, 13 , 21, 31, ?
The series follows the rule of $n^2 - (n - 1)$ hence the next term will be 43.

3. 3, 7, 15, 31, 63,.
In this series, each term is equal to one more than twice the previous term.

4. 19, 119, 1119, 11119,
This is too easy and the answer is 111119.

5. 7, 19, 37, 61, 91,
The law followed here is $(n + 1)^3 - n^3$, i.e, $7^3 - 6^3 = 127$

Alternate method: The difference in the consecutive terms increases by multiples of 6.
7, 7+12, 19+18, 37+24, 61+30, 91+36……….

6. 1, 3 , 7, 13, 21,31
Here again as difference between the terms are multiple of 2 so the ans is 31 + 12 = 43.
Alternate method: This series follows a rule: $n^2 – (n-1)$.

7. 1, 4, 27, 256, 3125,
The term is n^n so the term must be $6^6 = 46656$.

8. 11, 13, 17, 23, 29,
Each term is nothing but prime number so the next term must be 31.

9. 1! , 2!, 720!
This is an interesting series we see factorial signs nothing can be interpretted from first two terms but as soon as third term is seen we can say 720 is nothing but 6! Again 6 can be written as 3!. So the whole series can be considered as
1! , Then factorial of factorial 2, the third term which is factorial of factorial of factorial 3, so the next term will be factorial of factorial of factorial of factorial 4.

10. 1, 6, 15, 20, 15, 6,
This nothing but the series of the coefficients of binomial expansion with the power 6, so the next term will be 1.

Concepts 10: PERMUTATIONS AND COMBINATIONS.

1. The first person can take a seat in 6 ways, the second person in 5, the third in 4, and the fourth in 3 ways. ∴ no. of ways in which four can take their places = 6 x 5 x 4 x 3 = 360.
Alternatively, we can solve directly as the no of ways = ${}^6P_4 = 360$.

2. The Thousand’s place can be filled in 6 ways, Hundred’s place can be filled in 5 ways, Ten’s place can be filled in 4 ways, Unit’s place can be filled in 3 ways. Number of ways of making 4 digit numbers = 6 x 5 x 4 x 3 = 360. Alternatively, ${}^6P_4 = 360$. Numbers greater than 4000 : Thousand’s place can be filled in 4 ways (4,5,6,7), Hundred’s place can be filled in 5

ways (remaining three digits and 2,3), Ten's place can be filled in 4 ways and Unit's place can be filled in 3 ways. Total number of ways = 4 x 5 x 4 x 3 = 240

3. * B1* B2 * B3 * B4 * B5 * Five boys can be seated in 5! ways. The girls can occupy any of the places marked *. Thus the girls can be seated in ${}^6P_4$ = 360 ways.
$\therefore$ total number of ways = 5! x 360 = 43200.

4. (F1, F2, F3), (S1, S2), (T1).
The first year students can be arranged among themselves in 3! ways, second year in 2! ways and the third year student can stand in one way.
For each arrangement of the students in their respective groups, the three groups can be arranged in 3! ways.
$\therefore$ Number of ways of arrangement = 3! x 2! x 1 x 3! = 72 ways.

5. This is equal to arranging 5 persons at a time by choosing from 8 persons.
$\therefore$ no of ways = ${}^8P_5$ = 6720.

6. R.H.S. = $\frac{3}{5}\ {}^7C_4 = \frac{3}{5}\cdot\frac{7!}{3!\,4!} = \frac{7!}{2!\,5!} = {}^7C_5$
Now, comparing with ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$ we get r = 5,
and n + 1 = 7 $\Rightarrow$ n = 6. $\therefore$ ${}^nC_r = {}^6C_2$ = 15.

7. n! / [(n - 12)! 12!] = n!/ [(n - 8)! 8!] $\therefore$ n - 12 = 8 $\therefore$ n = 20.
$\therefore$ ${}^{20}C_{17} = \frac{20!}{(20-17)!\,17!}$ = 1140.
$\therefore$ ${}^{22}C_{20}$ = 22! / [(22-20)! 20!] = 231

8. (a) one book is always to be included. Thus the remaining 4 books can be chosen out of the remaining 11. $\therefore$ no. ways of selection = ${}^{11}C_4$ = 330.
(b) One book to be excluded. $\therefore$ selection of 5 books to be made from the remaining 11 $\therefore$ no. ways of selection = ${}^{11}C_5$ = 462

9. The required no of ways is the ways of selecting 11 things out of 14. = ${}^{14}C_{11}$ = 364.

10. The captain can be chosen in 11 ways, Vice captain can be chosen in 10 ways
Required no. of ways = 11 x 10 = 110.

11. (a) The hundred's place can be filled in 5 ways (0 cannot come in the first place), the ten's in 5 ways, and the one's in 4 ways. $\therefore$ no. of ways = 5 x 5 x 4 = 100.
(b) The hundred's place in 5 ways, the ten's in 6 ways, and the one's in 6 ways.
$\therefore$ no of ways = 180.

12. 4 teachers to be assigned to 4 classes out of 6. i.e. arrangement of 4 things out of 6 things i.e. ${}^6P_4$ = 360.

13. Each of the six digits can be allotted in 9 ways. $\therefore$ total number of ways = 9^6 = 531441

14. A straight line can be drawn through 2 given points, in this case ${}^8C_2$ = 28 lines can be drawn. Out of these the points selected from the bunch of 4 collinear points will give only one line i.e. ${}^4C_2$ = 6 lines are not distinct and must be counted as only one line. $\therefore$ number of lines = 28 - 6 + 1 = 23.

15. The required numbered of selections is the number of ways of selecting one or more objects from n different objects = 2^n - 1.
In this case, no. of selections = 2^5 - 1 = 31.

16. Since all one rupee coins are identical, 1, 2, 3, 4, or 5 coins of one rupee can be selected in only one way each. $\therefore$ different number of ways of selecting one rupee coins = 5, Similarly 50 Ps. and 25 Ps. coins can be selected in 4 and 3 different ways respectively.
$\therefore$ Total number of ways = $5 \times 4 \times 3 = 60$.

17. Odd digits are 1, 1, 3, 3 & even are 2, 2, 4
The even digits are arranged with a gap in between _ 2 _ 2 _ 4 _ This arrangement of the even digits can be done (considering that 2 is repeated) in (3!) / (2!) = 3 ways. Odd digits can be arranged in odd places i.e. the blanks (3 and 1 are repeated) in (4!) / (2! 2!) = 6 ways.
$\therefore$ total number of ways = 3 x 6 = 18

18. $\therefore$ 10! / [(10-r-1)!] **:** 11! / [(11-r)!] **::** 30 **:** 11
$\therefore$ (11 - r)! / [11 (9-r)!] = 30 / 11
$\therefore$ (10 - r) (11 - r) = 30, since (11-r)! = (9-r)! (10 - r) (11 -r)
$\therefore$ 110 - 21r + r^2 = 30.
$\therefore$ (16 - r) (5 - r) = 0 $\therefore$ r = 16 or r = 5.
But r cannot exceed 9 as ${}^{10}P_{r+1}$ is defined.

19. 2 men can be chosen out of 8 in ${}^8C_2$ = 28 ways.
The wives of these two cannot play. $\therefore$two ladies have to be selected from 6 ladies in ${}^6C_2$ = 15 ways. If G_1 , G_2, L_1, L_2 are the gents and ladies selected then two games can be played i.e. $G_1 L_1$ Vs G_2L_2 or G_1L_2 Vs G_2L_1 in 2 ways.
$\therefore$ total number of matches = 2 x 28 x 15 = 840.

20. He may vote for 1, 2, 3, or 4 candidates.
$\therefore$ he can vote in 7 + ${}^7C_2$ + ${}^7C_3$ + ${}^7C_4$ = 98 ways.

21. The two vowels can be arranged in 5 x 4 = 20 ways.
The 17 consonants can take the two places in 17 x 17 = 289 ways, since the same consonant can appear in both places.
$\therefore$ Total number of four-letter words of the required kind = 20 x 289 = 5780.

22. Total number of ways of arranging 6 papers = 6! = 720.
We consider the two mathematics papers to be together, i.e. as a single paper.
$\therefore$ ways of arranging 5 papers = 5! x 2 = 240, since for every arrangement of 5 papers, the two mathematics papers can be arranged amongst themselves in 2 ways.
$\therefore$ no. of arrangements in which the mathematics papers are **not** together = 720 - 240 = 480.

Alternate method: * _ * _ * _ * _ *
Four non-mathematics papers can be arranged in the places of dashes(_ _ _) in 4! number of ways and two mathematics papers can be arranged in the places of the asterick (* * *) in ${}^5P_2$ number of ways. So, the total number of ways = 4!. ${}^5P_2$ = 480

23. $\therefore$ (2n)! / [(2n - 3)!3!] **:**n! / [(n-2)! 2!] **::** 44 **:** 3. $\therefore$ (2n-2)(2n-1)2n / [(n-1) n x 3] = 44/3
$\therefore$ 2n - 1 = 11 $\therefore$ n = 6.

24. The number of ways is same as the number of ways in which m+n+p things can be divided into three groups containing m, n, and p respectively, which is, (m+n+p)! / [m! n! p!]
$\therefore$ the required number of different allotments = 45 ! / [(10)!(15)!(20)!]. Further these three different groups can be allotted to 3 different districts in 3! Ways.
Hence the answer is 3!(45 ! / [(10)!(15)!(20)!])

25. $\therefore$ 28! / [(28 - 2r) !(2r) !] **:** 24! / [(24 - 2r + 4)!(2r-4)!] **::** 225 **:** 11
$\therefore$ 25 x 26 x 27 x 28 / [(2r-3)(2r-2)(2r-1)(2r)] = 225 / 11
After this point, you should try to get the answer by substituting the various alternatives provided, since trying to solve a 4-degree equation in r will lead to a waste of valuable time.

26. Each of the n things can be given to any of the p persons, i.e. it can be given in p ways. Each of the remaining n-1 things can also be given out in p ways each.
$\therefore$ Required number of ways = p x p x p ... n times = p^n

27. Total number of different attempts = 15 x 15 x 15 = 3375.
$\therefore$ Only one attempt of the above is successful.
$\therefore$ maximum number of unsuccessful attempts = 3374.

28. For every constant value of a variable, there are 4 constant values of each of the other 4 variables. Also the positions of the variables in the arrangement can be changed in 5! ways.
$\therefore$ The total number of possible codes = 4 x 4 x 4 x 4 x 4 x 5!.= 1024 x 5!.

29. $\therefore$ Every individual shakes hands with 14 other individuals.
We have, in all, 15 individuals.
$\therefore$ number of handshakes = 14 x 15 = 210.
But the above number includes handshakes counted twice i.e. cases of A shaking hands with B, and B shaking hands with A.
$\therefore$ Actual number of handshakes = 210/2 = 105.

Alternate method: For a handshake, two people are required. Two people out of five can be selected in ${}^{15}C_2$ ways, which comes to 105 ways.

30. Soft-drinks can be chosen in 2^3 - 1 ways, Chinese dishes in 2^4 - 1, and ice-creams in 2^2 - 1 ways. $\therefore$ Required number of selections = (2^3 - 1)(2^4 - 1)(2^2 - 1) = 315 ways.

31. Consider the chairman and the vice-chairman as one individual. So, instead of 15 members to be arranged, we now have only 14. These 14 members can be arranged around a round table in 13! ways. Also, the chairman and the vice-chairman can be arranged in 2! ways. $\therefore$, the total number of ways is 2x13!.
If the chairman and the vice-chairman are never to be seated together, then the total number of ways is 14! – 2x13! = 12x13!.

32. 7 boys can be arranged in 7! Ways. 5 girls can then occupy any of the places denoted by *.
*B**B*B*B*B*B*B* there are 8 such places which can be filled in ${}^8P_5$ ways.
Hence the answer is $7!\text{x}{}^8P_5$

33. Akshay can call one, two, three, four or five of his friends at a time and arrange them accordingly. So, the required number of ways is $({}^5C_1\text{x}0!) + ({}^5C_2\text{x}1!) + ({}^5C_3\text{x}2!) + ({}^5C_4\text{x}3!) + ({}^5C_5\text{x}4!)$=5+10+20+30+24 = 89.

34. <u>Case 1</u>: 4 batsmen, 6 bowlers, 1 wicket-keeper.
The number of ways is ${}^8C_4\text{x}{}^6C_6\text{ x}{}^2C_1$ =140.
<u>Case 2:</u> 5 batsmen, 5 bowlers, 1 wicket-keeper.
The number of ways is ${}^8C_5\text{x}{}^6C_5\text{ x}{}^2C_1$= 672.
<u>Case 3</u>: 6 batsmen, 4 bowlers, 1 wicket-keeper.
The number of ways is ${}^8C_6\text{x}{}^6C_4\text{ x}{}^2C_1$ =840.
$\therefore$, the total number of ways is 140+672+840 = 1652.

35. 10 people can be arranged in 10! Ways. Let us find the number of ways of arrangements where no 2 girls sit together. 5 boys can be arranged in 5! Ways. 5 girls can then occupy any of the places denoted by *. *B**B*B*B*B* there are 6 such places which can be filled in ${}^6P_5$ ways.
Therefore the number of ways of arrangement where no 2 girls sit together is $5!\text{x}{}^6P_5$ = 5!x6!.
Hence the answer is 10!-(5!x6!)

36. The required number of ways is (33!) / (3! x 11! x 11! x 11!).

37. There are 2 C's, 2 O's, 2 R's, 3 E's, 2 N's, 1 S, 1 P and 1 D. the total number of consonants is 9 and the total number of vowels is 5. Five vowels and a group of consonants can be arranged in (6!) / (3! x 2!) Ways. The 9 consonants can be arranged among themselves in (9!) / (2! x 2! x 2!) ways.

Therefore, the total required number of ways is $\frac{6!}{(3! \times 2!)} \times \frac{9!}{(2! \times 2! \times 2!)}$.

38. Numbers less than 10,000 can be single digit, two, three or four digit numbers. There are two single digit numbers – 4 and 8.

<u>2-digit numbers</u>: For these numbers to be even, their units digit will have to be either 4 or 8. The tens digit of these numbers can be chosen from among four digits in four ways. So, there are 2 x 4 = 8 such numbers.

<u>3-digit numbers</u>: For these numbers to be even, their units digit will have to be either 4 or 8. The tens digit of these numbers can be chosen from among four digits in four ways. The hundreds digit of these numbers can be chosen from among four digits in four ways. So, there are 2 x 4 x 4 = 32 such numbers.

4<u>-digit numbers</u>: For these numbers to be even, their units digit will have to be either 4 or 8. The tens digit of these numbers can be chosen from among four digits in four ways. The hundreds and the thousands digits of these numbers can be chosen from among four numbers in four different ways each. So, there are 2 x 4 x 4 x 4 = 128 such numbers.

Therefore, there are (2 + 8 + 32 + 128) = 170 numbers.

39. 5 boys can be arranged in 5! Ways. 3 girls can then occupy any of the places denoted by *.
*B**B*B*B*B* there are 6 such places which can be filled in $^6P_3$ ways.
Hence the answer is $5! \times {}^6P_3 = 14400$.

40. For the number to be an even number, the units digit must be 4. The tens and the hundreds digits can be chosen from among three digits in 3 ways each. So, there are 3 x 3 x 1 = 9 such numbers.

41. We require that the particular person is found in every group of six people from out of a total of ten. As this person is common to all groups of six, we will have to consider the number of ways of choosing the remaining five persons from out of the other nine. This can be done in $^9C_5$ ways = 126 ways.

42. Since we require at least one lady on the committee, we could have one, two, three or four ladies on the committee. A committee of 5 from among 10 people can be chosen in $^{10}C_5$ ways = 252 ways. These ways include committees where there are no ladies. A committee with no ladies on it will have five men from among six, i.e. $^6C_5$ ways = 6 ways. So, the required number of ways is 252 – 6 = 246.

43. 4 men can be chosen from among 6 in $^6C_4$ ways and 3 women from among 4 can be chosen in $^4C_3$ ways, i.e. 60 ways. If the two women, W1 and W2 are on the same committee, then the third woman can be chosen in 2 ways. In this case, the number of ways will be $^6C_4 \times 2 = 30$ ways. Therefore, the number of ways of forming the committee if W1 and W2 are never on the same committee is 60 – 30 = 30.

44. There are 2 U's, 1 S, 1 R, 1 P, 1 E, 2 A's, 3 N's, 1 T, 1 I and 1 O. There are 7 vowels. The number of ways I which this group and 7 consonants can be arranged is (8! x 7!) / (3! x 2! x 2!). If the vowels form one group and the same letters are always together, the number of ways of arranging them is (6! x 5!). Therefore, the total required number of ways is (8! x 7!) / (3! x 2! x 2!) - (6! x 5!).

45. If the captain and the vice-captain are always included, the number of ways is $^9C_4 \times 6!$. But, these photographs can be taken by the photographers in 4 different ways. So, the required number of ways is $^9C_4 \times 6! \times 4$.

If the captain and the vice-captain are never included, the number of ways is $^9P_6$. But, these photographs can be taken by the photographers in 4 different ways. So, the required number of ways is $^9P_6 \times 4$.

46. Since we are taking letters three at a time, the total number of three letter words is 6 x 5 x 4 = 120. The number of words not containing A and E is 4 x 3 x 2 = 24. Therefore, the required number of words is 96.

47. Since we require that there is exactly one 3, this digit could occupy any of the five places. If the number does not start with 3, then the first digit must necessarily be a digit other than 3 and 0. In such a case, the first digit can be chosen in eight ways and the remaining digits can be chosen in 9 ways each. The numbers will be of the following forms: 3XXXX, X3XXX, XX3XX, XXX3X, XXXX3. Correspondingly, the total number of ways is $(9^4) + (4 \times 8 \times 9^3) = 29889$.

48. By the fundamental principle of multiplication total number of types is equal to 10x6x4x4 = 960.

49. (a)First card i.e an Ace can be drawn in 4 ways. Second card i.e. not a queen can be drawn in 51 - 4 i.e. 47 ways. Therefore the total number of ways is equal to 4x47 i.e. 188 ways.

(b)Case1. Let the first card be Spade queen. Therefore the total number of ways = 1x48 = 48. Case2. Let the first card is a spade card but not the queen. Therefore the total number of ways = 12x47 = 564. Therefore the answer is 48+564 = 612.

50. Out of the 36 possible outcomes, 12 outcomes will yield a score divisible by 3, i.e. a score of either 3, 6, 9 or 12. The 12 outcomes will be various arrangements of (1, 2), (2, 4), (3, 3), (3, 6), (4, 5), (5,1), and (6, 6).

51. The first and the last letters can be filled in five ways each. The remaining letters can be filled in 26 ways each. Therefore the total number of ways in which a four letter word can be formed is 5x26x26x5 = 16900.

52. We require that the 1^{st} and the 5^{th} numbers are the same and the 2^{nd} and the 4^{th} numbers are the same. If the 1^{st} and the 2^{nd} numbers are chosen, the 5^{th} and the 4^{th} numbers will also have been chosen. As the 1^{st} number cannot be 0, it can be chosen in 9 ways. The 2^{nd} and the 3^{rd} numbers can be chosen in 10 ways each. Therefore, the total number of ways is $(9 \times 10^2) = 900$.

53. When a pair of dice is rolled twice, it effectively amounts to rolling 4 dice at the same time. The six numbers on the faces of the dice can now each be repeated 4 times. Therefore, the total number of outcomes is 6^4.

54. Each letter in the licence plate can be chosen in 26 ways. So the total number of ways is 26^3. As we are considering a licence plate number,the number could start with a 0. Each of the numbers in the licence plate can thus be chosen in 10 ways. So, the total number of ways is 10^3. These two blocks can now be arranged in two diferent ways. Therefore, the total required number of ways is $(2 \times 26^3 \times 10^3)$.

55. The four letters, A, B, C and D can be repeated 5 times each. So, there are 4^5 such words. Consider the arrangement BAD--. The letters A, B, C and D can be repeated in any of these two places, i.e. in 4^2 ways. Therefore, the required number of ways is $4^5 - 4^2 = 1008$.

56. Total number of ways = ${}^4C_2 \times {}^3C_1 \times {}^5C_2 = 180$.

57. ${}^{10}P_r = 10!/(10 - r)! = 604800$. ${}^{10}C_r = 10!/[r!(10 - r)!] = 120$. So, $604800/r! = 120$. Therefore, r = 7.

58. Take the first key. To get the maximum number of trials one has to check with seven locks giving all negative results.So we can conclude that this key belongs to eighth lock. Thus there are 7 trials . Similarly second key requires 6 trials, third key requires 5 trials and so on. Also the last key and the last lock remaining don't require any trial. So the maximum number of trials required in order to determine which key belongs to which lock, is equal to 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28.

59. There are ${}^{20}C_{14}$ ways the clerks can be appointed. Since each clerk gets equal opportunities, a particular clerk may be selected in ${}^{19}C_{13}$ ways.

60. We have $[{}^{2x+1}C_1 + {}^{2x+1}C_2 + \ldots\ldots\ldots + {}^{2x+1}C_x] + [{}^{2x+1}C_{x+1} + {}^{2x+1}C_{x+2} + \ldots\ldots\ldots + {}^{2x+1}C_{2x+1}] = 2^{2x+1} - 1$. Therefore by using the property, ${}^nC_r = {}^nC_{n-r,}$ we have $[{}^{2x+1}C_1 + {}^{2x+2}C_2 + \ldots\ldots\ldots + {}^{2x+1}C_x] + [{}^{2x+1}C_x + {}^{2x+1}C_{x-1} + \ldots\ldots\ldots + {}^{2x+1}C_1 + {}^{2x+1}C_0] = 2^{2x+1} - 1$, Therefore $2\,[{}^{2x+1}C_1 + {}^{2x+2}C_2 + \ldots\ldots\ldots + {}^{2x+1}C_x] = 2^{2x+1} - 2$. Therefore $[{}^{2x+1}C_1 + {}^{2x+2}C_2 + \ldots\ldots\ldots + {}^{2x+1}C_x] = 2^{2x} - 1 = 63$. On solving we get x = 3

Concepts 11: BINOMIAL THEOREM.

1. $\{a + (a^2 - 1)^{1/2}\}^7 + \{a - (a^2 - 1)^{1/2}\}^7$
Whenever we have expansion in the form $(x + y)^n + (x - y)^n$ then twice of odd terms will be there in the expansion. $2\,(x^n + {}^nC_2\, x^{n-2}.y^2 + {}^nC_4\, x^{n-4}.y^4 + {}^nC_6\, x^{n-6}.y^6$ ....). By using this formula we have the answer as
$2(a^7 + {}^7C_2\, a^5 (a^2 - 1)^1 + {}^7C_4\, a^3 (a^2 - 1)^2 + {}^7C_6\, a^1 (a^2 - 1)^3)$.

2. $(x^3 + 1/x^6)^{36}$ we can re write this expression as $\{x^3(1 + 1/x^9)\}^{36}$ T_{r+1} term of the given expansion is $x^{108}\, {}^{36}C_r\, (1/x^9)^r$. Now the term has to be free of x so we $(1/x^{9r}).\, X^{108} = 1$
So 9r = 108, so by solving this r = 12. Hence the term is 13^{th}.

3. The coefficient of x^4 in the expression is going to be zero because only odd terms will be there.

4. $(ax^4 - bx)^9$ we have to find the coefficient x^{18} in the expression, again as same in the question no. 2 we have $(r + 1)^{th} = x^{36}\ {}^9C_r\, a^{9-r}\, (b/x^3)^r$. In order to have 18 as the power of x we should have 36 – 3r = 18 and on solving this r comes out to be 6. Hence the seventh term is coefficient is ${}^9C_6\, a^3\, b^6$.

5. $(x + 1/x)^n$ we should have the power of x as r and the general term $T_{p+1} = {}^nC_p\, x^{n-p}.(1/x)^p$
${}^nC_p\, x^{n-2p}$. Now the index has to be r so n – 2p = r, therefore p = (n – r)/2. Substituting the value of p coefficient comes out to be n! / { [(n – r) / 2] ! . [(n + r) / 2] ! }.

6. $(x + \sqrt{2})^4 + (x - \sqrt{2})^4$
The value of the expression will be . $2\,(x^4 + {}^4C_2\, x^2.\, \sqrt{2}^2 + {}^4C_4\, \sqrt{2}^4)$, substituting the value as x = 4 in the expression the value comes out to be 904.

7(I). $(T_{r+1})/T_r = (n - r + 1)/r\ (y/x) > 1$
On substituting the value of n , y and x, we get
(31 – r)/r (2/3) > 1
(31- r) (2) > 3r
62 > 5r, the integer value of r comes out to be 12
So 13^{th} term is the greatest.

(II). $(2a + b)^{14}$ which can be written as $(2a)^{14}(1 + b / 2a)^{14}$
Applying the same formula as above we have
(15 –r) / r . (b / 2a) > 1 substituting the value of a and b in this and solving we have r = 8
So the 9^{th} term will be the greatest.

(III) (16 – r) / r . (2x / 3) > 1 substituting the value of x = 3 / 2 we get 16 > 2r which gives r as an integer value as 8, this means 8^{th} and 9^{th} are equal and both are the greatest term in the expansion.

8. As the index is 2n, there will be 2n + 1 terms and the middle term will be $(n + 1)^{th}$ term, for which the coefficient will be given as ${}^{2n}C_n =$ 2n ! / (n ! . n !)............(1)

2n ! = 2n (2n – 1) (2n – 2) (2n – 3) (2n – 4) (2n – 5)............4. 3. 2.1
= { (2n – 1)(2n – 3)(2n – 5)(2n – 7) ..5.3.1 } . { 2n(2n – 2) (2n – 4).........6.4.2 }
= (2n – 1)(2n – 3)(2n – 5)(2n – 7) ..5.3.1 .n(n – 1)(n – 3)(n – 5)...... $3.2.1.2^n$

Substituting the value of 2n ! in (1)

We get coefficient as $(2n – 1)(2n – 3)(2n – 5)(2n – 7) ..5.3.1.\ 2^n / n!$

9. If we consider the R.H.S which is $n(2^{n-1})$, which can be written as $n(1 + 1)^{n-1}$
$n(1 + {}^{n-1}C_1 + {}^{n-1}C_2 + {}^{n-1}C_3 + \ldots\ldots\ldots)$
$n(1 + (n-1) + (n-1)(n-2)/2 + (n-1)(n-2)(n-3)/3.2.1 \ldots\ldots\ldots)$
Now, solving the L.H.S we get same result as above . Hence proved.

10. $(x + y)^n$, $T_2 = {}^nC_1 x^{n-1} y = 240$, $T_3 = {}^nC_2 x^{n-2} y^2$, $T_4 = {}^nC_3 x^{n-3} y^3 = 1080$
$T_3/T_2 = {}^nC_2 x^{n-2} y^2 / {}^nC_1 x^{n-1} y = 720 / 240$ ....(1)
$T_4 / T_3 = {}^nC_3 x^{n-3} y^3 / {}^nC_2 x^{n-2} y^2 = 1080 / 720$.............(2)
Dividing equation (1) by (2) we get $(n – 1) / (n – 2) = 4/3$
We get n = 5 substituting n = 5 and solving the above two equations we get x = 2 and y = 3 as answers.

Concepts 12: PROBABILITY.

1. Total number of outcomes = $6^3 = 216$
Total number of favorable outcomes (6,6,6), (6,6,5), (6,6,4), (6,5,6), (6,4,6), (5,6,6), (4,6,6) = 7
$\therefore$ probability = 7/216

2. Total number of outcomes = $6^2 = 36$.
(a) favorable outcomes : 1+4, 4+1, 2+3, 3+2, i.e. 4 ways. $\therefore$ req. chance = 4/36 = 1/9.
(b) favorable outcomes : 1+5, 5+1, 2+4, 4+2, 3+3, i.e. 5 ways. $\therefore$ req. chance = 5/36.

3. Total number of outcomes = $2^4 = 16$
Number of favorable outcomes = (HHTT), (HTHT), (HTTH), (TTHH), (THTH), (THHT) = 6
Probability = 6/16 =3/8

4. A will get all blanks in ${}^9C_3 = 84$ ways, and draw 3 draws in ${}^{12}C_3 = 220$ ways
A's chance of success, i.e. at least one prize = 1 - (84 / 220) = 34/55.
Similarly, B will get all blanks in ${}^6C_2 = 15$ ways, and have two draws in ${}^8C_2 = 28$ ways.
$\therefore$ B's chance of success = 1 - (15 / 28) = 13 / 28.
$\therefore$ A's chance / B's chance = (34/55) / (13/28) = 952 / 715

5. N is repeated once. Total number of ways in which the letters can be placed = 7!/2!
Number of ways in which the vowels are together = 2 x 6!/2! = 6!
Probability = 6!/(7!/2!) = 2/7

6. Total number of outcomes = ${}^{15}C_3 = 455$
Number of favorable outcomes = ${}^9C_3 = 84$
$\therefore$ probability = 84/455 = 12/ 65

7. Total no. of outcomes = $6^2 = 36$.
(a) The perfect squares that can occur are 4 and 9.
To achieve this, the following cases are probable : 1+3, 3+1, 2+2, 3+6, 6+3, 5+4, 4+5, i.e. 7 outcomes. $\therefore$ required prob. = 7/36.
(b) multiples of 3 that can occur are 3, 6, 9, and 12, which can occur in 2 + 5 + 4 + 1 = 12 ways
required probability = 12 / 36 = 1/3.

8. Required probability = First shows 6 and second does not **or** Second shows 6 and first does not = $(1/6) \times (5/6)$ **+** $(5/6) \times (1/6) = 2(5/36) = 5/18$.

9. Total number of outcomes = ${}^9C_2 = 36$.
(a) Number of favorable outcomes = $({}^3C_1)({}^2C_1) = 6$. $\therefore$ prob. = 6/36 = 1/6.
(b) Probability that a white ball is chosen = $[{}^3C_2 + {}^3C_1\ {}^6C_1]/{}^9C_2 = 21/36$
Probability that no white ball is chosen = 1 - 21/36 = 15/36 = 5/12
(c) Number of favorable outcomes = $({}^4C_1\ {}^5C_1) = 20$ $\therefore$ prob. = 20/36 = 5/9

10. Total number of possible outcomes = ${}^{12}C_3 = 220$
Number of ways of selecting three defective bulbs from 6 defective bulbs = ${}^{6}C_3 = 20$
Probability that the room is **not lighted** = 20/220 = 1/11
Probability that the room is lighted = 1 - 1/11 = 10/11

11. If ‘p’ is the probability of A winning the game and ‘q’ is the probability of B winning the game, then, p = 3/4 q = 1/4 n = 5, r = 3
$P(r) = {}^{n}C_r\, p^r\, q^{n-r} = {}^{5}C_3\,(3/4)^3.\,(1/4)^2 = 135/512$

12. Number of days in a leap year = 366 = 52 x 7 + 2 i.e. 52 weeks & 2 extra days
These extra days may be (Monday, Tuesday), (Tuesday, Wednesday), (Wednesday, Thursday), (Thursday, Friday), (Friday, Saturday), **(Saturday, Sunday)**, **(Sunday, Monday)**
Number of possible cases = 7
Number of favorable cases for an extra Sunday = 2
∴ required probability = 2/7.

13. Probability of selecting a teacher = P(T) = 15/50 = 3/10
Probability of selecting a woman = P(W) = 20/50 = 2/5
Probability of selecting a Hindi knowing candidate = P(H) = 10/50 = 1/5
Required Probability = P(T)P(W)P(H)
= (3/10)(2/5)(1/5)
= 3/125.

14. The probabilities that Rahul, Shashank, and Charudatta will **not** solve the problem are 0.8, 0.3, and 0.4 respectively. The problem will not be solved if and only if all three do not solve it.
∴ the chances of this happening are : (0.8)x(0.3)x(0.4) = 0.096.

15. Let x be the number of red balls. ∴ total no. of balls = 5 + x.
The probability of both the drawn balls being blue is : ${}^{5}C_2$ / [${}^{5+x}C_2$] = 5! / [3!2!] . 2!(3+x)! / [(5+x)!]
∴ (5)(4) / [(4+x)(5+x)] = 5 / 14 ∴ $x^2 + 9x - 36 = 0$. ∴ (x + 12)(x - 3) = 0.
∴ x = -12, or x = 3. The negative value is naturally not admissible.

16. Total number of outcomes with two dice = $6^2 = 36$.
Same digit will appear in the following cases : 1-1, 2-2, 3-3, 4-4, 5-5, 6-6, i.e. 6 outcomes.
∴ required probability = 6/36 = 1/6.

17. 8 boys and girls can be seated on 8 chairs in 8! different ways.
G B G B G B G B G B G
As shown above, 5 boys can be seated at 5 places in 5! ways. After this, 3 girls can be seated in the remaining 6 places in ${}^{6}P_3$ ways.
∴ favorable outcomes = 5! x ${}^{6}P_3$
∴ probability = 5! / 3!x 6! / 8!= 5/14.

18. The probability of the sun shining = 1/3, and the probability of the sun being hidden = 2/3.
The required probability = ${}^{5}C_4\,(1/3)^4(2/3) + {}^{5}C_5\,(1/3)^5(2/3)^0$
∴ the required probability = 11/243.

19. ∴ the probability that a vessel will be wrecked is 1/10.
The required probability = ${}^{5}C_4(9/10)^4\,(1/10) + {}^{5}C_5(9/10)^5(1/10)^0$
∴ the required probability = 45927/ 50000

20. Since each institute is equally likely to be taken, the chance of selecting the first is 1/3, and the chance of taking marketing is 1/4, thus the prob. of choosing marketing here is 1/12.
Similarly, the chance in case of the second institute is 1/3 x 2/6 = 1/9.
Similarly, the chance in the third case is 1/3 x 3/4 = 1/4.
∴ The required probability = 1/12 + 1/9 + 1/4 = 4/9.

21. The probability that yarn selected is green in colour is 25% i.e. ¼. For the green yarn to be defective the probability is 7% i.e. $^7/_{100}$. Hence the probability that selected yarn is defective and green in colour is ¼ . $^7/_{100} = {}^7/_{400}$
Similarly the probability that selected yarn is red in colour and is defective = $^2/_{100} \,.\, ^{40}/_{100} = {}^1/_{125}$
And the probability that selected yarn is blue in colour and is defective = $^5/_{100} \,.\, ^{35}/_{100} = {}^7/_{400}$
Therefore the probability that selected yarn is defective = $^7/_{400} + {}^1/_{125} + {}^7/_{400}$
Probability that the defective yarn selected is green is
= (Number of favourable cases of selecting defective green yarn) / (Total number of cases of selecting defective yarn).
Hence the answer is $(^7/_{400}) / (^7/_{400} + {}^1/_{125} + {}^7/_{400}) = {}^{35}/_{86}$

The probability that yarn selected is Red in colour is 40% i.e. $^2/_5$. For the red yarn to be defective the probability is 2% i.e. $^2/_{100} = {}^1/_{50}$. hence for the red yarn to be non defective the probability is $1 - {}^1/_{50} = {}^{49}/_{50}$. The probability that the selected yarn is non defective and red in colour is $^2/_5 \times {}^{49}/_{50} = {}^{49}/_{125}$.

22. Since we need at least two white balls, we can have 2, 3, 4 or 5 white balls. The remaining balls required to complete the set of 5 can be of any other colour. The number of ways of choosing at least 2 white balls is $({}^7C_2 \times {}^{20}C_3) + ({}^7C_3 \times {}^{20}C_2) + ({}^7C_4 \times {}^{20}C_1) + ({}^7C_5)$. The total number of ways of choosing 5 balls from among 27 is ${}^{27}C_5$. Therefore, the required probability is $\{({}^7C_2 \times {}^{20}C_3) + ({}^7C_3 \times {}^{20}C_2) + ({}^7C_4 \times {}^{20}C_1) + ({}^7C_5)\} / {}^{27}C_5$.

23. The total number of outcomes is 2^9. The number of ways in which he can get 0, 1, 2, 3 or 4 heads will be the same as the number of ways in which we can obtain at least five tails. The number of ways in which 0, 1, 2, 3 or 4 heads can be obtained is $({}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4) = 256$. Therefore, the required probability is $256 / 2^9 = ½$.

24. To find the number of points in the sample space, we will have to find the total number of four digit odd numbers. The numbers could be of the following types: XX35, X35X, 35XX or any number not containing the digit 3. (Please note that we cannot have a number with the last digit as 3 because 3 is always followed by 5). It should also be noted that the first digit can never be 0 as we are considering four digit numbers.

Consider the number XX35. The first digit cannot be 0,3 or 5. This digit can be chosen in 7 different ways. The second digit can now be chosen in 7 different ways. So, there are 7 x 7 =49 numbers of the form XX35.

Consider the numbers of the form X35X. The last digit has to be one of 1, 7 or 9 and can be chosen in 3 different ways. The first digit of these numbers can be chosen in 6 different ways. So, there are 6 x 3 = 18 such numbers.

Consider the numbers of the form 35XX. The last digit of these numbers has to be one of 1, 7 or 9 and can be chosen in 3 different ways. The last but one digit of these numbers can be chosen in 7 different ways. So, there are 3 x 7 = 21 such numbers.

Consider numbers which do not contain the digit 3. The last digit of these numbers must be one of 1, 5, 7 or 9 and can be chosen in 4 different ways. The first digit of these numbers can be chosen in 8 different ways. The other two digits can be chosen in 7 and 6 different ways.

So, there are 7 x 7 x 6 x 4 = 1176 such numbers.

The sample space, thus, has a total of 1176 + 21 + 18 + 49 = 1264. Of these numbers, exactly one number is 2695. Therefore, the required probability is 1/1264.

25. Since two dice are thrown a score of 5 can be obtained in 4 ways (1+4, 2+3, 3+2, 4+1). A score of 9 can be obtained in 4 ways (3+6, 4+5, 5+4, 6+3). Since the number of ways in both cases is the same, their respective probabilities must be the same.

26. Since the last three digits are different, these can be chosen in 10 x 9 x 8 = 720 ways. Of these, exactly one choice will be the correct number. So, the required probability is 1/720.

27. 5 balls from out of 15 can be chosen in ${}^{15}C_5$ ways. The number of ways in which only one ball drawn will be white is ${}^{10}C_1$ x ${}^{5}C_4$. Therefore, the required probability is$({}^{10}C_1$ x ${}^{5}C_4)/({}^{15}C_5)$ =50/3003.

28. P(A) = probability that A solves the problem = 1/5. ∴, P(A') = 4/5.
P(B) = probability that A solves the problem = 1/6. ∴, P(B') = 5/6.
(a) The probability that the problem is solved = 1 – (probability that the problem is not solved) = 1 – [P(A') x P(B')] = 1 – [(4/5) (5/6)] = 1- 4/6 = 1/3.
(b) The probability that B solves the problem but A does not = P(A') x P(B) = (4/5) x (1/6) = 2/15.
(c) The probability that the problem is not solved is [P(A') x P(B')] = [(4/5) (5/6)] = 2/3.

29. We require either a King or a Spade. We have 4 Kings and 13 Spades. However, we have included the King of Spades in these 17 cards. So, in effect, we have a total of only 16 cards. So, the requird probability is 16/52 = 4/13.

30. A: the first die shows 4, 5 or 6 dots. B: the second die shows 5 or 6 dots.
P(A) = 3/6 = ½. P(B) = 2/6 = 1/3.
So, the required probability is P(A) x P(B) = ½ x 1/3 = 1/6.

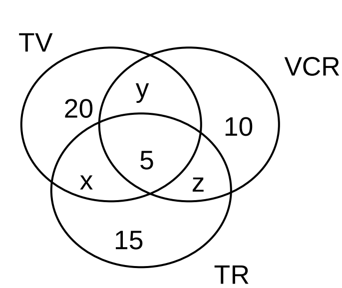
31. Case 1 : If the ball drawn is white, the first box will now have 5 white and 3 black balls. The probability in this case is (6/9) x (5/8) = 5/12.
Case 2 : If the ball drawn is black, the first box will now have 6 white and 2 black balls. The probability in this case is (3/9) x (6/8) = 3/12.
Therefore the required probability is (5/12) + (3/12) = 2/3.

32. Required probability = $({}^{7}C_1$ x ${}^{9}C_1)/({}^{16}C_2)$ = 21/40

33. Since there are two different bags, the probability of choosing any one bag is ½.
Suppose the first bag is chosen. The probability of choosing a red ball from this bag is ½ x 5/12 = 5/24.
Suppose the second bag is chosen. The probability of choosing a red ball from this bag is ½ x 1/5 = 1/10.
Therefore the required probability is 5/24 + 1/10 = 37/120.

34. Probability (Score is even) = 2/3. Probability (Score is odd) = 1/3. Therefore in long run on average in each 9 throws the player gets twice the score of 2, 4, 6 and once the score of 1, 3, 5. Therefore the player receives the total money equal to 2(2+4+6)+(1+3+5) = Rs.33 in 9 throws. So on average the player gets 33/9 i.e Rs. 11/3 per throw. Therefore the student should pay 11/3 - 2/3 = Rs. 3. for each roll.

35. From the Venn diagram, 20+10+15+5+(x+y+z) = 70.
Therefore x+y+z = 20.
Therefore the required probability = 20/70 = 2/7.



36. Case (1): Event A: First ball is white; second ball is black. Then P(A) = (7/21) x (14/20) = 7/30.
Case (2): Event B: First ball is black; second ball is white. Then P(B) = (14/21) x (7/20) = 7/30.
So, the required probability is P(A) +P(B) = 7/15.

Alternate solution: $({}^{7}C_1 x {}^{14}C_1) / {}^{21}C_2 = 7/15$

37. We have 000, 001,004, 009, 016, … 961. (Note that $31^2 = 961 < 1000$ and $32^2 = 1024 > 1000$). Thus there are 32 perfect square numbers which can be formed using 3 digits. From 32 available choices the person will select 1 hence the probability is 1/32.

38. Since A, B or C could solve the problem independently, these are mutually exclusive events. Therefore, the required probability is (1 - 0.5) x (1 - 0.2) x (1 - 0.3) = 0.28.

39. The probability of winning a lottery ticket = 0.25 = ¼. The probability of not winning a lottery ticket is 0.75 = ¾. So, the probability that at least one of the four lottery tickets will win is
$1 - {}^{4}C_0(1/4)^0 \times (3/4)^4 = 1 - (81/256) = 175/256$

40. There are a total of 7 floors, including the ground floor. The four people can get off on any of the remaining 6 floors in 6^4 ways. The number of ways in which the four people will get off the elevator on different floors is ${}^{6}P_4$. So, the required probability is ${}^{6}P_4 / 6^4$.