C o n c e p t s 1

Ratio & Proportion

Ratio :
The ratio of two quantities (in same units) is the fraction that one quantity is of the other. e.g. the ratio 2 to 3 is 2/3 & is written as 2 : 3. The first term is called the **antecedent** and the second term is known as the **consequent**. Multiplication of each term of a ratio by the same number does not affect the ratio. e.g. 14/28 is the same as 2/4 is the same as 1/2.

Proportion
The equality of two ratios is called proportion. e.g. 2 : 3 = 4 : 6 ∴ we write 2 : 3 :: 4 : 6 and say that 2, 3, 4 & 6 are in proportion. 2, 3, 4 & 6 are called the first, second, third & fourth **proportional** respectively. The first & fourth proportionals are called the **extreme terms**. The second & the third proportionals are called the **mean terms**.

1. Product of means = Product of Extremes.

2. If **a : x :: x : b** i.e. x^2 = ab, then x is known as **mean proportion** of a and b.

3. If **a : b :: b : x** i.e. b^2 = ax, then x is known as **third proportion** of a and b.

4. Duplicate ratio of a : b is $a^2 : b^2$ and **Sub duplicate ratio** of a : b is √a : √b.

5. Triplicate ratio of a : b is $a^3 : b^3$ and **Sub triplicate ratio** of a : b is $a^{1/3} : b^{1/3}$.

Ex. A man 1.4 metres tall casts a shadow 1.2 metres long and at the same time a building next to him casts a shadow 5.4 metres long. What is the height of the building ?

Sol. Let the height of the building be h metres.
1.4/1.2 = h/5.4
∴ h = 1.4 x 5.4/1.2 = 6.3 metres
∴ the height of the building is 6.3 metres

Ex. Find three numbers in the ratio 2 : 3 : 5 such that the sum of their squares is 608.

Sol. Let the numbers be 2x, 3x, 5x
$4x^2 + 9x^2 + 25x^2$ = 608
or $38x^2$ = 608
∴ x^2 = 16 or x = ± 4
∴ the numbers are 8, 12, 20 or - 8, -12, - 20.

Compound Proportion

The method of finding out the fourth proportional when the other three are given is called the **rule of three** or **simple proportion.** Repeated use of simple proportion is called **compound proportion.** Two quantities are said to be **directly proportional** if the increase or decrease of one causes the increase or decrease of the other to the same extent. Two quantities are said to be **inversely proportional** if the increase of one causes the decrease in the other to the same extent.

Rules on Proportion

1. Alternendo Rule : a : b :: c : d then a : c :: b : d.
2. Componendo Dividendo Rule : a/b = c/d ⟹ (a + b) / (a - b) = (c + d) / (c - d)

3. If a/b = c/d = e/f then each ratio equals to (a + c + e) / (b + d + f)

Ex. If 20 men can build a wall 112 metres long in 6 days, what length of wall can be built by 25 men in 3 days ?

Sol. More the number of men longer is the length of wall built. (Direct Proportion).
Similarly, lesser the number of days, shorter will be the length of wall built.
$(M_1 T_1) / W_1 = (M_2 T_2) / W_2$
∴ (20 x 6)/112 = (25 x 3)/x
∴ x = 70
∴ the length of the wall built = 70 metres.

Ex. If 17 men can dig a ditch 26 m long in 18 days working 8 hours a day how many more men should be engaged to dig a similar ditch 39 m long in 6 day, each man now working 9 hours a day.

Sol. As in the above example, $(M_1 R_1 T_1) / W_1 = (M_2 R_2 T_2) / W_2$
$M_1 = 17$ $R_1 = 8$ $T_1 = 18$ $W_1 = 26$ $M_2 = ?$ $R_2 = 9$ $T_2 = 6$ $W_2 = 39$
(17 x 8 x 18)/26 = (M_2 x 9 x 6)/39
∴ $M_2 = 68$
∴ the number of extra men to be employed is (68 - 17) = 51

Ex. A contractor undertook to build a road in 200 days. He employed 140 men. After 60 days, he found that only ¼ of the road was built. How many additional men should he employed to complete the work in time ?

Sol. We use, $(M_1 T_1) / W_1 = (M_2 T_2) / W_2$
$M_1 = 140$ $T_1 = 60$ $W_1 = 1/4$ $M_2 = ?$ $T_2 = (200 - 60) = 140$ $W_2 = 3/4$
(140 x 60)/(1/4) = (M_2 x 140)/(3/4)
∴ $M_2 = 180$
∴ Additional men to be employed = (180 - 140) = 40

Ex. 10 men 6 boys do a piece of work in 20 days while 12 men and 4 boys can do this work in 18 days. What time will 8 men and 8 boys take to do the work?

Sol. (10 men + 6 boys) complete the work in 20 days. Thus (10 x 20 men + 6 x 20 boys) complete the work in one day. ∴ **(200 men + 120 boys) complete the work in one day.**
Similarly, (12 men + 4 boys) complete the work in 18 days. Thus (12 x 18 men + 4 x 18 boys) complete the work in one day. ∴ **(216 men + 72 boys complete the work in one day)**
∴ (200 men + 120 boys) = (216 men + 72 boys)
∴ 16 men = 48 boys or 1 man = 3 boys
∴ (10 men + 6 boys) = 36 boys & (8 men + 8 boys) = 32 boys
Thus 36 boys complete a piece of work in 20 days,
∴ 32 boys complete the work in (30 x 20)/36 = 22.5 days

Ex. A garrison of 3300 men has provisions for 32 days, when given at a rate of 850 grams per head. At the end of 7 days a reinforcement arrives and it was found that now the provisions will last 8 days less, when given at the rate of 825 grams per head. How many more men can it feed ?

Sol. When the reinforcement arrives after 7 days, 3300 men taking 850 gm. per head have provisions for (32 - 7) or 25 days. So, the number of men taking 825 gm. per head, having provisions for 17 days = ?
Let the number of men be M_2
∴ (825 x 17 x M_2) = 850 x 25 x 3300

∴ $M_2 = 5000$
∴ the number of extra men = (5000 - 3300) = 1700

Variation

Apart from being directly proportional or inversely proportional to one another, some quantities may vary directly or inversely in proportion to the power of another variable.

Ex.   Two balls have their radii in the ratio 1:3. What is the ratio of their volumes ?

Sol.   The volume 'V' of a sphere is given by the equation $V = 4/3 \, \pi \, r^3$.
Since $4/3 \, \pi$ is a constant, $V \propto r^3$
$$\therefore V_1 : V_2 = (r_1)^3 : (r_2)^3$$
$$\therefore V_1 : V_2 = 1 : 27.$$

Ex.   The parameters a and b of an object are such that
$$a = 9/(143 \, b^2).$$
Two such similar objects of different sizes have their parameter B in the ratio 2: 3. What is the   ratio of the parameters a?

Sol. $a_1 : a_2 = \dfrac{1}{b_1^2} : \dfrac{1}{b_2^2}$

$\therefore a_1 : a_2 = \dfrac{1}{4} : \dfrac{1}{9}$

$\therefore a_1 : a_2 = 9 : 4$

Exercise 1

1.   If 3:21 :: x : 63, then find the value of x.

2.   If a : b = 8:15, b : c = 5:8 and c : d = 4:5 then find a : d

3.   If a : b = 3:4, b : c = 8:9, c : d = 15:16 then compare a, b, c, d.

4.   Find a fraction which bears the same ratio to 1/27 that 3/7 does to 5/9.

5.   Find the mean proportional to .9 and 4.9.

6.   Find the third proportional of 6 and 30.

7.   Find the fourth proportional of 1,2,3.

8.   Compare the ratios 5:6 and 7:8.

9.   Write the following in ascending order: 3:4, 7:8, 5:6, 17:19.

10.   Write the following ratios in descending order: 2:5, 4:7, and 16:19.

11.   Which of the following ratios is the greatest and which is the smallest?
        0.8:0.4, 3.5:2.5, 15:20, 4:7.

12.   The ratio between two quantities is 5:7. If the first quantity is 155, find the other.

13.   The ratio between two quantities is x:21. If the original quantities are 200 litres and 840 litres, find x.

14.   The shadow of Qutab Minar, which is 72 metres high is 8 metres at a particular time on a day. Find the height of an electric pole which casts a shadow of 10 meters, under the similar conditions on the same day and time.

15.   Two car-drivers take 3 1/4 hours and 4 hours respectively to cover a distance of 650 km from a city P to Q. Find the ratio of the average speeds of the two cars.

16.   Two numbers bear a ratio of 2:7. If each of them is increased by 14, then their ratio becomes 4:7, Find the numbers.

17.   Two numbers are in the ratio 2:7. Their L.C.M. is 70. Find the numbers.

18.   Two numbers are in the ratio 5:8 and the difference between them is 24. What are the two numbers?

19.   Two numbers are in the ratio of 7:4. The difference of their squares is 132. Find the numbers.

20.   If 16 men construct a wall 60 meters in length in 6 days, then, how many days will be taken by 28 men in constructing a wall 70 meters in length?

21.   If 16 men working 7 hours a day can plough a field in 48 days, in how many days will 28 boys working 12 hours a day plough the same field, if one man does a work as 2 boys?

22.   If 75 men can dig a trench 300 m long. 9 m wide and 3 m deep in 6 days working 5 hours a days, in how many days will 50 men dig another trench 540 m long, 6 m wide and 4 1/2 m deep working 4 hours a day?

23. One man or two boys complete a work in 40 days. How long will 3 men and 4 boys take to complete ½ of that work?

24. 2 men and 7 boys complete a piece of work in 14 days. 3 men and 8 boys complete that work in 11 days. How long will 8 men and 6 boys take to complete a job 3 times the original work?

25. A besieged garrison of 600 men has provisions to last for 35 days at a certain daily rate of issue per day. After 10 days, 300 men reinforced the garrison. For how many days in all would the provisions last if the daily consumption of provisions per man reduced in the ratio 6:5?

26. 1500 soldiers in a fort have provisions for 48 days. After 13 days, some soldiers join them and then the food lasts for 25 days. How many soldiers joined later?

27. 3500 cadets have provisions for a 20 days training camp and 5 days of social work at the rate of 500 gm per day per cadet. If the provisions are to last for 10 days more and the ration is reduced to 400 gm per day, how many cadets must leave the camp?

28. In an army there are 840 soldiers. They have enough food for 70 days. After 5 days, 210 soldiers join the army. For how many days will the remaining food last?

29. A contractor undertook to finish a certain work in 124 days and employed 120 men. After 64 days, he found that he had already completed 2/3rd of the work. How many men can the contractor discharge so that he may complete the work on time?

30. A contractor undertook to finish 6 km of railway track in 130 days and employed 72 men to complete the job. After 50 days he found that only $1^1/_2$ km of the track had been completed. What is the number of men that the contractor must now hire so that the work may be completed on time?

31. A contractor has 20 men working 8 hours a day engaged on a piece of work to be finished in 28 days. After 18 days, only 1/3rd of the work had been completed and the contractor decided to hire some more men to complete the work. If all the men now work 9 hours a day, What is the least number of men that the contractor must hire in order that the work is completed on time?

32. 12 men and 8 women can do a piece of work in 20 days, while 36 men and 48 women can do the same work in 5 days. How many days will 12 men and 12 women take to finish the same piece of work?

33. The length and breadth of a rectangle are increased in the ratio 3:4 and 4:5 respectively. What is the ratio of the old area to the new one?

34. A man went to the market to buy fruits. He bought apples, mangoes and dates. The ratio of the weight of apples to that of mangoes bought is the same as the ratio of the weight of mangoes to that of dates bought. If he bought 7.2 kg of apples and 5 kg of dates, find the weight of mangoes bought.

35. A grey hound pursues a hare and takes 4 leaps for every 5 leaps of the hare; but 3 leaps of the hound are equal to 4 leaps of the hare. Compare the rates of the hound and hare.

36. A mixture consists of three substances whose volumes are in the ratio 5:6:8. The weights of equal volumes of these substances are in the ratio 4:5:3. What is the ratio of the weight of the three substances in the mixture?

37. Three pots have same volume. The ratios of milk and water in first, second and third pot are respectively 3:2, 7:3 and 11:4. If the milk of all the three pots is mixed, then find the ratio of milk and water in the mixture?

38. The force of gravity at the surface of a planet is jointly proportional to the density of the planet and its radius. The planet Jupiter has radius equal to ten times the radius of the earth. The densities of Jupiter and Earth are respectively 1.75 and 5.67. If a man can jump to a height of 1500 cm, on earth how high can he jump on the surface of Jupiter, given that the height to which a man can jump is inversely proportional to the force of gravity?

39. The distance of the horizon at sea varies as the square root of the height of the eye above the sea level. When the distance is 14.4 kilometers, the height of the eye is 18 metres. Find in kilometers the distance when the height of the eye is 8 metres, and find in metres the height of the eye when the distance is 7.2 kilometres.

40. The pressure of a certain mass of a gas varies inversely as its volume. When p (measured in kg weight per square decimetre) is 20 then v (measured in cubic decimetre) is 450. Find in kg weight per square decimetre the pressure when the volume is 300 cubic decimetres. Also find the volume when the pressure is 60 kg weight per square decimetre.

Answers

1. 9

2. a : d = 4 :15

3. 30:40:45:48

4. 1/35

5. 2.1

6. 150

7. 6

8. 7:8 > 5:6

9. 3:4, 5:6, 7:8, 17:19

10. 16:19, 4:7, 2:5

11. greatest 0.8 : 0.4 , smallest 4:7

12. 217

13. 5 litres

14. 90 metres

15. 16 : 13

16. 6 and 21

17. 10 and 35

18. -40 & -64

19. ±8 & ±14

20. 4 days.

21. 32 days

22. 20 $\frac{1}{4}$ days

23. 4 days

24. 21 days

25. 30 days

26 600 soldiers

27. 375 cadets

28. 52 days

29. 56 men

30. 63 men

31. 44 men

32. 17 1/7 days

33. 5:3

34. 6 kg

35. 16:15

36. 10:15:12

37. 61:29

38. 486 cm

39. 9.6 km, 4 .5 m

40. 30, 150

C o n c e p t s 2

Percentage

Percentage is a fraction whose denominator is 100. In other words, it can be said that percentage is a ratio with the consequent (i.e. denominator) as 100. Consider the following example.

e.g. 25% = 25/100 = ¼.

4/5 = (4/5) x 100 = 80%.

In the above example, 25% is expressed as a ratio ¼. All percentages can be represented as a ratio with the denominator as 100. The conversion of fractions to percentages and vice - versa is most useful for calculations. Some commonly required fractions and their conversions are as follows:

Fraction	Percent	Fraction	Percent	Fraction	Percent	Fraction	Percent
1/1	100%	1/5	20%	1/9	11.11%	1/13	7.69%
½	50%	1/6	16.66%	1/10	10%	1/14	7.14%
1/3	33.33%	1/7	14.28%	1/11	9.09%	1/15	6.66%
¼	25%	1/8	12.5%	1/12	8.33%	1/16	6.25%

The use of the above table will be made clear with the help of the following examples.

Ex. What is 14.28% of 560?

Sol. We know that 14.28% = 1/7. So, 14.28% of 560 = (1/7) x 560 = 80.

Ex. What is 37.5% of 720?

Sol. We know that 12.5% is 1/8. So, 37.5% = (3 x 12.5%) = (3 x 1/8) = 3/8. Therefore, the required answer is (3/8) x 720 = 270.

Ex. Express 7/8 as a rate percent.

Sol. 7/8 = (7/8) x 100 = 175/2 = 87.5%

Ex. A earns 25% more than B. By how much percent is B's income less than that of A?

Sol. Let B's income be Rs. 100. Then A's income = Rs. 125
If A's income is Rs. 125, B's income = Rs. 100
∴ if A's income is Rs.100, B's income = Rs. (100/125)100 = Rs. 80
∴B's income is less than that of A by (100-80)% = 20%

Ex. In an examination 36% are the pass marks. If a student gets 17 marks and fails by 10 marks, what are the maximum marks?

Sol. Total pass marks = 17 + 10 = 27
∴ 36% of maximum marks = 27
∴ maximum marks = 27 x 100/36 = 75

Ex. In an election, one of the two candidates gets 42% of the total votes and still loses by 368 votes. What is the total number of votes?

Sol. Let the total number of votes be 100
Then, the votes secured by the defeated candidate = 42
Votes secured by the winning candidate = 100 - 42 = 58
Difference in the votes = 58 - 42 = 16

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If the difference in the votes is 16, total number of votes is 100

∴ if the total number of votes is 368, total number of votes = 368 x 100/16 = 2300

Ex.   Sugar is now being sold at Rs. 15 a kg. During last month its rate was Rs. 13 per kg. By how much percent should a family reduce its consumption so as to keep the expenditure fixed ?

Sol.   Let the earlier consumption be 1 kg

∴ the earlier expenditure = Rs. 13

Now Rs. 15 gives 1 kg

∴ Rs. 13 gives 13/15 kg

Reduction in consumption = 1 - (13/15) = 2/15 kg

∴ reduction percentage = (2/15) x 100 = $13\frac{1}{3}$ %

Exercise 2

1. Find the number whose 30% are 225.

2. Find 8% of 325.

3. What per cent of 88 are 33?

4. A man Expenses 85% of his income, if his saving is Rs.37.50, find his salary.

5. Divide Rs. 1100 into two parts such that one is 10% of the other.

6. A's Salary is 25% higher than that of B. By how much percent is B's salary lower than that of A.

7. A's salary is 20% below B's salary. By how much per cent is B's salary above A?

8. The price of cloth is increased by 20%. By how much percent must a family reduce the consumption of cloth so that their expenditure may remain the same.

9. A reduction of 40% in the price of eggs would enable a purchaser to purchase 48 eggs more for Re. 1. Find the reduced price per dozen. Also find the original price.

10. A reduction of 20% in the price of oranges enables a man to buy 5 oranges more for Rs.10. What is the reduced price per orange? What is the price per orange before reduction?

11. If the price of sugar falls by $12\frac{1}{2}$% a person can buy 9 kilograms more of sugar for Rs. 126 than before. If the price had risen by $12\frac{1}{2}$%, how much less sugar could he have bought for the same sum?

12. A candidate secured 20% marks in an examination and failed by 10 marks. Another secured 42% marks and got 1 mark more than the marks required to pass the examination. Determine the maximum number of marks and the percentage necessary to pass the examination.

13. In an examination 75% of the candidates passed in English and 70% of the candidates passed in Mathematics, 23% candidates failed in both subjects. If 136 candidates passed in both the subjects, find out the number of candidates that appeared in the examination.

14. The population of a town increases by 10% every year for two years and then decreases by 10% every year for two years. If population just before 4 years was 100000, what was it after four years?

15. The population of a town decreases per year at the rate of 5%. If after 2 years, the population of the town is 902500, then what is the population of the town at present?

16. At an election, a candidate secures 40% of the total votes but is defeated by the other candidate by 300 votes. Find out the total number of votes polled.

17. 10% of the soldiers of an army are killed in the battle. 10% of the remaining soldiers died of disease and 10% of the remaining men were disabled. Now only 729000 soldiers are left in the army. How many soldiers were there in all in the army in the beginning.

18. In a direct election between two contestants for the post of secretary, 4% of the total votes cast are declared to be illegal. One contestant secures 55% of the valid votes and wins with a majority of 240 votes. Find the total number of votes cast.

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19. Two numbers are greater than the third number by 25% and 20% respectively. What per cent of first number is the second number?

20. The number of boys and girls who appeared at an examination were in the ratio 16 : 9 and the number of boys and girls passing the examination were in the ratio 4:3. If 75% of the girls passed the examination, find the % of boys who passed the examination and the % of all the candidates passing the examination.

21. Entry fee in an exhibition was Re. 1/-. Later this was reduced by 25% which increased the sale by 20%. Find the percentage increase in the number of visitors.

22. Two candidates contested the election. At the election 10% of people on the voting list did not vote, and 60 votes recorded were rejected as illegal. The majority of the successful candidates was 308 and it was found that he had been supported by 47% of the whole number on the voter's list. What was the number of legal votes recorded by each candidate?

23. In an examination paper of 5 question, 5 percent of the candidates answered all of them and 5 percent none. Of the rest, 25 percent answered only 1 question and 20 percent answered 4. If $24\frac{1}{2}$ percent of the entire number answered only 2 questions and 200 candidates answered only 3 questions, how many competed?

24. A mans taxable income is Rs. 28600 .In a certain year, the income tax for taxable income over Rs.25,000 but not exceeding Rs.30,000 is computed according to the following rule:
"Total income Tax = Tax + Surcharge where Surcharge = 15% of the tax and tax = Rs.3200 + 30% of the amount by which taxable income exceeds Rs.25000".
Determine the tax he must pay.

25. The marks obtained at an examination by three candidates A, B, C in English, Mathematics and General Knowledge each with maximum marks 150, were as follows:

	A	B	C
English	99	79	105
Mathematics	107	91	120
G.K.	68	93	101

(i) By what percentage A obtained more marks in English than B;
(ii) By what percentage C obtained more marks in Mathematics than in General Knowledge?

26. The population of a village is 2000. In the first year it increases at the rate of 10% and in the second year it decreases at the rate of 10%. Find the population of the village after two years.

27. The population of the town increased 3% in a given interval but it would have been 1500 less if there had been a decrease of 2%. Find the original population.

28. A man gave 30% of his money to his wife, 40% of the remainder to his son and the remaining money equally to his three daughters. If each daughter gets Rs. 224, what does the wife get?

29. A man spends 80% of his income. With the increase in the cost of living his expenses also increase by $37\frac{1}{2}$% and his income increases by $16\frac{2}{3}$%. Find his present percentage saving.

30. The tax on a commodity is diminished by 10% and its consumption increases by 10%. Find the decrease percent in the revenue derived from it. With what increase percent in its consumption, revenue will remain the same?

31. A working man has a rise of a $12\frac{1}{2}$% in his wages rate per hour. There is a drop of 8% in the number of hours worked per week. If his original weekly wages for a week of 50 hours

was Rs.120, find (i) his new wages per hour (ii) the percentage increase in the total weekly wages.

32. A sells his goods 10% cheaper than B and 10% dearer than C. If a customer of B purchases goods worth for Rs.100 from C, how much will he save?

33. At an election, 10% of the total voters did not cast their votes. 400 votes were declared illegal. There were only two contestants A and B. A defeated B by 640 votes. It was found than 60% of the legal votes had been secured by A. Find the total number of votes.

34. The incomes of A and B are in the ratio 5:3, their expenses are in the ratio 2:1. If the total annual savings of A and B are Rs. 3600 each, find the monthly income of A and B.

35. The measure of one side of a rectangle is too long by 4% of its true length and that of the other is short by 5% of its true length. Find the error per cent in measure obtained for the area of the rectangle.

36. A person can save only 6% of his income. Two years later his income is increased by 15% and he still saves the same amount. Calculate the increase percent in his expenditure.

37. The freight of machine amounts to 20% of its price. Had the price been 10% less than what it is, the total cost of the machine would have been Rs.480 less. What is the price of the machine?

38. At an exam the full marks were 1000. A got 10% less than B, B 25% more than C and C 20% less than D. If A got 720, what percentage of full marks was got by D?

39. Find a single discount equivalent to the discount series 33%, 12 $\frac{1}{2}$% and 4%.

40. Which bargain is better: discount series of 20%, 15% and 10% or discount series of 25%, 12% and 8%?

Answers

1. 750	**2 .** 26	**3.** 37 1/2	**4.** Rs.250	**5.** 1000,Rs100
6. 20%	**7.** 25%	**8.** 16 2/3%	**9.** 10 ps, 16 2/3 ps	**10.** 40ps, 50 ps
11. 7 kg	**12.** 50,40%	**13.** 200	**14.** 98010	**15.** 10,00,000
16. 1500	**17.** 10,00,000	**18.** 2500	**19.** 96%	**20.** 56.25%,63%
21. 60%	**22.** 2914, 2606	**23.** 800	**24.** 4922	**25.** 25.3%,18.8%
26. 1980	**27.** 30,000	**28.** 480	**29.** 5 5/7 %	**30.** 1%, 11 1/9%
31. Rs2.7/hr,3.5%	**32.** 22 2/9%	**33.** 4000	**34.** Rs. 18000 and Rs.10800	
35. − 1.2%	**36.** 15 45/47%	**37.** 4000	**38.** 80%	**39.** 43.72%
40. 2 nd				

Concepts 3

Proportional Division

If a given quantity is divided into parts which bear a given ratio to one another, such parts are called proportional parts and the process of obtaining these parts is called proportional division.

Ex. Divide 81 in the ratio 7 : 2.

Sol. Given ratio = 7 :2

Sum of the ratio = 7 + 2 = 9

First part = $\frac{7}{9}$ X 81 = 63

Second part = $\frac{2}{9}$ X 81 = 18

Ex. Divide 1162 in three parts such that 4 times the first part is equal to 5 times the second and 7 times the third.

Sol. 4 x (1st part) = 5 x (2nd part) = 7 x (3rd part) = x say

(1st part) = x/4   (2nd part) = x/5   (3rd part) = x/7

∴ ratio of divisions = x/4 : x/5 : x/7

= 1/4 : 1/5 : 1/7 = 35 : 28 : 20

Sum of ratios = 35 + 28 + 20 = 83

∴ (1st part) = 1162 x 35/83 = 490

(2nd part) = 1162 x 28/83 = 392

(3rd part) = 1162 x 20/83 = 280

Ex. Divide Rs. 680 among A, B & C such that A gets 2/3 of what B gets & B gets 1/4 of what C gets.

Sol. Let C get Re. 1. Then B gets Re. 1/4 & A gets Rs. (2/3) of (1/4) i.e. Rs. 1/6

Ratios of A, B & C's shares = 1/6 : 1/4 : 1 = 2 : 3 : 12

∴ A's share = Rs. 680 x 2/17 = Rs. 80

B's share = Rs. 680 x 3/17 = Rs. 120

C's share = Rs. 680 x 12/17 = Rs. 480

Resolving an Equation

Any arithmetic equation can be expressed as N X R = V where N is the number, R is the rate and V is the final value. Some common examples are : If one buys 4 kgs of potatoes at Rs. 5 per kg, total rupees spent = Rs. 20. If one travels for 5 hours at 10 km per hour, total distance traveled is 50 kilometers.

Ex. Between two stations the first, second and third class fares are in the ratio 9:7:2. The passengers travelling in a day are in the ratio 5:3:2 respectively in the above classes. If the sale of tickets generated revenue of Rs. 98000 that day and if 200 passengers travelled by third class, what was the fare for a first class ticket.

Sol. Ratio of number of passengers=5:3:2

Let the number of passengers travelling by first class be x. We know that the number of third class passengers is 200.

∴ 5:2 = x : 200; ∴ x = 500

Ratio of number of passengers = 5 :3:2

Ratio of fares   = 9:7:2

∴ Ratio of amounts collected   = 5 X 9 : 3 X 7 : 2 X 2 = 45:21:4

Sum of ratios 45 + 21 + 4 =70; But total amount = Rs. 98000.

∴ Amount collected from first class passengers = (45/70) x Rs. 98000 = Rs. 63000.

But N X R = V   ∴ R = V/N.

Rate of first class ticket = $\frac{Rs. 63000}{500}$ = Rs. 126.

Exercise 3

1.	Divide Rs. 90 in the ratio 7:11.

2.	Divide Rs. 1690 in the ratio 1/2 : 1/3: 1/4

3.	Divide Rs. 90 in two parts such that 5/8 of first part bears a ratio of 5:4 with 2/5 of second part.

4.	The ratio of milk and water in a 20 liters mixture is 3:1. How much milk must be added so that this ratio becomes 4:1?

5.	Rs. 232 are divided among 480 children such that each boy gets 40 paise and each girl gets 60 paise. How many girls were there?

6.	A certain sum of money is distributed among two friends in the ratio 5:11. If one of them got Rs. 1200 more than the other, what was the total sum?

7.	Divide Rs. 395 among A,B and C such that B gets 25% more than A and 20% more than C.

8.	Divide Rs.4300 in A,B,C and D such that when A gets Rs.9 then B gets Rs.8; when B gets Rs.6 then C gets Rs.5 and when C gets Rs.4 then D gets Rs.3.

9.	Divide Rs. 730 among Sudha, Kamla and Kamal such that if Sudha gets Rs.3, then Kamla gets Rs.4 and if Kamla gets Rs.3.50 then Kamal gets Rs.3.

10.	Divide Rs. 558 in A,B,C such that if Rs.4, 6 and 8 are decreased respectively from their shares then the ratio of their shares is 2:3:7.

11.	Divide Rs.1215, in Manish, Sangeeta and Umesh such that Sangeeta gets double of Manish and Umesh gets three times of what Sangeeta gets.

12.	Divide Rs.1180 in A,B,C such that 5 times of A, 6 times of B and 8 times of C are equal.

13.	Divide 510 in A,B, C such that A gets 2/3 of what B gets and B gets 1/4 of what C gets. Find the share of each.

14.	Divide Rs.12540 in A, B and C such that A shall receive 3/7 of B and C together and B may receive 2/9 of what A and C together receive.

15.	Divide Rs.600 among A,B, and C so that Rs.40 more than 2/5 of A's share, Rs.20 more than 2/7 of B's share and Rs.10 more than 9/17 of C's share are equal.

16.	Divide Rs.258 in 8 men, 6 women and 10 children such that a woman gets 3/2 of a child and a man gets twice of a woman. What is the share of each man, woman and child?

17.	Rs.430 are divided among 45 persons consisting of men women and children. The combined shares of all men, all women and all children are as 12:15:16 whereas the individual share of a man, a woman and a child are as 6:5:4. Find what one man, one woman and one child gets.

18.	A bag contains fifty paise, twenty five paise and 10 paise coins in the ratio of 2:5:10 If the total amount is Rs.52, find the number of coins of each kind.

19.	A bag contains 1 rupee, 50 paise and 10 paise coins. The ratio of 1 rupee and 50 paise coins is 2:5. The ratio of 50 paise and 10 paise coins is 4:9. If total money is Rs.1125, find the number of each type of coins.

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20. A purse contains 378 coins consisting of rupees, 50 paise and 25 paise coins. Their values are in the ratio 13:11:7. Find the number of each coin.

21. Between two stations the first, second and third class fares were fixed at first in the ratio of 8:6:3, but afterwards the first class fares were reduced by 1/6 and the second class by 1/12. In a year, the number of first, second and third class passengers were respectively 9:12:26 and the money at the booking offices was Rs.1088. How much was paid by first class passengers?

22. Between two railway stations the I, II and III class fares were in the ratio 10:8:3 and in an year the ratio of passengers of I, II & III classes was 3:4:10 respectively. If the total sale proceeds of the tickets during an year was Rs.8050, then find the amount for which the tickets of the II class were sold during that year.

23. Three persons A,B,C finished a piece of work. A worked at it for 5 days, B for 7 days and C for 9 days. Their daily wages were in the proportion of 4:3:2 and the earning amounted to Rs.118. What were the daily wages of each?

24. If Rs.150 be divided among 6 men, 12 women and 17 boys so that 2 men receive as much as 5 boys and 2 women as much as 3 boys, how much will each man, woman and boy receive?

25. Men, women and children are employed to do a work in the proportion of 1:2:3: and their wages are 6:3:2. When 50 men are employed, total wages of all the hands amount to Rs.450. Find the weekly wages paid to a man, a woman and a child.

26. Annual incomes of A and B are in the ratio 4:3 and their annual expenses bear the ratio of 3:2. If each of them saves Rs.600 at the end of the year, Find the annual income of each.

27. A and B earn in the ratio 2:1. They spend in the ratio of 5:3 and save in the ratio of 4:1. If the total monthly savings of both A and B is Rs.500, find the monthly income of each.

28. A student appeared in an examination with Hindi, English, Physics, Chemistry and Mathematics as subjects. He got the marks in these subjects in the ratio of 5:3:4:6:7. The maximum marks in each subject were 100. He got 70% marks in aggregate in all the subjects. If 60% marks are essential for getting first class in each subject, then point out the subjects in which he got first division.

29. In a competitive examination, there are four subjects. The maximum marks of all the subjects are same. The ratio of marks of a student in different subjects is 3:4:6:7. If he gets 60% marks in all, find out the number of subjects in which he gets more than 50% marks.

30. The annual incomes of A, B, and C taken together is Rs.33600. A spends 80% of his income, B spends 87 $\frac{1}{2}$% of his income and C spends 90% of his income. If their annual savings are as 16:17:12, find the annual saving of each.

31. At the beginning of a term, the ratio of the number of boys in a school under 15 years to those over 15 years age was 5:4. At the end of term the ratio was 7:8 as 20 of the boys had reached the age of 15 years during the term. Find the total number of boys in the school, given that no boy left or was admitted during the term.

32. It was intended that Rs.19.50 should be divided between three boys A, B and C in the ratio of 2:3:4, but by mistake the distribution was made in the proportion of 1/2 : 1/3 : 1/4. How much does B gain or lose by error?

33. A man gives 3/8 of his property to one son and 30% of the remainder to another. He then distributed the remaining property among three charities in the proportion 2:5:7:. The difference of his sons' share is Rs.1800. What was the value of his property and how much was given to each charity?

34. Three persons A,B and C agree to pay their total bill in the proportion 4:5:6. A pays the first day's bill which amounts to Rs.45.75; B the second days' bill which amounts to 59.60 and C the third day's bill which amounts to Rs.74.65. How must they settle their accounts?

35. A employer reduces the number of his employees in the ratio of 9:8 and increases the wages in the ratio of 14:15. Find in what ratio the wage bill is increased or decreased and find the difference in the amount of the bill, if it was previously Rs.1680.

36. A sum of Rs. 2368 was divided among 8 men, 10 women and 12 children in such way that each man got 25 percent more than a woman and each woman got 25 percent more than a child. How much did each woman get?

37. A certain amount is divided among 2 boys and 2 girls. 1st boy gets 1/4 of the whole where as the second gets 1/5 of the remaining. The balance amount is distributed equally between both the girls. Find the share of each girl.

38. A man, his wife and son worked in a garden, The man for 2 days, his wife for $3\frac{1}{2}$ days and the boy for 4 days. Their daily wages were, man's to woman's in the ratio of 7:4 and the man's to boy's in the ratio of 7 to 3. Their total earnings amounted to Rs.60. Find the daily wages of each.

39. A tin is full of wine. Out of this tin, 4 gallons of wine are taken out and 4 gallons of water are added. After this 4 gallons of mixture were taken out and again 4 gallons of water were added. Now the ratio of wine and water in the tin 36:13. Find the volume of tin?

40. If an election in which each elector may vote for two candidates, half of the electors vote for A but divide their other votes between B, C and D in the ratio 3:2:1 . Of the remainder, half vote for B and divide their votes between C and D, in the ratio of 2:1 .Of the remainder, half vote for C and D, and the remainder 840 in number do not vote. How many votes does each candidate get?

Answers

1. Rs.35, Rs.55   **2.** Rs.780, Rs.520, Rs.390   **3.** Rs.40, Rs.50

4. 5 litres   **5.** 200   **6.** Rs.3200

7. Rs.120, Rs.150, Rs.125   **8.** Rs.1350, Rs.1200, Rs.1000, Rs.750

9. Rs.210, Rs.280, Rs.240   **10.** Rs.94, Rs.141, Rs.323   **11.** Rs.135, Rs.270, Rs.810

12. Rs.480, Rs.400, Rs.300   **13.** Rs.60, Rs. 90, Rs.360   **14.** Rs.3762, Rs.2280, Rs.6498

15. Rs.150, Rs.280, Rs.170   **16.** Rs.18, Rs.9, Rs.6   **17.** Rs.12, Rs.10, Rs.8

18. 32, 80, 160   **19.** 400, 1000, 2250   **20.** 78, 132, 168

21. Rs.320   **22.** Rs.2800   **23.** Rs.8, Rs.6, Rs.4

24. Rs.7.50, Rs.4.50, Rs.3   **25.** Rs.21, Rs.10.5, Rs.7   **26.** Rs.2400, Rs.1800

27. Rs.1400, Rs.700   **28.** Hin 70, Chem 84, Maths 98   **29.** two subjects.

30. 1600, 1700, 1200   **31.** 225   **32.** Rs.0.50 loss

33. Rs.9600, Rs.600, Rs.1500, Rs.2100   **34.** A to C Rs.2.25, B to C Rs.0.40

35. 21:20, Rs.80   **36.** Rs.80   **37.** 3/10

38. Rs.10.50, Rs.6, Rs.4.50   **39.** 28 gallons.   **40.** 3360, 3360, 3080, 1960

C o n c e p t s 4

Time & Work

- If A can do a piece of work in x days, then the work done by A in one day is equal to 1/x of the entire work. e.g. if A can finish a job in 10 days then A can finish 1/10 of the job in a day.

- If A is twice as good a workman as B, then A will take half of the time taken by B to do a certain piece of work.

- If the number of people engaged to do a certain work be increased (or decreased) in a certain ratio, the time required to do the same work is decreased (or increased) in the same ratio. e.g. if the number of men is changed in the ratio 3:4, the time taken to finish the work is changed to 4:3.

Ex. Tina can do a piece of work in 12 days and Mina alone can do it in 15 days. What time will they take to complete the job if they work together ?

Sol. In one day Tina can complete 1/12 of the work & Mina can complete 1/15 of the work.
∴ in one day Tina & Mina can together complete (1/12 + 1/15) = 3/20 of the work
∴ both can complete the work together in 20/3 days.

Ex. A can do a piece of work in 25 days and B can do it in 20 days. They work together for 5 days and then A goes away. In how many days will B finish the work ?

Sol. A & B together complete (1/25 + 1/20) = 9/100 of the work in a day
∴ in 5 days they will complete 5(9/100) = 9/20 of the work
Remaining work = 1 - 9/20 = 11/20
1/20 work is completed by B in a day.
∴ 11/20 work will be completed by B in (20 x 11)/20 = 11 days

Ex. A is thrice as good a workman as B and can therefore finish a piece of work in 60 days less than B. Find the time in which they can do it working together.

Sol. Let B finish the work in x days. A can finish it in (x - 60) days.
But A is thrice as good a workman as B, so time taken by A is 1/3 the time taken by B.
(x - 60) = x/3 or x = 90
Time taken by B = 90 days , time taken by A = 30 days
A & B together can finish (1/90 + 1/30) = 2/45 work in a day
∴ A & B together finish the whole work in 45/2 = 22.5 days

Ex. Certain number of men can finish a piece of work in 60 days. If there were 8 men more the work could be finished in 10 days less. Find the original number of men.

Sol. Let the original number of men be x
x men can complete the work in 60 days and (x + 8) men can complete the work in 50 days.
(x + 8) : x :: 60 : 50
(x + 8)/x = 60/50
∴ x = 40
∴ the original number of men = 40

Ex. A can do a piece of work in 10 days, B in 12 days and C in 15 days. They all start the work together, but A leaves the work after 2 days and B leaves 3 days before the work is completed. How many days did the work last ?

Sol. A, B & C work together for 2 days, C works alone for 3 days, B & C finish the remaining work together.
A, B & C finish in 2 days : 2(1/10 + 1/12 + 1/15) = 1/2 of the work
C finishes in 3 days : 3(1/15) = 1/5 of the work

Remaining work = 1 - (1/2 + 1/5) = 3/10
B & C can complete in a day : (1/12 + 1/15) = 3/20 of the work
Thus, B & C complete 3/20 of the work in a day,
they complete 3/10 of the work in (20/3) x (3/10) = 2 days
∴ total time taken = 2 + 3 + 2 = 7 days

Pipes & Tanks

A pipe connected with a tank is called an inlet pipe or an outlet pipe accordingly as it fills it or empties it respectively.

Note : Conventionally, filling of a tank is treated as a positive work and emptying it as a negative work.

Net Work Done = (Sum of work done by inlets) - (Sum of work done by outlets)

Ex. Two pipes can fill a cistern in 14 & 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom 32 extra minutes are taken for the cistern to be filled up. If the cistern is full, in what time will the leak empty it ?

Sol. Work done by both the pipes in an hour = (1/14) + (1/16) = 15/112
Time taken by both the pipes to fill the cistern(without leakage)=112/15hours=7 hours 28mins
Due to leakage time taken = (7 hrs. 28 minutes) + (32 minutes) = 8 hours
Net work done by the pipes and the leak in 1 hour = 1/8
Work done by leak in 1 hour = (15/112) - (1/8) = 1/112
∴ time taken by the leak to empty the cistern = 112 hours

Ex. Two pipes A & B can fill a cistern in 20 minutes and 25 minutes respectively. Both are opened together, but at the end of 5 minutes, B is turned off. How much longer will the cistern take to fill.

Sol. Cistern filled by A & B in 5 minutes.
        = 5(1/20 + 1/25) = 9/20
Part left unfilled = (1 - 9/20) = 11/20
1/20 part is filled by A in one minute
∴ 11/20 part will be filled by A in 20 x 11/20 = 11 minutes
∴ total time taken to fill the cistern = 5 + 11 = 16 minutes

Ex. Three pipes, A, B, & C are attached to a cistern. A & B can fill it in 20 & 30 minutes respectively while C can empty it in 15 minutes. If A, B & C are kept open successively for 1 minute each, how soon will the cistern be filled ?

Sol. Work done in 3 minutes = (1/20 + 1/30 - 1/15) = 1/60
Thus 55/60 part of the cistern gets filled in 3 x 55 = 165 minutes
Out of the remaining 5/60 cistern, 1/20 is filled by A in 1 minute and (5/60 - 1/20) = 1/30 will be filled by B in 1 minute.
∴the total time taken to fill the cistern = 165 + 1 + 1 = 167 minutes

Exercise 4

1. A alone can do a piece of work in 6 days and B alone can do it in 8 days. How long would it take for A and B to finish the work?

2. Mohan can do 1/2 of a work in 8 days while Sohan can do 1/3 of the same work in 6 days. How long would it take for Mohan and Sohan to finish the work?

3. A and B can do a piece of work in 10 days and A alone can do it in 15 days. In how many days can B alone do it?

4. A can do a piece of work in 45 days. He works at it for 5 days and then B completes it in 20 days. How long will A and B together take to complete the work?

5. A and B together can complete a work in 8 days. B and C together can complete the same work in 12 days. If A alone can do that work in 16 days, then how long would it take for A and C together to complete that work?

6. A and B can do a piece of work in 24 days. B and C can do a piece of work in 30 days. C and A can do a piece of work in 40 days. How long would they all take to do the same work? In what time can each do it separately?

7. A and B can do a piece of work in 30 days while B and C can do the same work in 24 days and C and A in 20 days. They all work for 10 days when B and C leave. How many days more will A take to finish the work?

8. Ram finished 3/5 of a work in 9 days and the remaining work he finished in 4 days with the assistance of Shyam. Find in how many days Shyam alone can finish that work?

9. Mohan does a work in 12 days and Sohan does the same work in 20 days. Mohan alone started the work and after 4 days, Sohan began to do work with Mohan. Find
   (i) how much work did Mohan do in 4 days
   (ii) the Balance work.
   (iii) in how many days did both together finish the balance work?

10. Ram and Shyam together can finish a work in 8 days. Both together started the work. After 3 days, Ram fell ill. Shyam alone finished the remaining work in 15 days. How many days would Ram take to finish that work alone?

11. A and B can do a work in 45 days and 40 days respectively. Both started to do the work simultaneously but after few days, A left the work. Then B alone completed the work in 23 days. After how many days did A leave the work?

12. A can do a piece work in 120 days and B can do it in 150 days. They work together for 20 days. Then B leaves and A continues the work alone, 12 days after that C joins A and the work is completed in 48 days more. In how many days can C do it if he works alone?

13. A and B can do a piece of work in 12 days. B and C together do it in 15 days. If A is twice as good a workman as C, find in what time B alone will do it?

14. A and B could do a piece of work in 40 days; after working for 10 days they are assisted by C, and the work is finished in 20 days more. If C does as much work in 2 days as B does in 3 days, in how many days could each of them do the same work alone?

15. Ashok takes twice as much time as Arun and thrice as much as Alok to finish a piece of work. They together finish the work in one day. Find the time taken by each to finish the work.

16. A, B and C respectively can finish a work in 12,15 and 24 days respectively. A, B and C together finished the work and received Rs.920. Find the share of each in the total remuneration?

17. A can do a piece of work in 16 days and B in 24 days. They take the help of C and the three together finish the work in 6 days. If the total remuneration for the work is Rs.40, find the amount each will receive in proportion to work done by each.

18. Three taps empty a tank in 3 hours. The first tap alone can empty it in 6 hours and second tap alone can empty it in 9 hours. How many hours would third tap alone take to empty the tank?

19. Two pipes A and B would fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes being opened, find when the second pipe must be turned off, so that the cistern may be filled in half an hour?

20. A cistern is provided with two supply pipes A and B and a waste pipe C. A and B fill it in 12 and 20 minutes respectively and C discharges 15 gallons of water per minute. If all the pipes are opened at once, the empty cistern is filled in 2 hours. Find the capacity of the cistern.

21. A cistern can be filled separately by two pipes A and B in 45 minutes and 36 minutes respectively. A tap C at the bottom can empty the full cistern in 30 minutes. If the tap C is opened 7 minutes after the two taps A and B are opened, find when the cistern will be full?

22. There are two taps which can fill a cistern in 60 minutes and 75 minutes respectively and there is another exhausting tap to it. If all the taps are opened at the same time, the cistern becomes full in 50 minutes. In what time can the third tap empty the cistern when full?

23. A pipe can fill a bath in 20 minutes and another can fill it in 30 minutes. A person opens both the pipes simultaneously. When the bath should have been full, he finds that the waste pipe was open. He then closes the waste pipe and in 3 minutes more, the bath is full. In what time, would the waste pipe empty it?

24. A, B and C can do a piece of work in 6,8 and 10 days respectively. They begin to work together; A continues to work till it is finished; B leaves off 1 day before and C leaves off 1/2 day before the work is finished. In what time is the work finished?

25. 3 men or 5 women can do a job in 12 days. How long will 6 men and 5 women take to finish the job?

26. A, B and C are employed to do a piece of work for Rs.529. A and B together are supposed to do 19/23 of the work and B and C together 8/23 of the work. What should A be paid?

27. Two men undertake to do a piece of work for Rs.240. One could do it alone in 5 days and the other in 8 days. With the help of a boy and a girl they finished it in 3 days. If the work done by the boy be twice that of the girl, how should the money be divided amongst the workers?

28. A contractor receives a certain sum that he uses for paying wages. His capital together with the weekly subsidy would just enable him to pay 42 men for 52 weeks. If he had 60 men at the same wages, his capital together with the weekly subsidy would just suffice for 13 weeks. How many men can be maintained for 26 weeks?

29. Two pipes A and B can fill up a tank in 6 minutes and 7 minutes respectively. If they are opened for one minute alternately, A beginning, in how much time will they fill up the tank?

30. There is a leak in the bottom of a cistern. When the cistern had no leak, it was filled in $2\frac{1}{2}$ hours. It now takes half an hour longer. If the cistern is full of water, how long would it take in leaking itself empty, in case the water leaks out at double the rate after half the cistern becomes empty?

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31. Two taps can separately fill a cistern, when the waste pipe is closed in 10 and 12 minutes, respectively. When the waste pipe is open they together fill it in 15 minutes. How long does it take the waste pipe to empty the cistern, when the taps are closed.

32. A cistern can be filled by one of the two pipes in 30 minutes and by the other in 36 minutes. Both pipes are opened for a certain time but being partially clogged, only 5/6 of the full quantity of water flows through the former and only 9/10 through the later. The obstructions, however, being suddenly removed, the cistern is filled in 15 $\frac{1}{2}$ minutes from that moment. How long was it before the full flow of water began?

33. Two taps can fill a cistern separately in 60 and 80 minutes respectively. To fill the empty cistern, both the taps are kept open for 20 minutes and then the second tap is turned off. Find the time required to fill the cistern.

34. A bath can be filled by the cold water pipe in 10 minutes and by the hot water pipe in 15 minutes. A person leaves the bathroom after turning on both pipes simultaneously and returns at the moment when the bath should be full. Finding, however, that the waste pipe has been open, he now closes it. In 4 minutes more the bath is full. In what time would the waste pipe empty it?

35. A supply of water lasts for 200 days if 10 gallons leak off every day, but only for 180 days if 16 gallons leak off daily. Find the total quantity of water in the supply.

36. A tap drips at an average rate of 2 drops in 3 seconds. It takes 2080 drops of water thus collected to completely fill a vessel of hemispherical shape (radius 8.3 cm.). How many times can the vessel be filled in this manner in 13 hours?

37. A photocopy machine copies 1500 workbooks in 8 hours while another machine takes 12 hours to do the same job. What is the total number of hours that it would take both the machines working together, to complete the 1500 workbooks?

38. A is thrice as good a workman as B and therefore able to finish a job in 60 days less than B. Working together they will finish this job in how many days?

39. Pipe A can fill a tank in 4 minutes whereas pipes A & B together can fill the same tank in 3 minutes. How much time will pipe B take to fill the tank?

40. Two pipes A & B fill 12 litres and 15 litres of water, respectively into a tank in one hour. Another pipe C working independently empties the tank in 12 hours. The three pipes together fill the tank in 6 hours. What is the capacity of the tank?

Answers

1. 24/7 days **2.** 8 $^8/_{17}$ days **3.** 30 days **4.** 15 days
5. 12 days **6.** (A+B+C)=20 days A=60 days B=40 days C=120 days
7. 18 days **8.** 30 days **9.** (i) 1/3 (ii) 2/3 (iii) 5 days
10. 12 days **11.** 9 days **12.** 240 days **13.** 20 days
14. C = 80 days, B = 120 days, A = 60 days **15.** Ashok = 6 days, Arun = 3 days, Alok = 2 d
16. Rs.400, Rs.320, Rs.200 **17.** Rs.15, Rs.10, Rs.15 **18.** 18 hours
19. after 9 minutes **20.** 120 gallons **21.** 46 minutes **22.** 100 minutes
23. 48 minutes **24.** 3 days **25.** 4 days **26.** Rs.345
27. A=Rs.144, B=Rs.90, Boy=Rs.4, Girl=Rs.2 **28.** 48 men **29.** 6 $^3/_7$ minutes
30. 11 hours 15 minutes **31.** 8 mins 34 secs **32.** 1 minute **33.** 45 minutes
34. 9 minutes **35.** 10800 gallons. **36.** 15 times **37.** 4.8 hours
38. 22 ½ days **39.** 12 minutes **40.** 108 litres

C o n c e p t s 5

Averages and Alligation

Averages

Average = Sum of quantities / Number of quantities

If the weights of five people are 60 kg., 70 kg., 80 kg., 90 kg., & 100 kg., then the average weight is given by, (60 + 70 + 80 + 90 + 100)/5 = 80 kg.

Ex. 13 ducks and 9 pigeons were bought for Rs. 1291.85. If the average price of a duck is Rs. 74, what is the average price of a pigeon?

Sol. The average price of a duck = Rs. 74
∴total price of 13 ducks = Rs. 13 x 74 = Rs. 962
Total price of 13 ducks and 9 pigeons = Rs. 1291.85
∴ Total price of 9 pigeons = Rs. 1291.85 - 962 = Rs. 329.85
∴ average price of a pigeon = Rs. (329.85/9) = Rs. 36.65

Ex. The average weight of 34 men is 46.5 kg. If one more man is added to the group the average weight rises by half a kg. What is the weight of the new man?

Sol. Average weight of 34 men = 46.5 kg.
∴ Total weight of 34 men = 46.5 x 34 = 1581 kg.
Average weight of 35 men (34 + 1 new) = 46.5 + 0.5 = 47 kg.
Total weight of 35 men = 47 x 35 = 1645 kg.
∴ weight of the new man = (1645 - 1581) = 64 kg.

Ex. The average of 11 numbers is 50. If the average of the first six numbers is 49 and that of last six is 52, what is the sixth number ?

Sol. The total of the 11 numbers = 11 x 50 = 550
The total of the first six numbers = 6 x 49 = 294
The total of the last six numbers = 6 x 52 = 312
∴ Sixth number = 294 + 312 - 550 = 56

Ex. A batsman increases his average by 2 runs when he makes 63 runs in his 12th innings. What is his average after 12 innings ?

Sol. Average score after 11 innings = 63 - (2 x 12) = 39
∴ average score after 12 innings = 39 + 2 = 41

Ex. The average weight of a group of 10 men is increased by 1.5 kg when one of the men of weight 68 kg. is replaced by a new man. What is the weight of the new man ?

Sol. Total increase in weight = 10 x 1.5 = 15 kg.
∴ weight of the new man = 68 + 15 kg. = 83 kg.

Alligation

- **Alligation** is the method to determine the ratio in which two quantities at different rates should be mixed to get a mixture of desired rate.
- If two quantities with different rates R_1 & R_2 are mixed to get the mean rate of the mixture as R_M then the ratio in which two quantities should be mixed is given by

$$N_1 / N_2 = (R_2 - R_M) / (R_M - R_1)$$

This equation is represented as follows :

R₁ .... R₂

R_M

(R₂ – R_M)   :   (R_M – R₁)

$$N_1 / N_2 = (R_2 - R_M) / (R_M - R_1)$$
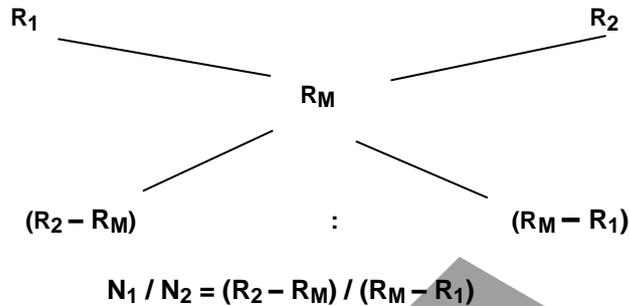
Ex. What is the proportion in which rice at Rs. 3.10 per kg must be mixed with rice at Rs. 3.60 per kg, so that the mixture is worth Rs. 3.25 per kg ?

Sol.

R_1 = 310 paise R_2 = 360 paise

R_M = 325 paise

35   15

$$N_1 / N_2 = 35 / 15 = 7 / 3$$
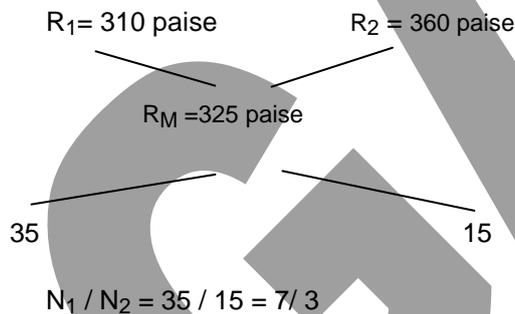
Where N_1 = Number of kg's of rice at R_1 and N_2 = Number of kg's of rice at R_2
Hence the required ratio is 7 : 3.

POINTS TO REMEMBER

- All three values that are being alligated should represent the same variable (either R or N). All three values should have same units.
- Alligation of 3 values of R (rate) gives the ratio in terms of N (number) & vice – versa.
- In case of problems based on mixing of two solutions R represents the concentration.
- In some questions, 2 values of cost price and selling price of the mixture is given. In such cases first find out the cost price of the mixture & then alligate the 3 values of cost price.

Ex. What quantity of sugar costing Rs. 6.10 per kg must be mixed with 126 kg of sugar priced at Rs. 2.85 per kg, so that 20% may be gained by selling the mixture at Rs. 4.80 per kg ?

Sol. S.P. of 1 kg of mixture = Rs. 4.80 Gain = 20%
C.P. of 1 kg of mixture = (100/120) x 4.80 = Rs. 4.00

R_1 = 285 paise R_2 = 610 paise

R_M = 400

210   115

$$N_1 / N_2 = 210 / 115 = 42 / 23$$
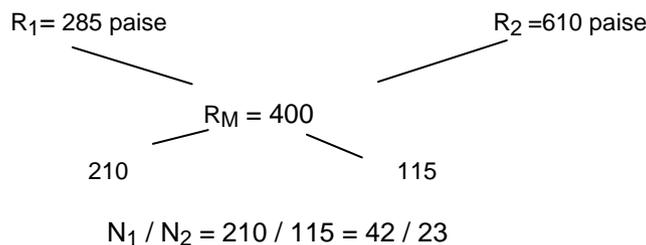
If sugar at 285 paise per kg. is 42 kg then sugar at 610 paise per kg. = 23 kg
∴ If sugar at 285 paise per kg. is 126 kg then sugar at 610 paise per kg. = 69 kg

Exercise 5

1. The sum of seven numbers is 235. The average of first three numbers is 23 and the average of last three is 42. Find the fourth number.

2. 30 horses were purchased for Rs.12000. The average cost of 12 horses out of them is Rs.250. Find the average cost of the remaining horses.

3. The average marks of 500 students in an examination are 45. Among them the average marks of the last 150 are 25 and that of the top 150 students are 75. What are the average marks of the remaining 200 students.

4. There are 40 students in a class. Out of these students, one student whose weight is 100 kg goes away and at the same time a new student gets admitted into the class. Thus the average weight of the students increases by 1/4 kg. Find the weight of new student.

5. The average temperature of Monday, Tuesday, Wednesday and Thursday is 62°. The average temperature of Monday and Tuesday is 56°. The ratio of temperatures of Wednesday and Thursday is 15:19. Find the temperatures of Wednesday and Thursday.

6. In a cricket match, 6 players had a certain average of their runs. Seventh player makes a score of 112 runs, thereby increasing the average of their runs by 10. Find the average of first 6 players.

7. The average weight of boys in a class is 43 kg. Later four boys join them, whose weights are respectively 42 kg, 36.5 kg, 39 kg and 42.5 kg. The average now becomes 42.5 kg. Find the original number of boys in the class.

8. The average age of 8 men increases by 2 years when two women are included in a place of two men of ages 20 and 24 years. Find the average age of the women.

9. The average of 8 results is 20. The average of first two results is 15 $\frac{1}{2}$. The average of next three results is 21 $\frac{1}{3}$. The sixth result is 4 less than seventh and 7 less than 8th. Find the last result.

10. The average salary per head of the entire staff of an office including the officers and clerks is Rs.90. The average salary of the officers is Rs.600 and that of clerks is Rs.84. If the number of officers is 12, find the no. of clerks in the office.

11. A man had 7 children. When their average age was 12 years, the child who was 6 years of age, died. What was the average of the surviving children 5 years after the death of his child?

12. A motorist covers the distance of 60 km from A to B at a speed by 20 km per hour and return journey from B to A at a speed of 30 km per hour. Determine the average speed of the motorist for the entire journey.

13. A car starts from rest and traverses first 1/20 th of 1 km with a speed of 9 km/hr and next three 1/20 th of 1 km with a speed of 12,18 & 36 km/hr respectively. Two students calculated the average speeds of this car during first 1/5th of 1 km as 18.75 & 14.4 km/hr. Which of these answers is correct and why?

14. The average speed of a train including stoppage times at stations is 27 km/hr and the average speed of a train excluding stoppage times at stations is 41 km/hr. Find how many minutes does a train stop in 1 hour?

15. In each row of a hand written book of 180 pages, there are 12 words on an average and each page consists of 16 rows on an average. After printing, in each row of the book these are 18

words on average and each page contains 20 rows on average. Find the number of pages of the printed book.

16. Visitors to a show were charged Rs.15 each on the first day, Rs. 7.5 on the second, Rs. 2.5 on the third and the total attendance on three days were in the ratio 2:5:13 respectively. Find the average charge per person for the whole show.

17. A ship, 40 km from the shore, springs a leak which admits $3 \frac{3}{4}$ tonnes of water in 12 minutes. 60 tonnes would suffice to sink her, but the ship's pumps can throw out 12 tonnes of water in one hour. Find the average rate of sailing, so that she may reach the shore just she begins to sink.

18. A mixture of 70 litres of wine and water contains 10 per cent of water. How much water must be added to make water 37 per cent of the resulting mixture?

19. In what ratio should water be added to a liquid costing Rs.12 per litre so as to make a profit of 25% by selling the diluted liquid at Rs.13.75 per litre?

20. How many kg of tea selling at Rs.10.40 per kg should be mixed with tea selling at Rs. 8.80 per kg to make a mixture 15 kg at Rs.146.40?

21. A litre of water weighs a kilogram and a litre of another liquid weighs 1.340 kilograms. A mixture of two weighs 1.270 kilograms per litre. Determine the volume of water in a litre of mixture.

22. A merchant has 100 kg of sugar, part of which he sells at 7 p.c. profit and the rest at 17 p.c. profit. He gains 10 p.c. on the whole. Find how much is sold at 7% profit?

23. A man invested Rs.11,000 partly in 6% stock at 125 and partly in 5% stock at 120. The total annual dividend received by him was Rs. 490. Find the amount invested in 5% stock.

24. The average salary per head of all the workers and officers in a workshop is Rs. 95. The average salary of 15 officers is Rs. 525 and the average salary of the workers is Rs. 85. Find the number of workers in the workshop.

25. A tea merchant buys two kinds of tea, the price of the first being twice that of the second. He sells the mixture at Rs.14 per kg, thereby making a profit of 20 per cent. If the ratio of the first and second kinds of tea in the mixture be 2:3, find the cost price of each kind of tea.

26. A cup of milk contains 3 parts of pure milk and 1 part of water. How much of the mixture must be withdrawn and water substituted in order that the resulting mixture may be half milk and half water.

27. A mixture contains wine and water in the ratio 3:2. Another contains wine and water in the ratio 4:5. How many gallons of the later must be mixed with 3 gallons of the former so that the resulting mixture may contain equal quantities of wine and water?

28. One alloy of metal contains 90 % copper and 10 % tin. Another alloy contains 93 % copper 4% tin. If they are mixed so that the mixture may contain 9 % of tin , what per cent of copper will it contain?

29. One alloy contains silver and copper in the ratio of 5:1 and the other contains them in the ratio 7:2 respectively. What weights of the two must be melted together, so as to make a 5 lb. mass with 80% silver?

30. Nine litres are drawn from a cask full of wine and it is then filled with water. Nine litres of the mixture are drawn and the cask is again filled with water. The quantity of the wine now left in the cask is to that of the water in it as 16:9. How much does the cask hold?

31. A vessel contains mixture of spirit and water. Spirit is 18%. 8 litres of mixture is taken out of the vessel which is again filled with water. If the present percentage of spirit is 15%, find the quantity in litres of the mixture in the vessel.

32. Gold is 19 times as heavy as water and copper 9 times. In what ratio should these metals be mixed that the mixture may be 15 times as heavy as water?

33. A man buys milk at 85 paise per litre and mixes water in it. He sells the mixture at the same rate and thus gains $11\frac{1}{9}$%. Find the quantity of water per litre of the mixture?

34. Two liquids are mixed in the ratio 5:3 and by selling the mixture at Rs.12.25 / litre, a profit of $16\frac{2}{3}$ per cent is made. If the fist liquid costs Rs.4 / litre more than the second, find the price per litre of each liquid.

35. A mixture composed of 4 parts of brandy and 1 part of water. 1 kg of water is added, and the resulting mixture contains 3 times as much brandy as water. Find the quantity of brandy in the original mixture.

36. Three equal glasses are filled with mixtures of spirit and water. The proportion of spirit to water in each glass is as follows: in the first glass as 2:3, in the second glass as 3:4 and in the third glass as 4:5. The contents of the three are emptied into a single vessel. What is the proportion of spirit and water in it?

37. The ratio of petrol and spirit in three liquids is 2:3, 3:4 and 4:5 respectively. A motor owner mixes 20 litres of first liquid, 21 litres of second liquid and few litres of third liquid. If in the resulting mixture the ratio of petrol and spirit is 29:39, find how many litres of the third liquid were taken to prepare resulting mixture?

38. There are two mixtures, one containing wine, water and alcohol in the proportion 3:5:2 and the other containing wine and water in the ratio 4:5. If 1 litre of the first be mixed with 2 litres of second, what fraction of the resulting mixture will be wine?

39. A butler stole wine from a butt of sherry which contained 30% of spirit and he replaced what he had stolen by wine containing 12% of spirit. The butt was then of 18% strength only. How much of the butt did he steal?

40. A lump of two metals weighing 18 grams is worth Rs.74 but if their weights be interchanged, it would be worth Rs.60.10. If the price of the gold be Rs.7.20 per gram, find the weight of the other metal in the mixture .

Answers

1. 40	**2.** Rs.500	**3.** 37.5	**4.** 110 kg	**5.** 60, 76 degrees
6. 42	**7.** 20	**8.** 30 years	**9.** 25	**10.** 1020
11. 18 years	**12.** 24 km/hr	**13.** 14.4 km/hr	**14.** 20.488 mins	**15.** 96 pages
16. Rs. 5	**17.** 4.5 km/hr	**18.** 30 litres	**19.** 11 : 1	**20.** 9 kg.
21. 7/34 lt.	**22.** 70 kg	**23.** Rs. 6000	**24.** 645	**25.** Rs $16\frac{2}{3}$ / kg, Rs $8\frac{1}{3}$ / kg
26. 1/3 part	**27.** 5.4 gallons	**28.** 90.5 %	**29.** 2 lb, 3 lb	**30.** 45 litres
31. 48	**32.** 3 : 2	**33.** 1/10	**34.** Rs 12& Rs 8	**35.** 12 kg
36. 401 : 544	**37.** 27 litres	**38.** 107/270 ltr	**39.** 2/3	**40.** 8 grams

C o n c e p t s 6

Time & Distance

Speed of a moving body is the distance traveled by it in a unit time.

$$\text{Speed} = \frac{\text{Distance Travelled}}{\text{Time Taken}}$$

Variation

1. Distance remaining constant, speed is inversely proportional to time and vice versa. If the speed of a body is changed in a certain ratio, say 3:4, then the time taken to travel a certain distance changes in an inverse ratio i.e. 4:3.

2. Speed remaining constant, time is directly proportional to distance and vice versa.
    If the time for which a body is moving at a certain rate has increased in the ratio, say 5:6, then the distance traveled has increased in the same ratio i.e. 5:6.

3. Time remaining constant, speed is directly proportional to distance and vice versa.
    If the speed of a body is changed in a certain ratio, say 3:4, then in a certain time the distance traveled increases in the same ratio i.e. 3:4.

4. If two persons A and B start at the same time in opposite directions from two points, each to the others point of starting, and reach their destinations in time t_a and t_b respectively after having met, the ratio of speeds Sa and Sb is given by the proportion

    $$S_a \quad : \quad S_b \quad : : \quad \sqrt{t_b} \quad : \quad \sqrt{t_a}$$

5. If the same distance is traveled at two different speeds S_1 and S_2, the average speed is given by

    $$S_a = \frac{2\ S_1\ S_2}{(S_1 + S_2)}$$

Ex. A person walking 6/7 of his usual rate is 25 minutes late. What is his usual time ?
Sol. Let the usual time be x minutes. The speed is 6/7 of the usual speed. Therefore the time now taken will be 7/6 of the usual time.
Given $(7/6)x = x + 25$ $\qquad \therefore \quad x = 150$ minutes $\quad \therefore$ Usual time = 2 hours 30 minutes

Ex. A boy starts from his house for college at a certain fixed time. If he walks at the rate of 5 kmph he is late by 7 minutes. If he walks at the rate of 6 kmph he reaches the college 5 minutes earlier than the scheduled time. Find the distance of the college from his house.
Sol. If the distance be 1 km, time taken to cover 1 km at 5 kmph = 1/5 hr & time taken to cover 1 km at 6 kmph = 1/6 hr ie. the difference of time is (1/30) hr
Difference of time in covering whole distance 'd'= (7 + 5) = 12 minutes = 1/5 hr
If for time difference (1/30) hr the distance is 1 km, the distance for time difference (1/5) hr will be d = 1 (1/5) / (1/30)km = (30)/5 km = 6 km
Distance between college & his house is 6 km

Ex. Two boys move in opposite directions, one from A to B and the other from B to A. The boy from A reaches his destination 16 hours and the boy from B reaches his destination 25 hours after they meet. If the former speed is 20 kmph, what is the speed of the latter.
Sol Let the boy from B travel at speed S_b.
By the formula $\qquad S_a \quad : \quad S_b \quad : : \quad \sqrt{t_b} \quad : \quad \sqrt{t_a}$
Here $t_a = 16$ hrs , $t_b = 25$ hrs , $S_a = 20$ kmph
$\therefore S_b = \dfrac{20 \times 4}{5} = 16$ kmph

Relative Speeds

When two bodies are in motion their speeds with respect to each other are different from their individual speeds. This speed is called relative speed.

1. If two bodies A and B are moving in the opposite direction, the relative speed is given by

$$S_R = S_1 + S_2$$

2. If two bodies A and B are moving in the same direction, the relative speed is given by

$$S_R = S_1 - S_2$$

Ex. Two boys start from the same place and walk at the rate of 4 kmph and 4.5 kmph respectively. When will they be 17 km apart if (i) they walk in opposite directions & (ii) they walk in the same direction

Sol. (i) If they walk in opposite directions they are 4 + 4.5 = 8.5 km apart in one hour.
Thus they are 8.5 km apart in 1 hour.
They are 17 km apart in (1/8.5)17 = 2 hrs
(ii) If they walk in the same direction they are 4.5 - 4 = 0.5 km apart in one hour.
Thus they are 0.5 km apart in 1 hour
They are 17 km apart in (1/0.5)17 = 34 hrs

Ex. Two towns P & Q are 110 km apart. A motorcycle rider starts from P towards Q at 7 a.m. at a speed of 20 kmph. Another rider starts from Q towards P at 8 a.m. at a speed of 25 kmph. Find when will they cross each other.

Sol. Let the two riders meet after x hours, from the time the first rider starts from P.
The distance traveled by the first rider in x hours = 20x km.
The distance traveled by second rider in (x - 1) hours = 25(x - 1) km
20x + 25(x - 1) = 110 i.e. x = 3 hours
Thus they meet at 10 a.m.

Trains

1. When a train passes a stationary object without considerable length
e.g. stationary man, pole, stationary car etc.
S is the speed of the train
t is the time taken by the train to completely pass the object from the time of encounter
l is the length of the train

$$S = \frac{l}{t}$$

2. When a train passes a stationary object with considerable length
e.g. stationary train, platform, bridge etc.
S is the speed of the train.
t is the time taken by the train to completely pass the object from the time of encounter **.**
l_1 is the length of the train.
l_2 is the length of the stationary object.

$$S = \frac{l_1 + l_2}{t}$$

3. When a train passes a moving object without considerable length
e.g. running man, moving car etc.
s_1 is the speed of the train
s_2 is the speed of the other object
t   is the time taken by the train to completely pass the object from the time of encounter
l   is the length of the train
S_R is the relative speed where

$$S_R = s_1 + s_2 \text{ if the two are moving in opposite directions.}$$
$$S_R = s_1 - s_2 \text{ if the two are moving in the same directions.}$$

$$S_R = \frac{l}{t}$$

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4. When a train passes a moving object with considerable length

e.g. another moving train, moving tram etc.

s_1 is the speed of the train **.**

s_2 is the speed of the other object **.**

t is the time taken by the train to completely pass the object from the time of encounter **.**

l_1 is the length of one train.

l_2 is the length of the other moving object.

S_R is the relative speed where

$S_R = s_1 + s_2$ if the two are moving in opposite directions.

$S_R = s_1 - s_2$ if the two are moving in the same directions.

$$S_R = \frac{l_1 + l_2}{t}$$

- **To convert speed from metre /second to kilometres/hour, multiply by 18/5.**

- **To convert speed from kilometres/hour to metre /second, multiply by 5/18.**

Ex. A boy standing at a platform observes that it takes 4.5 seconds for a train to pass him but it takes 9 seconds for the train to pass the platform. If the length of the train is 100 m, what is the speed of the train and the length of the platform?

Sol. When we consider the condition of train passing the boy

$S = l/t$ $\therefore S = \dfrac{100 \times 18}{4.5 \times 5}$ kmph

$= 80$ kmph

Speed of the train is 80 kmph

But when we consider the condition of train passing the platform

$S = \dfrac{l_1 + l_2}{t}$ $\therefore l_1 + l_2 = S \times t$

$\therefore 100 + l_2 = \dfrac{80 \times 5 \times 9}{18}$

$\therefore l_2 = 100$ m

Length of the platform is 100m

Boats & Streams

If the speed of the boat in still water be x kmph and the speed of the stream be y kmph then,

i) Speed of the boat with the stream (downstream) $\mathbf{S_d = (x + y)\ kmph}$

ii) Speed of the boat against the stream (upstream) $\mathbf{S_u = (x - y)\ kmph}$

iii) $\mathbf{x = (1/2)(S_d + S_u)}$

iv) $\mathbf{y = (1/2)(S_d - S_u)}$

Ex. A man rows with the stream at the rate of 14 kmph and against the stream at 9 kmph. What is the speed of the boat in still water and the speed of the stream ?

Sol. Speed of boat in still water = (1/2)(14 + 9) = 11.5 kmph

Speed of the stream = (1/2)(14 - 9) = 2.5 kmph

Ex. A man can row at 4.5 kmph in still water. It takes him twice as long to row up as to row down the river. What is the rate of the stream ?

Sol.   The man's rate downstream is twice the man's rate upstream. Let the man's rate upstream be
x kmph. Man's rate downstream be 2x kmph.
(1/2)(2x + x) = 4.5
x = 3 kmph
Man's rate upstream = 3 kmph
Man's rate downstream = 6 kmph
Rate of current = (1/2)(6 - 3) = 1.5 kmph

Ex.   A man rows at the rate of 5 kmph in still water. If the river runs at the rate of 1.5 kmph, it
takes him 1 hour to row to a place and back. What is the distance ?

Sol.   Man's rate downstream = (5 + 1.5) kmph = 6.5 kmph
Man's rate upstream = (5 - 1.5) = 3.5 kmph
Let the distance be x km.
Total time taken to row x km & back
= (x/6.5 + x/3.5)
(x/6.5 + x/3.5) = 1
x = 2.275 km
Thus the distance = 2 km 275 m

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Exercise 6

1. A train runs at a speed of 90 kilometer per hour. Express its speed in metres per second.

2. The speed of car is $16 \frac{2}{3}$ meters per second. Find the speed in kms per hour.

3. Walking at a speed of 3 kilometer per hour a man takes 8 hours to complete a journey. How much time will he require if his speed is 4 kms/hr to complete the journey?

4. Walking at 4 kms per hour, a clerk reaches his office 5 minutes late. If he walks at 5 kms per hour, he will be $2 \frac{1}{2}$ minutes early. Find the distance of his office from his house.

5. A man traveled a distance of 61 kms in 9 hr. partly on foot at the rate of 4 kms per hour and partly on bicycle at the rate of 9 kms per hour. Find the distance traveled on foot.

6. A person has to reach a place 40 kms away. He walks at the rate of 4 kms/hr for the first 16 kms and then travel in a tonga for the rest of the journey. However, if he had traveled by tonga for the 16 kms and covered the remaining distance on foot at 4 kms/hr, he would have taken an hour longer to complete the journey. Find the speed of the tonga.

7. I traveled one-third of a journey with a speed of 10 kms/hr, the next one-third with a speed of 9 km/hr, and the rest at a speed of 8 kms/hr. If I had traveled half the journey at a speed of 10 km/hr and the other half with a speed of 8 kms/hr, I would have been half a minute longer on the way. What distance did I travel?

8. Two places P and Q are 162 kms apart. A train leaves P for Q and at the same time another train leaves Q for P. The two trains meet at the end of 6 hours. If the train traveling from P to Q travels 8 kms an hour faster than the other, find the speed of the two trains.

9. A train 100 metres long meets a man going in opposite direction at the rate of 5 kms/hr and passes him in 7.2 seconds. At what rate is the train going?

10. A train 300 metres long over took a man walking along the line (in the same direction as the train) at the rate of 4 km an hour and passed him in 30 seconds. The train reached the station in 15 minutes after it had passed the man. In what time did man reach the station?

11. A goods train 158 metres long, and traveling at the average speed of 32 km/hr leaves Delhi at 6 a.m. Another mail train 130 metres long and traveling at the average speed of 80 km/hr leaves Delhi at 12 noon and follows the goods train. At what time and at what distance from Delhi will the mail train completely cross the goods train?

12. The distance between two cities A and B is 80 kms. A motor cycle rider starts from A towards B at 7 a.m. with a speed of 10 km/hr. Another motor cyclist starts from B towards A at 8 a.m. with a speed of 25 kms/hr. Find when will they cross each other?

13. A train of 24 carriages, each of 60 metres length with an engine of 60 metres length is running at a speed of 60 km/hr. Find out the time within which the train will cross the bridge measuring $1 \frac{1}{2}$ km in length.

14. A train running between two town arrives at its destination 10 minutes late when it goes 60 km per hour and 16 minutes late when it goes 40 km per hour. Determine the distance between the two towns.

15. A train traveling at the rate of 60 km per hour, while inside a tunnel, meets another train of half its length direction at 90 km/hr and passes completely in $4 \frac{1}{2}$ seconds. Find the length of the tunnel, if the first train passes completely through it in 4 minutes 37 $\frac{1}{2}$ seconds.

16. A train 150 metres long, traveling at 75 km per hour overtakes another train traveling in the same direction at 45 km per hour. In how many seconds does the first train pass a passenger sitting in the second train? If the first train passes the second completely in 30 seconds, find the length of the second train.

17. A train going at the rate of 72 km/hr over takes another train 192 metres long, going in the same direction on a parallel line at 54 km per hour and completely passes it in 1 $\frac{1}{2}$ minutes. Find the time in which the trains would have completely passed each other, if they had been going in opposite directions, and also the length of the faster train.

18. Two trains are running at a speed of 50 kms/hr and 30 kms/hr respectively in the same direction. The train running at 50 kms/hr crosses a man in the other train in 18 seconds. Find the length of the fast train.

19. A person rows a kilometer down the stream in 10 minutes and up the stream in 30 minutes. Find the velocity of the stream.

20. A boat's man takes his boat in a river against the stream from a place A to a place B where AB =21 km and again returns to A. Thus, he takes 10 hr. in all. The time taken by him down stream in going 7 km is equal to the time taken by him against stream in going 3 km. Find the velocity of the river.

21. A bird flying 400 kms covers the first 100 kms at the rate of 100 kms/hour, the second 100 at 200 kms/hr, the third hundred at the rate of 300 kms/hours and the last 100 at the rate of 400 kms/hour. Determine the average speed of the bird.

22. Two men together start on a journey in the same direction. They travel 9 and 15 km respectively daily. After traveling for 6 days the man traveling at 9 km per day doubles his speed and both of them finish the distance in the same time. Find the time taken by them to reach their destination.

23. Two trains start at the same time, one from A to B and the other from B to A. If they arrive at B and A respectively 5 hours and 20 hours after they passed each other, what is the ratio of speeds of the train starting from A and that of the one starting from B ?.

24. The driver of a car driving at the speed of 38 kms/hr locates a bus 40 m ahead of him. After 20 seconds, the bus is 60 m behind. Find the speed of the bus.

25. A train overtakes two persons walking at 2 and 4 km per hour respectively and completely passes them respectively in 9 seconds and 10 seconds. What is the length of the train and its speed in km/hr?

26. Two cars start from one point and move along two roads at right angle. If their speeds be respectively 36 km/hr and 48 km/hr, find the distance between them 15 seconds after the start.

27. A carriage driving in a fog passed a man who was walking at the rate of 3 kms an hour in the same direction. He could see the carriage for 4 minutes and it was visible to him upto a distance of 100 metres. What was the speed of the carriage?

28. A monkey climbing up a greased pole ascends 12 metres and slips down 5 metres in alternate minutes. If the pole is 63 metres high, how long will it take for him to reach the top?

29. A train without stoppages travels on an average 50 km an hour and with stoppages 40 kms an hour. Prove that the train stops on an average 12 minutes per hour.

30. A hare sees a dog 100 metres away from her and scuds off in the opposite direction at a speed of 12 km an hour. A minute later the dog perceives her and gives chase at a speed of

16 km per hour. How soon will the dog overtake the hare, and at what distance from the spot whence the hare took flight?

31. If one walks at the rate of 5 kmph the distance from F.C road to M.G road one would be late for a matinee show by 15 minutes. If one travels by auto rickshaw at 20 kmph one would be early by 1/2 an hour. How far is M.G road from F.C road ?

32. A wire 1mm in diameter can be drawn out from a 14 cm bar of gold in 7 minutes. A wire 0.5 mm can be drawn out from a 20 cm bar of gold in 5 minutes. In how much time will the gold bars be of equal length?

33. A man leaves Bombay at 2.20 p.m. and reaches Lonavala at 4.10 p.m. Another man leaves Lonavala at 2.40 p.m. and reaches Bombay at 4.52 p.m. When do they pass one another? Give your answer to the closest one minute.

34. A man goes from Delhi to the village Chhappanpanjeri traveling 300 km by air, 60 km by rail, and 30 km by tonga taking altogether $3\frac{1}{2}$ hours. His speed by air is ten times his speed by rail and the speed by rail is four times that by tonga. What is his speed by air?

35. A car after traveling for 25 km from a city increases its speed by one fourth its original speed and reaches his destination 40 minutes early. Had it increased its speed 10 km before, it would have reached its destination 5 minutes earlier. What is the distance traveled by the car?

36. A man started from his house to his workplace 8 km away at the rate of 4 kmph so as to reach just in time. After 5 minutes he realised that he had left some important documents at home, so he turned back, and now walking at an increased speed, still succeeded in reaching his workplace in time. What was his increased speed?

37. Dinky is picked up by his father by car from school everyday and they reach home at 5.00p.m. One day, since school got over an hour earlier than usual, he started walking towards home at 3 kmph. He met his father on the way and they reached home 15 minutes earlier than their usual time. What is the speed of the car?

38. If a cyclist increases his speed by 2 kmph, he reaches his destination 10 minutes early. If he decreases his speed by 4 kmph, he reaches his destination 30 minutes late. What is the usual speed?

39. A train after traveling for a certain distance develops a snag and decreases its speed to half its original speed and reaches his destination 45 minutes late. Had the snag occurred 30 km further on , it would have reached its destination 15 minutes earlier. What is the speed of the train?

40. The rate of a stream is 4 kmph. A boat goes 6 km and back to the starting point in 2 hours. What is the speed of the boat in still water ?

Answers

1. 25 m per sec.   **2.** 60 kms per hr.   **3.** 6 hr.   **4.** 5/2 kms   **5.** 16 kms
6. 8 kms/hr   **7.** 18 kms   **8.** 17 .5 kmph, 9 .5 kmph   **9.** 45 kms/hr
10. $2\frac{1}{2}$ hr.   **11.** 320.22 km.4 hr. 9 ¾ seconds.   **12.** 10 a.m.   **13.** 3 minutes
14. 12 kms   **15.** 4 .5 km   **16.** 18 secs, 100 metres   **17.** 258 m , $12\,{}^{6}/7$ secs
18. 100 metres   **19.** 2 kms/hr   **20.** 2 kmph   **21.** 192 kms/hr   **22.** 18 days
23. 2:1   **24.** 20 kms/hr   **25.** 50m, 22 kmph   **26.** 250 metres   **27.** $4\frac{1}{2}$ kms/hr
28. 16 minutes 35 secs   **29.** 12 minutes   **30.** 4.5 min,1.1 km   **31.** 5 kms
32. 3 minutes   **33.** 3.29 p.m.   **34.** 600 kmph   **35.** 105 km   **36.** $4\,{}^{8}/_{23}$ kmph
37. 21 kmph   **38.** 16 kmph   **39.** 120 kmph   **40.** 8 kmph

C o n c e p t s 7

Races & Games of Skill

Races

A contest of speed in running, riding, driving, sailing or rowing is called a race. The path on which the contests are held is called a race course. The point from which the race begins is called the starting point. The point where the race ends is called the winning post or the goal. The person who first reaches the winning post is called the winner. If all the contestants reach the goal at the same time then the race is called a dead heat race.

If Ram & Shyam are running a race, at the beginning of which Ram is at the starting point & Shyam is ahead of Ram by 20 metres, then it is said that **Ram gives Shyam a start of 20 metres or Ram gives Shyam 20 metres.** Thus if they are running a 1 km. race Ram will have to run 1 km. where as Shyam will have to run (1000 - 20) = 980 metres.

Similarly, in a 100 metre race, **Ram gives Shyam 25 metres**, or **Ram can give Shyam a start of 25 metres**, or **Ram beats Shyam by 25 metres**, means that Ram runs 100 metres while Shyam runs only 75 metres.

Ex. In a km. race Peter beats Tony by 35 metres or 7 seconds. What is Peter's time over the course?

Sol. Tony runs 35 metres in 7 seconds.
∴ Tony's time over the course = (7/35) x 1000 seconds = 200 seconds.
∴ Peter's time over the course = (200 - 7) seconds = 193 seconds = 3 minutes, 13 seconds

Ex. Tom, Dick & Harry are the contestants in a race in a km. race. If Tom can give Dick a start of 40 metres and Tom can give Harry a start of 64 metres. How many metres can Dick give Harry ?

Sol. Tom covers 1000 metres, Dick covers 960 metres & Harry covers 936 metres.
When Dick covers 960 metres, Harry covers 936 metres
∴ When Dick covers 1000 metres. Harry covers $(^{936}/_{960})$x 1000 = 975 metres.
∴ Dick gives Harry (1000 - 975) = 25 metres.

Games of skill

A contest of skill like a game of billiards has a finite number of points to be score by the contestants. The contestant who scores the maximum score fixed for the game wins

A game of 100 means a game in which the players agree that whoever first scores 100 points is the winner. In a game of 100 **A can give B 10 points** means that while A scores 100 points, B scores 100-10 = 90 points.

Ex. In a game of billiards P can give Q 12 points in 60 and he can give R 10 points in 90. How many can R give Q in the game of 70?

Sol. If P scores 60 points, Q scores 48 points.
If P scores 90 points, R scores 80 points.
∴ If P scores 60 points, then R scores (80/90)x 60 = 160/3 points
Thus when R scores 160/3 points, Q scores 48 points
∴ When R scores 70 points, Q scores (48 x 3 x 70)/160 = 63 points
Thus R can give 7 points in a game of 70.

Pankaj Gandhi's Academy/Arithmetic

Exercise 7

1. In 1760 metre race, A can beat B by 44 metres while in a 1320 metres race, B can beat C by 30 metres. By what distance will A beat C in an 880 meter race?

2. A and B run a 2 km race. A gives B a start of 100 m and still beats him by 20 seconds. If A runs at 20 km per hour, find B's rate in kilometer per hour.

3. In a km race, if A gives B 40 m start, A wins by 19 seconds, but if A gives B 30 seconds start, B wins by 40 m. Find the time that each takes to run a km.

4. At a game of chess Mohan can give Ramesh 20 points in 100 and Ramesh can give Dinesh 20 points in 100. How many points can Mohan give Dinesh in a game of 100?

5. Three men A, B and C go round a circle 1760 metres in circumference at the rates of 160, 120 and 105 metres per minute respectively. If they all start together, when will they first be together again?

6. A, B and C are participating in a km race. A can give B 4 m and C 16 m in a 100 m race and B beats C by 5 seconds in the same race. How long after A completes the race will C reach the finish line?

7. A and B are running around a circular track in opposite directions. They meet at a point 450 m from the starting point and continue running. They now meet at a point 300 m from the starting point, but in the opposite direction as before. What is the length of the track?

8. In a 1600 m race, A can beat B by 40 m, while B gives C 60 m in a 1200 m race. By what distance will A beat C in a 800 m race?

9. In a km race, B takes 3 minutes more than A and 1 minute less than C. Also B beats C by 12 sec. in a 200m race. If A runs at the speed of 10 kmph in the 1km. race, what is C's speed?

10. In a 200 m race, Arjun runs at 15 kmph. If Arjun gives Karna a start of 5 m and still beats him by 6 seconds, what is the speed of Karna in kmph ?

11. Shyam runs at a speed 25% more than that of Ram. If, Ram runs at 8 kmph, how much can Shyam give Ram in a 1500 m race so that he still wins by 9 seconds?

12. A can give B 120 m and C 40 m over a km race, whereas, C can give B 5 seconds over the course. How long would the three take to complete a three km race if they are part of the same relay team?

13. In a 400 m race, A gives B a start of 5 seconds and beats him by 15 m. In another 400 m race, A beats B by 7 1/7 seconds. What are their speeds?

14. In a km race, A gives B a start of 40 m and still beats him by 19 seconds. If A gives B a start of 30 seconds, B beats A by 40 m. What is the ratio of their speed?

15. Two bodies are moving in the same direction along a circular track. One of them completes the track 2 seconds earlier than the other and they meet every 12 seconds at the starting point. Find the time taken by each of them to complete one round of the track.

16. Two men are running in the same direction along a circle. They meet for the first time at a point diametrically opposite the starting point when the faster one is in his fifth round. Find the ratio of their speeds.

17.	Ram and Shyam start from the same point to run in opposite directions round a circular race course, 550 m in circumference. Ram does not start till Shyam runs 100 m. They pass each other when Ram has run 250 m. Who will reach the starting point first? What will be the distance between them when the first one reaches the starting point?

18.	Ranjan, Ritesh, Rakesh and Rajeev are playing pool. Ranjan can beat Rajeev by 10 points in a game of 100, whereas, Rakesh can beat Ritesh by 10 points in a game of 60. If Rajeev beats Rakesh by 15 points in a game of 75, how many points can Ranjan give Ritesh in a game of 400?

19.	Abhimanyu, Bharat, Chandraprakash and Devadatta are four participants in a snooker competition. Chandraprakash can beat Abhimanyu by 10 points in a game of 100, whereas, Bharat can beat Devadatta by 6 points in a game of 60. If Bharat beats Abhimanyu by 5 points in a game of 50, how should the participants team up so that both teams have the same score in a game of 200 points?

20.	Samar and Parija are playing poker. In a game of 50 points, Parija can give Samar 3 points. In a particular game, when Samar has scored 88 points, Anwar joins in. Anwar usually beats Samar by 8 points in a game of 28. If, at the end of the game, Anwar has scored 140 points, by how many points did Samar lose to Parija in that game?

Answers

1. 41 $\frac{1}{2}$ m   **2.** 18 km/hr   **3.** A 125 secs , B 150 secs.

4. 36 points.   **5.** 5 hours 52 mins.   **6.** 64 sec.

7. 1200 m.   **8.** 59m.   **9.** 6kmph.

1013 kmph.   **11.** 280m.   **12. 2**min, 47.8 sec.

13. 8 m/s, 7 m/s.   **14.** 6:5.   **15.** 4 sec, 6 sec.

16. 9:7.   **17.** Ram, 10 m.   **18.** 160.

19. (A & B) v/s (C & D) OR(A & C) v/s (B & D).   **20.** 12.

C o n c e p t s 8

Clocks & calendar

- The dial of a clock is a circle whose circumference is divided into 60 equal parts, called **minute spaces.** One minute space (m.s.) = 6°

- The hour hand of the clock (the shorter hand) goes over 5 minute spaces in an hour, while the minute hand (the longer hand) goes over 60 minute spaces. **Thus in an hour, the minute hand gains 55 minute spaces on the hour hand.**

- In an hour, both the hands coincide once.

- When the two hands are at **right angles, t**hey are 15 minute spaces apart. This position occurs twice every hour.

- When the hands are in **opposite directions**, they are 30 minute spaces apart. This occurs once every hour.

- The hands are in the same **straight line** when they are coincident or opposite to each other.

- In a clock which runs correctly, the minute hand gains 55 minute spaces over the hour hand in 60 minutes. To be together again, the minute hand must gain 60 minutes over the hour hand. 55 minute spaces are gained in 60 minutes.
  $\therefore$ **60 minutes are gained in (60 x 60/55) = 65 $^5/_{11}$ minutes**

- If a watch indicates 10:12 when the correct time is 10, it is said to be 12 minutes too fast. If it indicates 9:55 when the correct time is 10, it is said to be 5 minutes too slow.

- At any given time the angle between the hands of a clock are given by the equation

$$\theta = 6\left(x - \frac{11}{12} m \right)^0$$

Where
θ   is the angle between the hands of a clock at the given   time.

x   is the distance in m.s. between the hands of the clock at the earlier o' clock. (if the time is 3.20 p.m. then the earlier O'clock is 3 O'clock).

m   is the distance traveled by the minute hand in moving   from the earlier O'clock to the position indicated by the given time.

Ex.   When are the hands of a clock together between 3 and 4 O'clock ?

Sol.   At 3 O'clock the hour hand is at 3 and the minute hand is at 12. They are thus 15 minute spaces apart. To be together the minute hand must gain 15 minute spaces over the hour hand.
55 minute spaces are gained in 60 minutes
$\therefore$ 15 minute spaces will be gained in (60/55) x 15 minutes = 16 $^4/_{11}$ min.
Thus the hands will coincide at 16 $^4/_{11}$ minutes past 3.

Ex.   At what time between 8 and 9 O'clock will the hands of a clock be in the same straight line but not together ?

Sol.   At 8 O'clock the hour hand is at 8 and the minute hand is at 12.

∴ the two are separated by 20 minute spaces. To be in the same straight line but not together they will be 30 minute spaces apart. Thus, the minute hand will have to gain (30 - 20) = 10 minute spaces over the hour hand.

55 minute spaces are gained in 60 minutes

∴ 10 minute spaces will be gained in (60/55) x 10 = $10 \frac{10}{11}$ minutes

∴ the hand will be in the same straight line but not together at $10 \frac{10}{11}$ minutes past 8.

Ex. What is the angle between the hands of a clock at 7.23 a.m.

Sol Formula for angle between the hands of a clock

$\theta = 6(x - \frac{11}{12} m)^{\circ}$

Here x = 35 m.s., m = 23 m.s.

∴ $\theta = 6(35 - \frac{11}{12} 23)^{\circ}$

∴ $\theta = 83.5^{\circ}$

Ex. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of correct time. How much a day does the clock gain or lose ?

Sol. In a clock which runs correctly, the hands of a clock overlap every $65 \frac{5}{11}$ mins. In this clock the hands are together after 65 minutes.

∴ gain in 65 minutes = $(65 \frac{5}{11} - 65) = 5/11$ minutes

Gain in 24 hours = (5/11) x (60 x 24/65) minutes = $10 \frac{10}{143}$ minutes

- When a man goes out between any two consecutive clock hrs. **'a' & 'b'** and returns between any two consecutive clock hrs. **'b' & 'c'** to find the position of minute hand & hour hand exactly interchanged, then

  Time at which he went out **(T$_o$) = [x + (13x – 60) / 143]** minutes past **'a'** O'clock.

  Time at which he returned **(Tr) = [x + (13x – 720) / 143]** minutes past **'b'** O'clock.

  Here **x** represents distance in minute-spaces between hour hand & minute hand at **'b'** O'clock position.

Ex. A man, went out between two & three, returned between three & four and found that the hands of the watch had exactly changed positions. At what time did he go out?

Sol. Comparing the given statement with the above formula we get 'a' = 2, 'b' = 3 & 'c' = 4. At 3 O'clock distance between the minute-hand & hour-hand of the clock is 15 minute-spaces, so x = 15.

Time at which this man went out **(T$_o$) = [x + (13x – 60) / 143]** minutes past **'a'** O'clock.

= [15 + (195 – 60) /143] minutes past 2 O' clock

= [15 + (135 / 143)] minutes past 2 O' clock

= 15 135/143 minutes past 2 O'clock

C a l e n d a r

- The number of days more than the complete number of weeks in a given period, are called **odd days**.

- Every year which is divisible by 4, e.g. 1988 is a called a **leap year**.

- Century year is a leap year only if it is divisible by 400. Every fourth century year is a leap year but no other century year is a leap year. e.g. 800, 1200, 1600 etc. are leap years but 900, 1100 etc. are not leap years.

- An ordinary year has 365 days. That means it has 52 weeks plus one extra day. Thus an ordinary year has 1 odd day.

- A leap year has 366 days . Thus a leap year has 2 odd days.

- A century (100 years) has 76 ordinary years and 24 leap years.

∴ 100 years = 76 ordinary + 24 leap

           = (76x52 weeks + 76 days) + (24x52 weeks + 48 days)

           = (5217 weeks + 5 days)

           = 5 odd days

Thus,   100 years contain 5 odd days.

        200 years contain 10 i.e. 7 + 3 i.e. 3 odd days

        300 years contain 15 i.e. 14 + 1 i.e. 1 odd day

        400 years contain 20 + 1 i.e. 0 odd days

        In a similar fashion 800, 1200, 1600 etc. have zero odd days.

- The days are counted according to the number of odd days. i.e. Sunday for 0 odd day, Monday for 1 odd day, Tuesday for 2 odd days etc.

Ex.   Find the day of the week on July 16, 1776

Sol.   July 16, 1776 implies (1775 years + 6 months + 16 days)

      1600 years have 0 odd days

      100 years have 5 odd days

      75 years have 18 leap years & 57 ordinary years i.e. (36 + 57) i.e. 93 i.e. 2 odd days

      ∴ 1775 years have 0 + 5 + 2 = 7 i.e. 0 odd days

      Number of days from January 1, 1776 to July 16, 1776:

      Jan 31 Feb. 29 Mar 31 Apr. 30 May 31 Jun. 30 Jul. 16

      31 + 29 + 31 + 30 + 31 + 30 + 16 = 198 days i.e. 28 weeks + 2 days i.e. 2 odd days

      ∴ Total number of odd days = 0 + 2 = 2

      Thus the day on July 16, 1776 was a *Tuesday*

Ex.   What day of the week was January 12, 1979 ?

Sol.   January 12, 1979 = 1978 years + 12 days

      1600 years   :   0 odd days

      300 years   :   1 odd day

      78 years have 19 leap years + 59 ordinary years i.e. 38 + 59 i.e. 97 i.e. 6 odd days

      12 days of January have 5 odd days

      Total number of odd days = 0 + 1 + 6 + 5 = 12 i.e. 5 odd days

      ∴ the day on January 12, 1979 was a *Friday*

Exercise 8

1. At what time between 4 and 5 will the hands of watch:

    (i) coincide?

    (ii) be at right angles?

    (iii) Point in opposite directions?

    (iv) The minute hand be 13 minute spaces behind the hour hand.

    (v) The minute hand be 13 minute spaces ahead of hour hand?

    (vi) The two hands will be equidistant from the figure 5?

2. If the hands of a clock coincide every 66 minutes (true time) how much does the clock gain or lose every hour?

3. Two clocks are set correctly at 10 a.m. on Friday. The first clock gains 2 minutes per hour and gains twice as much as the second. What time will the second clock register when the correct time is 2 p.m. on the following Monday?

4. My watch, which gains uniformly, is 2 min. slow at noon on Saturday, and is 4 minutes 48 seconds fast at 00.00 hrs on next Monday. When was it correct?

5. How many times during a day will the hour hand and the minute hand of a clock be six minutes apart?

6. At what time between 4:15 a.m. and 5:05 a.m. will the angle between the hour hand and the minute hand of a clock be the same as the angle between the hands at 8:45 p.m.?

7. A clock is 5 minutes slow at 4:35 p.m. on Monday and 12 minutes fast at 5:45 a.m. the following Monday. If the clock gains uniformly, when did it show the correct time?

8. If the hour hand and the minute hand of a clock coincide every 72 minutes, how much does the clock gain or lose every day?

9. Arjun goes out at some time between 2 o'clock and 3 o'clock and returns between 4 o'clock and 5 o'clock. He notices that the hands of the clock have exactly interchanged their positions. At what time did Arjun go out?

10. A clock loses 12 minutes every 24 clock hours. It is set right at 7:25 p.m. on Friday. What will be the time when the clock shows 1:45 p.m. the following day?

11. Akash went out between 7 o'clock and 8 o'clock. After 8:30 p.m., he realised that the minute hand of the clock was as much behind the hour hand as the hour hand was behind the minute hand when he had gone out. At what time after he went out did Akash look at his watch?

12. At what time between 9 o'clock and 10 o'clock are the hands of a clock
    a. in the same straight line, but not together
    b. at right angles,
    c. eight minutes apart?

13. A clock, which gains uniformly, is 18 minutes slow at 7:25 p.m. on Tuesday. The clock would have shown the correct time at 00:00 hrs on Thursday. How much did the clock gain per day?

14. A clock is set right at 1:00 p.m. If it gains one minute per hour, what is the time when it shows 6:00 p.m. the same day?

15. A watch shows 6:59 at 7:00 a.m. At 7:00 p.m. the same day, it shows 7:02. What is the time when the watch shows 7:01 the next morning?

16. Find the time between 4 o'clock and 5 o'clock when the angle between the hands of the clock is 1/3 of a right angle?

17. Two clocks begin to strike 11 together. The first clock strikes its stroke in 30 seconds while the second strikes in 20 seconds. What is the interval between the first clock's fifth stroke and the second clock's seventh stroke?

18. Two clocks are set right at 6:10 p.m. on Saturday. The first clock loses 2 minutes per hour, whereas, the second clock gains twice as much as the first one loses per hour. What is the difference between the times shown by the two clocks at 12:00 noon the next day?

19. What day of the week was on 15th August, 1987?

20. If today is Friday, what will it be after 62 days?

Answers

1.(i) 21 $\frac{9}{11}$ minutes past 4

(ii) at 5 $\frac{5}{11}$ minutes past 4 and again at 38 $\frac{2}{11}$ past 4.

(iii) 54 $\frac{6}{11}$ minute past 4

(iv) 7 $\frac{7}{11}$ minutes past 4

(v) 36 minutes past 4

(vi) 27 $\frac{9}{13}$ minutes past 4.

2. 60/121

3. 3 hours 16 minutes

4. The watch was correct at 12 p.m. on Monday.

5. 44 times.

6. 20 5/11, 23 2/11

7. 2:48:30, Wednesday.

8. 130$\frac{10}{11}$

9. 22$\frac{98}{121}$

10. 1:35:50 p.m.

11. 7: 51$\frac{9}{11}$

12. 16$\frac{4}{11}$, 32$\frac{8}{11}$, (40$\frac{4}{11}$ or 57$\frac{9}{11}$)

13. 15$\frac{39}{343}$

14. 5: 55$\frac{5}{61}$

15. 6: 56$\frac{4}{241}$ a.m.

16. 16$\frac{4}{11}$, 27$\frac{3}{11}$

17. No interval.

18. 107 min.

19. Saturday

20. Thursday

Concepts 9

Partnership

- Two or more than two persons run a business jointly. They are called **partners** and the deal is known as **partnership.**

- **Simple Partnership** is the one in which the capitals of each of the partners is invested for the same time.

- **Compound Partnership** is the one wherein the periods of investment are unequal.

    - **Gains or losses in a partnership are divided among the partners in the ratio of their investments in case of a simple partnership.**

    - **In case of a compound partnership, equivalent capitals for a unit of time are calculated by multiplying the capital with the number of units it was in business.**

- A partner who manages the business is called a **working partner** while one who just invests the money but does not manage is called a **sleeping partner.**

Ex. A, B, & C enter into a partnership investing Rs. 3800, Rs. 4200, & Rs. 4000 respectively. How should a profit of Rs. 1800 be divided among them ?

Sol. Profit should be divided among the partners in ratio of their investments.
∴ the ratio of A, B & C's share is :
3800 : 4200 : 4000 = 19 : 21 : 20
Sum of the ratios = 19 + 20 + 21 = 60
A's share = Rs. (1800 x 19)/60 = Rs. 570
B's share = Rs. (1800 x 21)/60 = Rs. 630
C's share = Rs. (1800 x 20)/60 = Rs. 600

Ex. In a partnership deal, A contributes Rs. 3200 for 4 months, B contributes Rs. 5100 for 3 months and C contributes Rs. 2700 for 5 months. How should they divide a total profit of Rs. 1248 among themselves ?

Sol. We calculate equivalent capitals for one month :
A's capital for 1 month = Rs. (3200 x 4) = Rs. 12800
B's capital for 1 month = Rs. (5100 x 3) = Rs. 15300
C's capital for 1 month = Rs. (2700 x 5) = Rs. 13500
∴ the ratio of A, B & C's share :
12800 : 15300 : 13500 = 128 : 153 : 135
Sum of ratios = 128 + 153 + 135 = 416
A's share = Rs. (1248 x 128)/416 = Rs. 384
B's share = Rs. (1248 x 153)/416 = Rs. 459
C's share = Rs. (1248 x 135)/416 = Rs. 405

Ex. In a partnership deal A puts Rs. 2000 for the entire year, B puts Rs. 3000 initially and then increases it to Rs. 4000 at the end of 4 months and C puts in Rs. 4000 initially, but withdraws Rs. 1000 at the end of 9 months. How should they divide a profit of Rs. 8475 at the end of the year ?

Sol. We calculate equivalent capital for 1 month for each :
A's capital = Rs. (2000 x 12) = Rs. 24000
B's capital = Rs. (3000 x 4 + 4000 x 8) = Rs. 44000
C's capital = Rs. (4000 x 9 + 3000 x 3) = Rs. 45000

Ratio of capitals :
24000 : 44000 : 45000 = 24 : 44 : 45
Sum of ratios = 24 + 44 + 45 = 113
A's share = Rs. (8475 x 24)/113 = Rs. 1800
B's share = Rs. (8475 x 44)/113 = Rs. 3300
C's share = Rs. (8475 x 45)/113 = Rs. 3375

Ex. A & B entered into partnership with capitals in the ratio of 4 : 5. After 3 months A withdrew 1/4 of his capital and B withdrew 1/5 of his capital. The gain at the end of 10 months was Rs. 760. What are their shares in the profit ?

Sol. Let A invest Rs. 4 & B invest Rs. 5.
A withdraws (1/4)x4 = Re. 1
B withdraws (1/5)x5 = Re. 1
Calculating equivalent capital for 1 month :
A's capital = Rs. (4 x 3 + 3 x 7) = Rs. 33
B's capital = Rs. (5 x 3 + 4 x 7) = Rs. 43
Ratio of capitals = 33 : 43
∴ A's share = Rs. (760 x 33)/76 = Rs. 330
B's share = Rs. (760 x 43)/76 = Rs. 430

Exercise 9

1. A, B and C invested Rs.6300, Rs.4200 and Rs.10500 respectively, in a partnership business. Find their share in a profit of Rs.12100 after a year.

2. A, B, C and D enter into a partnership. A subscribes 1/3 of the capital. B 1/4, C 1/5 and D the rest. How should they divide a profit of Rs.2460?

3. Naresh, Suresh and Mahesh are the partners in a firm. Their investments are respectively Rs.10,000, Rs.12,500 and Rs.20,000. They divide the profit in the ratio of 4:5:6. If the interest on the capital is paid at the rate of 5% per annum, how should a profit of Rs.2,725 be divided among the three?

4. A and B together start a business, subscribing Rs.3600 and 3400 respectively. A being a sleeping partner, B manages the business and receives 8 $\frac{1}{3}$% of the profits for his extra labour and the remainder is divided by them. Find the share of each in profit of Rs.2100.

5. Ram, Tony and Khilona Singh entered into a partnership. Ram invested Rs.6400 for 10 months, Tony invested Rs.8100 for 8 months and Khilona Singh invested Rs.9200 for 6 months. Find the share of each in a profit of Rs.35259.

6. A, B and C entered into a partnership A invested Rs.6500 for 6 months, B invested Rs.8400 For 5 months and C invested Rs.10000 for 3 months. A is a working partner and gets 5% of the total profit for the same. Find the share of each in a total profit of Rs.7400.

7. A, B and C entered into a partnership. A subscribes one half the capital for one-third of the time, B one-third of the capital for one-fourth of the time and C the rest of the capital for the whole time. How should they divide a profit of Rs.1000?

8. A, B, C are partners in a business. Their shares are in the proportion 1/3 : 1/4 : 1/5. A withdraws half his capital at the end of 15 months and after 15 months more a profit of Rs.4340 is divided. Find the share of each.

9. A began business with Rs.8400. After 2 months B joined with a certain capital. At the end of the year, profits were divided in the ratio of 3:2. How much did B invest?

10. A, B & C rented a house together for 2 years at Rs.360 yearly rent. They remained together for 5 months and then C left. After 4 months more B also left. How much rent should each pay?

11. Four milkmen rented a pasture. A grazed his 12 cows for 6 months, B 10 cows for 5 months and C 14 cows for 10 months and D 9 cows for 7 months. If A's share of rent is Rs.36, find
   (i) the total rent of the field.
   (ii) the amount paid as rent by B, C and D separately.

12. A, B and C rent a pasture for a year. A puts in 500 sheep for the first half of the year, B 750 sheep for the second half, and C 400 sheep all the year round. If the feed in the first half of the year is considered to be worth twice as much as in the second half of the year, find what fractional part of the rent each person ought to pay.

13. A, B and C are partners. A receives 2/3 of the profits, B and C dividing the remainder equally. A's income is increased by Rs.200 when the rate of profit rises from 5 to 7 percent of total capital. Find the capital of B.

14. Three persons invested Rs.9000 in a joint business. The second person invested Rs.1000 more than the first and the third Rs.1000 more than the second. After two years, they gained Rs.5400. How much will each get?

15. A, B and C enter into partnership and decide to contribute the total capital of Rs.12000 in the following manner. A will contribute 25 percent of the total capital, B's contribution will be 80 percent of C's contribution which is 50 percent of the contribution of A and C together. What amount will each contribute towards the capital?

16. A and B enter into partnership with unequal sums of money, it being agreed that each is to receive 6 percent per annum of his capital, and any profit remaining after this will be divided equally. At the end of the year, A receives Rs.4,630 and B Rs.3730; and it is found that B thus gets Rs.650 more than he would have received if all the profits had been shared in the proportion to the capitals invested by each. Find what each man's capital is?

17. Three men are employed to do a piece of work. They work respectively 8,9 and 10 hours a day receive the same hourly wage. After three days, each works an hour more and receives proportionately increased wages and at the end of three days from that time, the work is completed. The whole sum paid to them is Rs.45.60. How much of it should each receive?

18. Three partners A, B and C subscribe Rs.3000, Rs.4000 and Rs.5000 respectively as capital for a business. A receives 15% of the profits as manager and B 10% as assistant manager after which the remaining profits are divided in proportion to capital subscribed by each. Find the share of B and C when A receives Rs.540.

19. A, B and C agree to receive profit from their business each in the proportion to his investment. A and B put back into the business their share of the profit each year but C does not. Initially, A invests Rs.5000, B Rs.10,000 and C Rs. 30,000. If the profit for the first years is Rs.9000 and for the second year is Rs.10000. Calculate the amount of money each has in the business during the third year.

20. A and B enter into partnership. A supplies the whole of the capital amount of Rs. 45,000 with the condition that the profits are to be equally divided and that B pay A, interest on half the capital at 10 percent per annum. But B receives Rs.120 per month for carrying on the concern. Find their total yearly profit when B's income is one-half of A's income.

Answers

1. Rs.3630, Rs.2420, Rs.6050   **2.** Rs.820, Rs.615. Rs.492, Rs.533

3. Rs.660, Rs.825, Rs.1240   **4.** Rs.990, Rs.1110

5. Rs.12264, Rs.12417.30, Rs.10577.70   **6.** Rs.2840, Rs.2660, Rs.1900

7. Rs.400, Rs.200, Rs.400   **8.** Rs.1550, Rs.1550, Rs.1240

9. Rs.6720   **10.** C = Rs.50, B = Rs.110, A = Rs.560

11. (i) Rs.162.50 (ii) Rs.25, Rs.70, Rs.31.50   **12.** 20/59, 15/59, 24/59

13. Rs.2500   **14.** Rs.1200, Rs.1800, Rs.2400

15. A = Rs.3000, C = Rs.5000, B = Rs.4000   **16.** A's = 36000, B's = 21000

17. Rs.13.60, Rs.15.20, Rs.16.80   **18.** Rs.560, Rs.500

19. Rs.7250, Rs.14500, Rs.30000   **20.** Rs.9180

C o n c e p t s 10

Profit & Loss

- **Cost Price (C.P.)** of an article is the price for which the article has been purchased.

- **Selling Price (S.P.)** of an article is the price for which the article is sold.

- If the S.P. is greater than the C.P., then it is said that the seller has made a **profit or gain.**

- If the C.P. is greater than the S.P. then the seller is said to have made a **loss.**

Profit & Loss are always reckoned on the Cost Price.
Profit or Loss on Rs. 100 is called the profit or loss percent respectively.

Formulae :

- **Gain = (S.P. - C.P.)   Loss = (C.P. - S.P.)**

- **Gain % = (Gain x 100)/C.P.   Loss % = (Loss x 100)/C.P.**

- Given the cost & the gain percent, **S.P. = (100 + gain %) x C.P. / 100**

- Given the cost & the loss percent, **S.P. = (100 - loss %) x C.P. / 100**

- Given the S.P. & the gain percent, **C.P. = (100 x S.P.)/(100 + gain %)**

- Given the S.P. & the loss percent, **C.P. = (100 x S.P.)/(100 - loss %)**

Ex.   A man sells a pen at a profit of 20%. Had he bought it at 20% less and sold it for Rs. 5 less, he would have gained 25%. What is the C.P. of the pen ?
Sol.   Let the C.P. be Rs. 100   Gain = 20% ∴ S.P. = Rs. 120   New C.P. = 20% less = Rs. 80
If gain is 25%, then S.P. = Rs. (125 x 80) / 100 = Rs. 100
∴ Difference in S.P. = Rs. (120 - 100)   = Rs. 20
∴ if difference in S.P. is Rs. 20, C.P. is Rs. 100
If the difference in S.P. is Rs. 5, C.P. is Rs. (100 x $^5/_{20}$) = Rs. 25

Ex.   If the S.P. of 10 articles is the same as the C.P. of 11 articles, find the gain percent.
Sol.   Let the C.P. of 1 article be Re. 1
∴ C.P. of 10 articles = Rs. 10   C.P. of 11 articles = Rs.11
∴ S.P. of 10 articles = C.P. of 11 articles = Rs. 11   ∴ gain % = (11 - 10) x 100 / 10 = 10%

Ex.   A man loses 10% by selling a book for Rs. 144. What should be his selling price to gain 15 ?
Sol.   S. P. = Rs. 144 Loss = 10%
∴ C.P. = Rs. (100 x 144)/(100 - 10) = Rs. 160   Gain expected = 15%
∴ S.P. = Rs. (115 x 160)/100 = Rs. 184

Ex.   A man sells two radios for Rs. 924 each. On one he gains 12% and on the other he loses 12%. How much does he gain or lose on the whole ?
Sol.   S.P. of one radio = Rs. 924 Gain = 12%
∴ C.P. of this radio = Rs. (100 x 924) / 112 = Rs. 825
S.P. of other radio = Rs. 924 Loss = 12%
∴ C.P. of this radio = Rs. (100 x 924)/88 = Rs. 1050
Total C.P. = Rs. (825 + 1050) = Rs. 1875, Total S.P. = Rs. (924 + 924) = Rs. 1848
∴ Loss % = (27 x 100)/1875 = 1.44%

Exercise 10

1. In selling one article for Rs.450, one has a loss of 10%. Find the cost price. For getting a profit of 10%, what must be its selling price?

2. The cost price of 12 pens is equal to S.P. of 10 pens, find gain percent.

3. In selling an article for Rs.9, a man gets as much percent loss as is the prime cost (C.P.) of that article. Find the C.P. of that article.

4. A person purchases 50 dozen eggs at Rs.4 per dozen. Of these, 40 eggs were found broken. At what price should he sell the remaining eggs in order to make a profit of 5%?

5. A merchant sold two radios for 120 each. One was sold at a loss of 25% of the cost and the other was sold at a gain of 25% of the cost. Determine the total amount of gain / loss in the entire transaction.

6. A person purchases 90 clocks and sells 40 clocks at a gain of 10% and 50 clocks at a gain of 20%. Had he sold all of them at a uniform profit of 15%, he would have got Rs.40 less. Find the C.P. of each clock.

7. A man sold his radio at a loss of 5%. Had he purchased it at 10% lesser cost price and sold it at Rs.42 more then he would have had a gain of 1/4 of the C.P. Find the C.P. of the radio.

8. A man purchases some oranges at the rate of 3 for Rs.4 and the same quantity at 5 for Rs.6. If he sells all the oranges at the rate of 3 for Rs.5, find his gain or loss percent.

9. A businessman sells 20% of his stock at 10% profit. 50% of the remainder, he sells at 10% loss. At what percent profit, he must sell the remaining stock, so as to get 5% profit on the total stock?

10. A merchant buys 40 bicycles and marks them at 25% above the cost price. He allows a discount on the marked price at 10% for cash sales, and at 5% for credit sales. If three-fourth of the stock is sold for cash and the rest for credit, and if the total profit be Rs.2025, what is the cost price of a bicycle?

11. A dealer allows 10% discount on the list price of certain articles and yet makes a profit of 25 percent on each article. Find the cost price of an article whose listed price is Rs.50.

12. A bicycle agent allows 20% discount on his marked price and then makes a profit of 20% on his outlay (C.P.). What is the marked price on which he gain Rs.40?

13. If price of sugar falls by $12\frac{1}{2}$% a person can buy 9 kg more of sugar for Rs.126 than before. If the price had risen by $12\frac{1}{2}$%, how much sugar would he have bought for the same sum?

14. A tradesman cheats by means of a false balance to extent of 10% in buying goods and to the same extent in selling goods. What percent does he gain or lose on his outlay by defraud?

15. A dishonest dealer professes to sell his goods at cost price but he used a weight of 900 gm for 1 kg. Find his gain percent.

16. A merchant sells 90 quintals of wheat at a profit of 8 percent and 50 quintals at a profit of 10 percent. If he had sold the whole at a profit of 9 percent, he would have received Rs.60 more than he actually did. How much did the merchant pay for the wheat per quintal?

17. A tradesman sell one kind of sugar at Rs.3 per kg and loses 20% and another kind of sugar at Rs.5 per kg and gains 25%. He mixes the two together in equal proportion and sells the mixture at Rs.6/kg. What is now the gain percent?

18. In what ratio should a businessman mix two kinds of tea of C.P. Rs. 5 & Rs.7 per kg respectively so that he may gain a profit of 20% when he sells the mixture at the rate of Rs.7.50 per kg?

19. A publisher printed 2000 copies of a book at cost of Rs.2400. He gave 500 copies free to heads of the institutions. He allowed a discount of 25 percent on the published price and gave one extra copy for every 24 copies bought at a time. He sold all the copies in that manner. If the published price is Rs.3.25, find his gain or loss percent.

20. A man purchases 5 horses and 10 cows for Rs.10,000. He sell the horses at 15% profit and the cows at 10% loss. Thus he gets Rs.375 as profit. Find the cost of 1 horse and 1 cow separately.

21. A man bought a horse and a carriage for Rs.500, and sold the horse at a gain of 20 percent, and the carriage at a loss of 10 percent, thus gaining 2 percent on his total outlay. For how much was the horse bought?

22. A person bought a horse and a carriage for Rs.1800. He sold the horse at a profit of 20%, and the carriage at a profit of 30%. His total profit was $25\frac{5}{6}$%. Find the cost of the horse.

23. A sells a watch to B at a gain of 20% and B sells it to C at a loss of 10%. If C pays Rs.216, what does it cost to A?

24. A manufacturer fixes the wholesale price of an article by adding together the cost of production, excise duty (which is 22% of the cost of production) and his profit (which is 18% of the cost of production). The wholesaler sells the articles to a retailer at a profit of 15%. The retailer marks the price at 25% above his cost price and allows 4% discount on the marked price for cash payment. Find the production cost of the article for which a customer makes a cash payment of Rs.483 to the dealer.

25. A manufacturer sells an item to wholesale dealer at a profit of 18%. The wholesaler sells the same to a retailer at a profit of 20%. The retailer in turn, sells it to the customer for Rs.30.09, thereby earning a profit of 25%. Find the cost of the manufacturer.

26. The list price of a book is Rs.2550. A publisher sells this book to the book-seller after allowing a trade discount of 25%. The other expenditure on the book for the book-seller comes out to be Rs.50. Find the profit of book-seller if he sells the book at list price.

27. An almirah is listed at Rs.1000. A retailer buys it with two successive discounts of 10% and 20% for cash. The other expenses are 10% of the cost of the almirah. At what price should he sell to earn a profit of 15%?

28. Which is more favorable to buyer and by how much: A discount series of 20%, 15%, 10% or the discount series of 25%, 12%, 8% on the list price of Rs.700?

29. The list price of a watch is Rs.160. A retailer pays Rs.122.40 for it. He gets successive discounts of 10% and another rate which is illegible. What is the second discount rate?

30. A scooter dealer bought an old scooter listed at Rs.2600 and got successive discounts of 5% and 10%. He spends some amount on its repairs and sells it for Rs.2750 thereby gaining 10%. Find the amount spent by the shopkeeper on repairs.

31. At the off season period a hotel offers a double bed room at a discount of 30%. If there is a successive discount of 10% to tourists in groups, a double bed-room becomes available for Rs.189 per day. Find the original room rent.

32. A tradesman fixes the sale price of his articles 30% more than their cost price. Half of the articles, he sells at this increased sale price and 1/4 portion, he sells at 15% discount of this sale price and the remaining, he sells at 30% discount of this sale price What per cent profit did occur to the tradesman?

33. A radio dealer demands a price which is 50% higher than its cost price. How much percent less should the buyer pay to the dealer so that the dealer makes no profit or loss on it?

34. Two dealer offer an article at the same list price. The first allows a discount of 25% and offers further a rebate of 15% on the discounted price for cash payment. The other dealer offers a discount of 10% and allows a further concession of 30% on the discounted price of cash payment. What is the better offer to the buyer?

35. A bicycle costs Rs.260 (including 4% sales tax) payable in 60 days. A discount of 6% on the selling price (excluding the sale tax) is given by the shopkeeper to a customer who makes cash payment on the spot. If a customer pays in 60 days, how much interest per annum is he paying on the transaction?

36. A merchant was selling his goods for 20 percent profit. When he allowed a discount of 5 paise per rupee on cash, his sale improved in the ratio 3:5. Compare the profits which he was and is now making.

37. A dealer buys a table listed at Rs.100 and gets successive discounts of 10% & 20%. He spends 10% of cost price on transport. At which price should he sell the table to earn a profit of 15%?

38. If a dealer were to diminish the selling price of his wares by 10 percent and thereby doubled his sale, he could make the same profit on the cost price as before. In what ratio would his profit be diminished if he were to increase the selling price by the same amount and thereby halve his sale.

39. At a cricket match, the contractor provided dinner for 24 persons, and fixed the price so as to gain $12\frac{1}{2}$% upon his total outlay. Three of the players being absent, the remaining 21 paid the fixed price for their dinner, and as such the contractor lost one rupee. What was the charge for the dinner?

40. A manufacturing firm has a capital of Rs.90000. The gross annual sales are Rs.37500. The cost of materials and manufacturing was 35% of the gross sales. Rent and taxes amount to Rs.3860; advertising to Rs.9400; office expenses Rs.1360; manager's salary Rs.2000, together with 1% of the gross sales. Then if the expenditure on advertising is doubled, and in consequence, the gross sale increased by 40%, what increase will there be in the annual profit?

Answers

1. Rs.500, Rs.550	**2.** 20 %	**3.** Rs.90 or 10	**4.** Rs.4.50/dozen	**5.** Rs.16
6. Rs.80	**7.** Rs.240	**8.** Gain Rs.31$^{11}/_{19}$%	**9.** 17.5%	**10.** Rs.360
11. Rs.36	**12.** Rs.300	**13.** 56 kg	**14.** 22 $^{2}/_{9}$%	**15.** 11 $^{1}/_{9}$%
16. Rs.150/Qntl.	**17.** 54 $^{26}/_{31}$%	**18.** 3:5	**19.** Gain 46 $^{1}/_{4}$%	**20.** Rs. 1100, Rs.450
21. Rs.200	**22.** Rs.750	**23.** Rs.200	**24.** Rs.250	**25.** Rs.17
26. Rs.587.50	**27.** Rs.910.80	**28.** 2nd is better	**29.** 15%	**30.** Rs.277
31. Rs.300	**32.** 15 $^{3}/_{8}$%	**33.** 33 1/3 **%**	**34.** second one	**35.** Rs. 90
36. 6 : 7	**37.** Rs.91.08	**38.** 4:3	**39.** Rs. 3	**40.** Rs. 200

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C o n c e p t s 11

Simple Interest

A person borrows certain amount of money for some one for some period, then the borrower has to pay some extra money called **interest.** The money borrowed is called the **Principal** & the total sum of the principal & interest is called **Amount.** If the interest on a certain sum borrowed for a certain period is reckoned uniform then it is called as simple interest.

Formulae: The simple interest (S.I.) on a principal P at R% per annum for T years is given by :

S.I. = (PxRxT) / 100

P = 100 x S.I. / (RxT)

R = 100 x S.I. / (PxT)

T = 100 x S.I. / (PxR)

Ex. What will be the simple interest on Rs. 625 at 6.5% per annum for 2.5 years.
Sol. S.I. = Rs. (625 x 6.5 x 2.5) / 100 = Rs. 101.56

Ex. What will be the simple interest on Rs. 700 at 9% per annum for the period from February 5, 1994 to   April 18, 1994 ?
Sol. In such problems number of days are computed by including the first & the last date.
Number of days : February 24, March 31, April 18
Total time = 24 + 31 + 18 = 73 days = 1/5 years = 0.2 year.
Simple Interest = Rs (700 x 9 x 0.2)/ 100 = Rs. 12.60

Ex. A man borrowed Rs. 2500 from two money lenders**.** He paid interest at the rate of 12% per annum for one loan and at the rate of 14% per annum for the other. The total interest he paid for the entire year was Rs. 326. How much did he borrow at each rate?
Sol. Let he borrow Rs. x at 12% and Rs. (2500 – x) at 14% per annum.
S.I. on Rs. x = Rs. (x)(12)(1/100) = Rs. 3x/25
S.I. on Rs. (2500 – x) = Rs. (2500 –x)(14)(1/100) = Rs. (17500 – 7x) / 50
Therefore x = Rs. 1200
Thus, he borrows Rs. 1200 at 12% and 1300 at 14%.

Ex. A sum was put at simple interest at a certain rate of interest for 2 years. It would have fetched Rs. 72 more had it been put at 3% higher rate. What was the sum?
Sol. Let the sum be Rs. 100 & original rate be r% per annum
Simple interest at r% = Rs. (100 x r x 2) / 100 = Rs. 2r
Simple interest at (r + 3)% = Rs { 100 x (r + 3) x 2 } / 100 = Rs. (2r + 6)
Difference in the interests = (2r + 6) – 2r = Rs. 6
If the difference in the interests is Rs. 6, sum = Rs. 100
Therefore, if the difference is Rs. 72, sum = Rs. (100 x 72) / 6 = Rs. 1200

Compound Interest

The borrower and lender fix up a unit of time, e.g. quarterly or half yearly or yearly to settle the previous account. In all such cases, the amount after the first unit of time becomes the principal for the second unit, the amount after the second unit becomes the principal for the third unit and so on. After a certain period the difference between the amount & the money borrowed is called the compound interest for that period.

Formulae:

- If principal = Rs. P, Time = t years and Rate = r% p.a. and compound interest is reckoned annually, then

    Amount after t years = $P(1 + r/100)^t$

- If principal = Rs P, rate = r% p.a. and the time is a fraction of the year then, the rate is considered to be that fraction of the year. e.g. if time is 5 2/5 years, then

    Amount = $P\{(1 + r/100)^5 (1 + 2r/5)\}$

- If principal = Rs. P, Time = t years and Rate = r% p.a. and compound interest is reckoned half yearly, then rate = (r/2)% per half year, time = 2t half years

    Amount after t years = $P[1 + r/(2 \times 100)]^{2t}$

- If principal = Rs. P, Time = t years and Rate = r% p.a. and compound interest is reckoned quarterly, then rate = (r/4)% per quarter of the year, time = 4t quarters

    Amount after t years = $P[1 + r/(4 \times 100)]^{4t}$

- If principal = Rs P, Time = n years, Rate = r_1% for the 1st year, r_2% for the 2nd year, r_3% for the 3rd year ...., r_n% for the nth year then

    Amount after n years = $P\{1 + r_1/100\}\{1 + r_2/100\}............\{1 + r_n/100\}$

- Present worth of a sum of Rs A due t years hence at r% p.a. compound interest

    Present worth = Rs $\{A / (1 + r/100)^t\}$

If a sum of Rs. 100 is borrowed at r% per annum compound interest reckoned quarterly or half-yearly, then the compound interest for 1 year is called the effective annual rate.

Note that the amount at the end of certain period increases as the time of reckoning compound interest decreases.

Ex. Find the compound interest on Rs 4000 for 9 months at 6% per annum, the interest being reckoned (I) quarterly (ii) half – yearly

Sol. (I) Principal = Rs 4000, Time = 9 months = 3 quarters, Rate = 6% per annum = 6 / 4 = 3/2% per quarter. Therefore, Amount = Rs. $[4000 \times \{1 + (3/2)/100\}^3]$ = Rs. 4182.71
Compound interest = Rs. (4182.71 – 4000) = Rs 182.71
(ii) Principal = Rs 4000, Time = 9 months = 1.5 half years, Rate = 6% per annum = 3% half yearly
Therefore, Amount Rs. $[4000(1 + 3/100)\{1 + (3/2)/100\}]$ = Rs. 4181.80
Compound interest = Rs (4181.80 – 4000) = Rs. 181.80

Ex. The difference between the compound interest & the simple interest on a certain sum at 7.5% per annum for 3 years is Rs. 110.70. What is the sum?

Sol. Let the sum be Rs 100.
Amount after 3 years at 7.5% p.a. compound interest = Rs. $[100 \times (1 + 15/200)^3]$
= Rs (79507 / 640)
Compound interest = Rs. (79507/640 – 100) = Rs (15507/640)
Simple interest on Rs. 100 for 3 years at 7.5% = Rs (100 x 3 x 15) / 200 = Rs 45/2
Difference between compound interest & simple interest = Rs.(1107 /640)
If the difference between C.I. & S.I. is Rs 1107/640, sum = Rs 100
If the difference between C.I. & S.I. is Rs. 110.7, sum = Rs. (100 x 640/1107) x 110.7 = Rs. 6400.

Ex. A sum of Rs. 8000 is borrowed at 5% compound interest & paid back in 3 equal installments. What is the value of each installment?

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Sol.   Let the value of each installment be Rs. x
Amount to be returned after 3 years = 8000 $(1 + 5/100)^3$ = 9261
Therefore, $x(21/20)^2 + x(21/20) + x = 9261$
i.e. 1261x = 9261 x 400 $\lceil$ x = Rs 2937.67

Ex.   The compound interest on Rs. 30000 at 7% per annum for a certain time is Rs 4347. What is the time?

Sol.   Let the time be t years.
30000 x $(1 + 7/100)^t$ = 30000 + 4347
$(107/100)^t = (34347/30000) = (11449/10000) = (107/100)^2$ i.e. t = 2
Therefore the time is 2 years.

Problems based on Population

- If the population of a town is P_1 and the annual increase is r%, then the population after t years is $P_2 = P_1 (1 + r/100)^t$
- If the population of a town is P_1 and the annual decrease (or depreciation) is r%, then the population after t years is $P_2 = P_1 (1 - r/100)^t$

Ex.   The population of a town increases 5% annually but decreases by emigration by ¼ %. What is the net increase percent in 3 years?

Sol.   Let the original population be 100
Net annual increase = $(5 - 1/4) = 19/4\%$
Therefore the population in 3 years = 100 x $(1 + 19/400)^3$ = 100 x $(419/400)^3$ = 114.93
∴ increase = 114.93 – 100 = 14.93%

Ex.   The value of a machine depreciates at the rate of 10% after every year. What was its purchase price if at the end of three years its value is Rs. 8748 ?

Sol.   Let the purchase price be P
$P(1 - 10/100)^3$ = 8748
∴ $P(9/10)^3$ = 8748
∴ P = Rs. 12000

Exercise 11

1. Suruchi deposited Rs. 2500 on January 5, 1984 with Punjab National Bank at 7 ½ % per annum. She withdrew the whole amount on May 30,1984. What interest did the bank pay ? Find also the amount.

2. A lends Rs. 3500 to B at 10% per annum and B lends the sum to C at 11.5% per annum. Find the gain of B in a period of 3 years.

3. Ram deposited a certain sum of money in a post office at 10% p.a. for 4 years & deposited an equal amount in a fixed deposit in a bank for 5 ½ years at 11% p.a. If the difference in the interest from the two sources is Rs. 563.75, find the sum deposited.

4. A sum of money becomes 7/4 of itself in 6 years at a certain rate of interest. Find the rate of interest.

5. Amrish borrowed Rs. 900 at 4% p.a. & Rs. 1100 at 5% p.a. for the same duration. He had to pay Rs. 364 in all as interest. Find time.

6. If Rs. 450 amount to Rs. 504 in 3 years, what will Rs. 615 amount to in 2 ½ years at the same rate ?

7. A sum of money doubles itself in 10 years. In how many years will it treble itself at the same rate ?

8. Two equal sums are lent at the same time at 6% and 5% simple interest respectively. The former is received 2 years earlier than the later, and the amount in each case is Rs. 2400. Find the sum and the times.

9. Two equal sums are lent at the same time at 9% and 8% simple interest respectively. The former is recovered 6 months earlier than the later, and the amount in each case is Rs. 17680. Find each sum and the times for which they are lent.

10. Gopal invests a part of Rs. 8000 at 4% p.a. & the remainder at 5% per year. His annual income from the investments is Rs. 350. How much has he invested at each rate ?

11. Divide Rs. 6000 into two parts so that S.I. on the first part for 2 years at 6% p.a. may be equal to the simple interest on the second part for 3 years at 8% p.a.

12. A loan of Rs. 12800 is repaid by 3 annual payments, the first payment being half of the second and one-third of the third. If the interest be 10% p.a. payable annually, find three payments.

13. Mahajan lend out Rs. 9 on the condition that the loan is payable in 10 months by 10 equal installments of Re. 1 each. Find the rate of interest per annum.

14. Ramesh took Rs. 1500 on loan from Man Singh at 14% per annum. After 5 years, he gave Rs. 2100 and a watch to Man Singh to clear the account. Find the cost of the watch.

15. A certain sum of money double itself in 8 years at a simple interest in a firm but trebles itself in 17 years in another firm at simple interest. Which of the two firms pays better interest. And what are the rates?

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16. A man lent Rs. 2500 at a certain rate of interest and Rs. 1500 at 1% higher. After 3 years, he received Rs. 525 as simple interest in all. Find the rates.

17. A, B & C borrow Rs. 29200 in all at 4% S.I. and pay back equal in full settlement of their respective debts after 5, 10 & 15 years respectively. Find the sum borrowed by each.

18. The sum of Rs. 725 is borrowed at the beginning of a year at interest. After 8 months have passed Rs.362 ½ more is borrowed at a rate of interest double that at which the former sum bears. At the end of the year , the sum of interests on both loans is Rs. 43.50. What is the first rate of interest per annum ?

19. A father left a will that his capital of Rs. 18750 should be divided between his two sons aged 12 years and 14 years respectively, so that when they attained majority at the age of 18 years; the amounts received by each at 5% simple interest were the same. Find the sum allotted to each.

20. A buys a house for Rs. 23000. There is a ground rent of Rs. 500 a year, and annual repairs come on an average to 1 ½ % on the purchase price of the house. At what monthly rent must he let the house so as to get a return of 8% per annum on the purchase price ?

21. Find the compound interest & the amount on Rs. 8000 at 5% p.a. for 3 years when interest is reckoned yearly.

22. Find out the C.I. on Rs. 5000 at 4% p.a. compounded half-yearly for 1 ½ years.

23. The C. I. on Rs. 800 in 2 years becomes Rs. 82. Find the rate of compound interest.

24. The difference between C.I. & S.I. on a certain sum of money for 3 years at 6 1/3 % p.a. is Rs. 184. Find the sum.

25. A sum of money, compound interest compounded annually ,amounts in 2 years to Rs 605 and in 3 years to Rs. 665.50. Find the rate of interest & original sum.

26. The C.I. of 2 years of some money is greater than the simple interest of 2 years of the same money by Rs 2 when the rate of interest in each is 5% p.a. Find the sum.

27. The C.I. and simple interest on certain sum for 2 years are respectively Rs 41 & Rs 40. Find the sum & rate per cent.

28. A sum when placed at C.I. doubles itself in 4 years. In how many years will it amount to eight times of itself?

29. A money lender borrows money at 4% per annum and pays the interest at the end of the year. He lends it at 6% p.a. half-yearly & receives the interest at the end of the year . In this way he gains Rs. 104.50 in that year. How much money does he borrow?

30. A sum of money is put out at C.I. for 2 years at 20% interest compounded yearly. It would fetch Rs. 48.20 more if the interest were payable half yearly. Find the sum.

31. A man borrowed same money every year at 10% C.I. If at the end of 3 years, he pays Rs. 2641, how much money had he borrowed each year?

32. A sum of Rs. 1230 is borrowed at 5% C.I. interest for 2 years on the condition that it is refunded in 2 equal installments one at the end of the first year & the other at the end of the second. Find the amount of each installment.

33. Divide Rs. 3903 between A & B such that A's share at the end of 7 years is equal to B's share at the end of 9 years at 4% p.a. rate of compound interest.

34. A loan of Rs. 2000 is to be paid back in 3 equal annual installments. How much is each installment, to the nearest whole rupee, if the interest is compounded annually at 12 ½ % p.a.?

35. A property decreases in value every year at the rate of 6 ¼ % of its value at the beginning of the year. Its value at the end of 3 years was Rs. 21093.95. Find the value at the beginning of the first year .

36. Find the least number of complete years in which a sum of money put out at 25% C.I. will be more than double of itself?

37. Find the compound interest on Rs 5000 in 3 years if the rate is 5% for the first year, 3% for the second year and 2% for the third year.

38. A certain sum of money is put out to compound interest for 4 years at 5% per year. The interest for the third year is greater than that of second year by Rs. 262.50. Find the total interest for the four years and also the original sum.

39. A's capital exceeds B's capital by 41/400 of B's capital. B invests his capital at compound interest for 3 years at 5% per annum. At what rate per cent per annum at simple interest must A invest his capital in order that at the end of 3 years, the two amount ma be the same.

40. The bacteria in a culture grows by 5% in the first hour, get destroyed by 5% in the second hour and again grows by 5% in the third hour. If the count of the bacteria at the end of he third hour is 8.379 x 10^8 , find the original count of bacteria in the sample.

Answers

1. Rs 75, Rs 2575   **2.** Rs 157.50   **3.** Rs 2750

4. 12 ½ %   **5.** 4 years   **6.** Rs 676.50

7. 20 years   **8.** (I)10 yrs, 12 yrs (ii) Rs 1500   **9.** (I)Rs 13000 (ii) 4 yrs (iii) 4 ½ yrs

10. Rs 5000 at 4% , 3000 at 5% **11.** Rs 4000, Rs 2000   **12.** Rs. 2600, Rs. 5200, Rs. 7800

13. 26 2/3%   **14.** Rs. 450   **15.** 12.5%, 11.7%

16. 4%, 5%   **17.** A (Rs 11200), B(9600), C(8400)

18. 4.5%   **19.** Rs 9000   **20.** Rs. 224 (approx)

21. Rs. 1261, Rs. 9261   **22. Rs.** 306.04   **23.** 5% p.a.

24. Rs. 14959   **25.** 10%, Rs. 500   **26.** Rs. 800

27. 5%, Rs. 400   **28.** 12 years   **29**. Rs. 5000

30. Rs. 2000   **31.** Rs. 725.35   **32.** Rs. 661.50

33. A = Rs. 2028, B = Rs. 1875 **34.** Rs. 840 (approx)   **35**. Rs. 25600

36. 4 years   **37.** Rs. 463.12   **38.** Rs. 21550.62, Rs. 100000

39. 5/3 %   **40**. 8 x 10^8

C o n c e p t s 12

Stock & Shares

Stock

The government sometimes raises a loan from the public, at a certain fixed rate of interest. Bonds or Promissory notes each of a fixed value are sold to the public. If a person purchases a bond of Rs. 100 at which 5% interest is fixed, then the holder of such a bond has '**Rs. 100 stock at 5%**'. Rs. 100, in this case, is called the **face value** of the stock. Stocks can be sold and bought in the open market, through brokers at stock exchanges. The broker's charge is called the **brokerage.** For All theoretical purposes brokerage is calculated on the face value.

From the exam point of view

1. If not mentioned otherwise, The face value of one unit of stock (i.e. bond/debenture/IOU/Promissory Note) should be taken as 100.
2. The Brokerage should be calculated on the basis of the face value.

- When stock is purchased, brokerage is added to the cost price.

- When stock is sold, brokerage is subtracted from the selling price.

- The selling price of a Rs. 100 stock is said to be at par, above par (or at a premium) and below par (or at a discount) according as the selling price of this stock is exactly Rs. 100, more than Rs. 100 and less than Rs. 100 respectively.

- When we say 'Rs. 900, 7% stock at 96', we mean a stock whose face value is Rs. 900, the market price of a Rs. 100 stock is Rs. 96 and the annual interest on this stock 7% of the face value.

Ex. What is the cost of Rs. 9100, $5\,^3/_4$% stock at 92 ?
Sol. Cost of Rs. 100 stock = Rs. 92 ∴ Cost of Rs. 9100 stock = Rs. (92 x 9100)/100 = Rs. 8372

Ex. What is the cash required to purchase Rs. 1600, 3.5% stock at 105 ? (brokerage 0.5%)
Sol. Cash required for purchasing Rs. 100 stock = Rs. (105 + 0.5) = Rs. 105.50
Cash required for purchasing Rs. 1600 stock = Rs. (1600 x 105.50)/100 = Rs. 1688

Ex. What is the income generated by investing Rs. 3000 in 6% stock at 120 ?
Sol. Investment of Rs. 120, gives an income of Rs. 6
∴ an investment of Rs. 3000, gives an income of Rs. (3000 x 6)/120 = Rs. 150

Ex. What rate percent is obtained by investing in 5.5% stock at 96 ? (brokerage 0.25%)

Sol. Investment of Rs. (96 + 0.25) gives an income of Rs. 5.5
Investment of Rs. 100 gives an income = Rs. (100 x 5.5)/96.25 = Rs. $5\,^5/_7$
∴ Rate = $5\,^5/_7$%

Shares

Some people together associate to form a company. The company invites the public to subscribe. The required capital is divided into small parts called **shares.** Each share is of a particular fixed value. Parts of the profits distributed among the shareholders are called **dividends.** The original value of a share is called its **nominal value.** The current price of the share in the market is called its **market value.**

Note : If the brokerage, premium, discount, dividend etc is not mentioned as a percentage, then take it directly in rupees per unit of the share or stock that is transacted.

- **Preference Shares** : On these shares a fixed rate of dividend is paid to their holders, subject to the profits of the company.

- **Ordinary or Equity Shares** : After paying the dividends of the preference shareholders, the equity shareholders are paid the dividends depending upon the profits of the company.

Ex. What will be the cost of 96 shares of Rs. 10 each at Rs.3/4 discount, brokerage being Rs.0.25 per share ?

Sol. Cost of 1 share = Rs. (10 - 3/4) + 0.25 = Rs. 9.5
Cost of 96 shares = Rs. 9.5 x 96 = Rs. 912

Ex. A man purchases Rs. 25 shares in a company which pays 9% dividend. The money invested by the person is that much as gives 10% on investment. At what price did he buy the shares ?

Sol. Face value of 1 share = Rs. 25
Dividend on 1 share = Rs. (9/100 x 25) = Rs. 9/4
Rs. 10 is an income on an investment of Rs. 100
∴ Rs. 9/4 is an income on an investment of Rs. (100 x 9/4)/10 = Rs. 22.50
∴ Price of 1 share = Rs. 22.50

Pankaj Gandhi's Academy/Arithmetic

Exercise 12

1. Which is the better investment? 3% stock at 86 or 4% stock at 110.

2. A man invests equal sums in 3% stock at 80 and 5% stock at 120. If the total yearly income from both the stocks is Rs.380, find his investment in each stock.

3. A man transfers his stock of Rs.7600 from 5% at 105 to 4 $\frac{1}{2}$%. Thus he loses Rs.2. Find the market value of the other stock.

4. A company gives 8% dividend. Mohan has 500 shares each of face value Rs.10 of this company. He sells these shares at 60% premium and purchases shares each of face value Rs.5 of another company which are being sold at 20% discount. If the other company gives 5% dividend, then find the difference of his gains from the dividends.

5. Divide Rs.5300 in to two parts such that if one part is invested in the 3 $\frac{1}{2}$% at 98 and the other in 4% at par, the resulting incomes are equal.

6. A man invests Rs. 2490. He puts in Rs. 900 in 3.5% stock at 75, Rs. 850 in 3% at 68 and the remainder in 6% stock. Total yield from his investment is 5%. At what price does he buy 6% stock ?

7. A company's semi-annual dividend is 7% of the par value of its shares. Ganesh has invested Rs. 15600 in this company. What is his annual income from his investment?

8. An income of Rs. 14765 is derived from an investment of Rs. 4788782 in 7.5% stock. If the brokerage is 2.5%, find the market value of the stock.

9. A man buys Rs. 1400 stock at 6%. His investment is such that he earns 8% on his investment. At what discount / premium did he buy the stock?

10. Shantanu owns Rs. 17500 in 7% stock. How much should he invest in 9% stock at 108 so that his annual income from both types is the same? (Brokerage = ½).

11. Arvind sells 300 shares at a market value of Rs. 475. He had purchased the shares two years ago at Rs. 425. His broker charges brokerage at 11.11% of the gross value of the transaction. Determine the net gain / loss over the period.

12. What is the annual income from 180 shares of common stock of quarterly dividend of Rs 3.75 per share?

13. What is the return on investment obtained by investing in 8.5% stock at 94?

14. Anuj sells his Rs. 162800, 9% stock at 155 and invests the amount in oil stock at a premium of 48. At what % must he invest in order to earn the same as before?

15. Which of the following is the better investment option: stock at 105 with a quarterly dividend of 2% of the par value, stock at 135 with a semi-annual dividend of 5%, stock at 115 with a bi-monthly dividend of 1.5% or 14% stock at 125?

16. Find the amount of cash required to purchase Rs. 2000, 3½ % stock at 105? (Brokerage ½).

17. What is the worth of stock which should be sold in order to realise Rs. 1221 from stock at 101¾?

18. Jaydeep invests a certain sum in 6% stock at 75, twice the sum in 7% stock at 80 and thrice the sum in 10% stock at 90. What is the average yield percent that Jaydeep gets on his investment?

19. A person having bought a certain amount of 2.75% stock at 95, afterwards sold it, and with the proceeds, bought 3.5% stock; he obtained Rs. 900 less stock than before, but his income remained the same. How much money did he initially invest?

20. A man invests Rs. 1980 in 3.5% stock at 99 and Rs. 3220 in 4.5% stock at 105. Find the average rate of return that he earns on his capital?

Answers

1. Rs. 3.5 and Rs. 3.6 approximately Choose 2nd stock.   **2.** Rs.4800.

3. Rs.95.   **4.** Rs.100.   **5.** Rs.2800, Rs.2500.   **6.** 98 $^{2}/_{3}$.

7. Rs. 2184.   **8.** Rs. 2430.   **9.** Discount of 25.   **10.** Rs. 14768.

11. Rs. 833 Loss.   **12.** Rs. 2700.   **13.** 9.04%.   **14.** 8.59%.

15. 14% stock at 125.   **16.** Rs. 2110.   **17.** Rs. 1200.   **18.** 9.8%.

19. Rs. 3990.   **20.** 4%.

Solutions
Exercise 1

1. $x/63 = 3/21$ ∴ $x = 9$

2. $a/b = 8/15$ $b/c = 5/8$ $c/d = 4/5$ ∴ $(a/b)(b/c)(c/d) = (8/15)(5/8)(4/5) = 4/15$
   a:d = 4:15

3.
a	b	c	d
3	4		
	8	9	
		15	16
---	---	---	---
90	120	135	144

   a:b:c:d = 30:40:45:48

4. Let the fraction be k: ∴$k/(1/27) = (3/7)/(5/9)$ ∴ $k = 1/35$

5. Let the mean be x ∴$0.9/x = x/49$ ∴$x = 2.1$

6. Let the third proportional be x ∴$6/30 = 30/x$ ∴ $x = 150$

7. Let the fourth proportional be x ∴$1/2 = 3/x$ ∴ $x = 6$

8. Let a = 5/6   b = 7/8 ∴$a:b = (5/6) : (7/8) = 20 : 21$ ∴ b > a or 7 : 8 > 5 : 6

9. $3/4 = 0.75$ $7/8 = 0.875$ $5/6 = 0.833$ $17/19 = 0.894$ ∴$3/4 < 5/6 < 7/8 < 17/19$

10. $2/5 = 0.4$ $4/7 = 0.571$ $16/19 = 0.842$ ∴$16/19 > 4/7 > 2/5$

11. $a = 0.8/0.4 = 2$ $b = 3.5/2.5 = 1.4$ $c = 15/20 = 0.75$ $d = 4/7 = 0.571$
    0.8 : 0.4 -greatest   4 : 7 -smallest

12. Let the other quantity be x ∴$155/x = 5/7$ ∴ $x = 217$

13. $x/21 = 200/840$ ∴ $x = 5$

14. Let the height be x metres ∴$x/10 = 72/8$ ∴ $x = 90$

15. Average speed : Car I = $650/(13/4) = 200$ kmph
    Car II = $650/4 = 162.5$ kmph
    Car I / Car II = $200/162.5 = 16/13$

16. Let the numbers be 2x & 7x ∴$(2x + 14)/(7x + 14) = 4/7$ ∴ $x = 3$
    The numbers are 6 & 21

17. Let the numbers be 2x & 7x ∴Their L.C.M. is 14x
    $14x = 70 \Rightarrow x = 5.$ ∴The numbers are 10 & 35

18. Let the numbers be 5x & 8x ∴$5x - 8x = -3x = 24$ ∴$x = -8$
    The numbers are -40 & -64

19. Let the numbers be 7x & 4x. ∴$49x^2 - 16x^2 = 132$ ∴ $x = \pm 2$
    The numbers are ± 14 & ± 8

20. Let the number of days required be x
$(16)(6/60) = (28x/70) \Rightarrow x = 4$

[(M1T1)/W1 = (M2T2)/W2]

21. Let the number of days required be x
$[(16)(7)(48)]/1 = [(14)(12)(x)]/1 \Rightarrow x = 32$

[(M1R1T1)/W1 = (M2R2T2)/W2]

22. Let the number of days required be x
$[(75)(5)(6)]/[(300)(9)(3)] = [(50)(4)(x)]/[(540)(6)(4.5)]$

[(M1R1T1)/W1 = (M2R2T2)/W2]

$\Rightarrow x = 20 \ 1/4$

23. 1 Man = 2 Boys   &   3 Men = 6 Boys
Let the number of days required be x
$(2)(40)/1 = (6 + 4)(x)/(1/2) \Rightarrow x = 4$

24. $(2M + 7B)14 = (3M + 8B)11$
$\therefore 1M = 2B$
One man does as much work as two boys
$(2M + 7B) = 11$ Boys   & $(8M + 6B) = 22$ Boys
Let the number of days required be x
$(11)(14)/1 = (22)(x)/3 \Rightarrow x = 21$

25. Let the number of days be x & let the earlier & later provisions be 6y & 5y
$(600)(6y)(10)/10 = (900)(5y)(x)/25$
$x = 20 \therefore$ The provision will last for $10 + 20 = 30$ days.

26. Let the number of soldiers who join be x
$(1500)(13)/(13/48) = (1500 + x)(25)/(35/48) \Rightarrow x = 600$

27. Let the number of cadets remaining be x
$(3500)(500)(25) = x(400)(35) \Rightarrow x = 3125$
The number of cadets who leave is $(3500 - 3125) = 375$

28. If the quantity of food utilised daily is q then the total quantity available is 70q
In 5 days 5q is consumed, remainder = 65q
Let the new quantity suffice for x days
$840(70)/70q = 1050x/65q \Rightarrow x = 52$

29. Let the new number of people be x 120 men finish 2/3 of the job in 64 days
$\therefore$ Remaining men have to finish 1/3 of job in 60 days
$120(64)/(2/3) = 60x/(1/3) \Rightarrow x = 64$
The number of men to be discharged is $(120 - 64) = 56$

30. Let the new number of men on the job be x
In 50 days, 1.5 km is done $\therefore$ In the remaining 80 days 4.5 km is to be done
$72(50)/1.5 = 80x/4.5 \Rightarrow x = 135$
Additional men required $= 135 - 72 = 63$

31. Let the new number of men on the job be x
After 18 days 1/3 work is done $\therefore$ x men have to do 2/3 work in 10 days
$(20)(8)(18)/(1/3) = x(9)(10)/(2/3) \Rightarrow x = 64$
Additional men required $= 64 - 20 = 44$

32. Suppose 1 man alone can do the work in m days and 1 woman alone can do the work in w days. So, $12/m + 8/w = 1/20$ and $36/m + 48/w = 1/5$. Solving these two equations, we get, m = 360 and w = 480. Therefore, $12/360 + 12/480 = 7/120$. Therefore, 12 men and 12 women can complete the work in 17 1/7 days.

Pankaj Gandhi's Academy/Arithmetic

33. Let the earlier & current lengths be 3x & 4x respectively & Let the earlier & current breadths be 4y & 5y respectively
Earlier area = A_1 = 3x(4y) = 12xy. Current area = A_2 = 4x(5y) = 20xy
Ratio between the two areas = A_1/A_2 = 12/20 = 3/5

34. Let the weight of mangoes be x
Apples/Mangoes = Mangoes/Dates ∴ 7.2/x = x/5
x^2 = 36, ∴ x = 6 kg

35. Let the leaps of the greyhound & the hare be 4x & 5x respectively.
Let the distances covered by the greyhound & the hare be 3y & 4y reps.
The rate of the greyhound = R_1 = 4x/3y. The rate of the hare = R_2 = 5x/4y
The ratio is R_1/R_2 = (4x/3y)/(5x/4y) = 16/15 = 16 : 15

36. Let the volumes be 5x, 6x & 8x
Let the weights of equal volumes be 4y, 5y & 3y
Weights of the substance in the mixture is 20xy, 30xy, 24xy
The ratio of the substances is 10:15:12

37. Let the volume of each pot be x

	Pot I	Pot II	Pot III	Mixture
Milk	3x/5	7x/10	11x/15	122x/60
Water	2x/5	3x/10	4x/15	58x/60

Ratio of milk & water in the mixture = (122/58) = 61/29

38. Let F be the force of gravity and as per the relation given F ∝ r δ
where r = radius of the planet and δ = density of the planet,
F_1/F_2 = $(r_1 \delta_1)/(r_2 \delta_2)$
Also given that h ∝ 1/F where h = height to which man can jump on planet.
h_2/h_1 = F_1/F_2 = $(r_1 \delta_1)/(r_2 \delta_2)$
Substituting the values we get $h_1/1500$ = (5.67/1.75)x(1/10)
∴ h_1 = 486 cm.

39. Distance d = $k(h)^{0.5}$
 I) 14.4 = $k(18)^{0.5}$ (1)
Let distance be x km when height is 8m
x = $k(8)^{0.5}$ (2)
Divide equation (1) by equation (2)
14.4/x = $(18/8)^{0.5}$ ⇒ x = 9.6 km

ii) 14.4 = $k(18)^{0.5}$ (i)
Let the height be y when the distance is 7.2 km
7.2 = $k(y)^{0.5}$ (ii)
Divide equation (i) by (ii),
14.4/7.2 = $(18/y)^{0.5}$
∴ Squaring both sides, y = 4.5 m

40. Pressure = p, Volume = v
p = k/v, 20 = k / 450. k = 20(450)
Let pressure be x when volume is 300
x = k/300   k = 300x
300x = 20(450)
∴ x = 30 kg weight per square decimeter
Let volume be v when pressure is 60

60 = k/v. k = 60v

      60v = 20(450)

Therefore, v = 150 cubic decimeter.

Exercise 2

1.   Let the number be x   30% of x is 30x/100

    30x/100 = 225

    x = 750

2.   8(325/100) = 26

3.   Let 33 be x% of 88

    (x/100)(88) = 33

    x = 37.5

4.   Let the income of the man be x . He spends (85x/100).   He saves (15x/100).

    15x/100 = 37.50

    x = 250

5.   Let one part be x

    Other part = 10% of x = (x/10)

    x + x/10   = 1100

    x   = 1000

    The two parts are 1000, 100

6.   Let B's salary be Rs. 100

    Let A's salary be Rs. 125

    A - B = 125 - 100 = 25

    B's salary is (25/125)100 = 20% lower than A's

7.   Let B's salary be 100   ∴A's salary = 80

    B's salary is (20/80)100 = 25% above A

8.   Let the original price of cloth be Rs. 100. Let the original consumption be 100

    Original expenditure = Rs. 10000

    Let the new price be Rs. 120. Let the new consumption be x

    New expenditure = Rs. 120x

    120x = 10000

    x = 10000/120 = 250/3

    Reduction in consumption = 100 - 250/3 = 50/3

    Percentage reduction in expenditure = 50/3 = $16\,^2/_3$%

9.   Let the original price be Re. 1 per dozen. The new price is Rs. 0.60 per dozen

    There is reduction of Rs. 0.40 per dozen

    For Re.1, one can buy 12 eggs + 48 more

    New price of eggs is Rs. 0.40 for 48 eggs i.e. 4 doz. = Rs. 0.10 per dozen

    Let x be the original price of eggs

    New price = (60x/100) = 0.10   ∴   x = $16\,^2/_3$

    Original price = $16\,^2/_3$ paise per dozen

    New price   = 10 paise per dozen

10.   a). Let the original price of oranges be Rs. 5   New price = Rs. 4 (20% drop)

    New price corresponding to an old price of Rs. 10 is 10(4/5) = Rs. 8

    i.e. a reduction of Rs. 2

    For Rs. 2 one can buy 5 oranges

    New price of orange = 200/5 = 40p

80% of old price of per orange = 40ps,
therefore Old price per orange = 50ps.

11.   a). Let the original price of sugar be Rs. 8   New price = Rs. 7 (12.5% drop)
New price corresponding to an old price of Rs. 126 is 126(7/8) = Rs.110.25
i.e. a reduction of Rs. 15.75
For Rs. 15.75 one can buy 9 kg of sugar
New price of sugar = 15.75/9 = Rs. 1.75/kg
Old price of sugar = Rs. 2.00
For Rs.126 one can buy 63 kg

b. If the price goes up by 12.5%, new price = Rs. 2.25 per kg
For Rs. 126, one can buy 56 kg
from a. & b. one can buy 7 kg less if price rises by 12.5 %

12.   Let the maximum marks be x Candidate1 gets 20x/100 = x/5
Candidate2 gets 42x/100 = 21x/50
passing marks = x/5 + 10
passing marks = 21x/50 - 1
x/5 + 10 = 21x/50 - 1
 x = 50
Maximum marks = 50   Passing marks = 20 i.e 40%

13.   75% passed in English   25% failed in English
70% passed in Maths   30% failed in Maths
As per the Venn diagram, 23% failed in both
23 + 7 + 2 = 32% failed in either Maths or English or both
68% passed in both the subjects
Let x be the total number of candidates
68x/100 = 136
x = 200

14.

Year	I	II	III	IV	V
Population	100000	110000	121000	108900	98010

15.   Let the population be x
After one year the population becomes 95x/100 = 19x/20
After two years the population becomes (95/100)(19x/20) = 902500
x = 1000000

16.   Let the total number of votes be x
Candidate1 secures (40x/100) votes
Candidate2 secures (60x/100) votes
60x/100 - 40x/100 = 300
 x = 1500

17.   Let the original number of soldiers be x
No. of soldiers killed = 10% = x/10
Remainder = 9x/10
No. soldiers died in disease = (1/10)(9x/10) = 9x/100
Remainder = 9x/10 - 9x/100 = 81x/100
No. of soldiers disabled = (1/10)(81x/100) = 81x/1000
Remainder = 81x/100 - 81x/1000 = 729x/1000
729x/1000 = 729000
 x = 1000000

18. Let the number of votes be x
    Illegal votes = 4x/100 = x/25
    valid votes = 24x/25
    winner gets (55/100)(24x/25)
    Loser gets (45/100)(24x/25)
    Majority of winning candidates
    (55/100)(24x/25) - (45/100)(24x/25) = 240
    x = 2500
    The number of votes are 2500

19. Let the numbers be 125, 120 & 100
    The first number is (120/125)100 % i.e. 96% of the second number.

20. Let the number of boys & girls be 16x & 9x reps.
    The number of girls passing = 6.75x
    Number of boys passing = 4(6.75x)/3 = 9x
    % of boys passing = (9x/16x)100 = 56 1/4%
    % of all the candidates passing
    = (15.75x/25x)100 = 63%

21. Earlier entry fee = Re. 1
    Earlier number of visitors = y
    Earlier sale = Rs. y
    New entry fee = Re. 0.75
    New number of visitors = x
    New sale = Rs. (0.75)x
    (0.75)x = (120/100)y
    x = (160/100)y
    The number of visitors rises by 60%

22. Let the successful candidate get x votes
    The other candidate gets x - 308 votes
    The total number on the voters list = 100x/47
    The number of people who voted = (90/100)(100x/47)
    Number of legal votes = 90x/47 - 60
    legal votes = x + (x - 308) = 2x - 308
    2x - 308 = 90x/47 - 60
    x = 2914
    Successful candidate gets 2914 votes, other candidate gets 2606 votes

23. Let the number of candidates be x
    The number of candidates who attempted :
    All questions = 5x/100 (5% of the total)
    none   = 5x/100 (5% of the total), (remainder = 90x/100)
    1 question   = 9x/40 (25% of the remainder)
    4 questions = 9x/50 (20% of the remainder)
    2 questions = 49x/200 (24.5% of the total)
    3 questions = x - (sum of all the above)
    = x/4 = 200
    x = 800

24. Taxable income = Rs. 28600
    Tax = Rs. 3200 + (30/100)(28600 - 25000) = Rs. 3200 + 1080 = Rs. 4280
    Surcharge = Rs. (15/100)(4280) = Rs. 642
    Total income tax = Rs. (4280 + 642) = Rs. 4922

25. i) Marks in English : A 99, B 79
    A gets 20 marks more than B

% increase = (20/79)100 = 25.3%
ii) C gets 120 marks in Maths, 101 marks in G.K.
∴ % increase = (19/101)100 = 18.8%

26.   Year I   Year II Year III
          2000   2200   1980

27.   Let the original population be x
          5% of x = 1500
          ∴ 5x / 100 = 1500 ⇒ x = 30,000

28.   Let the money be Rs. x
          Wife gets Rs. (30x/100) = Rs. 3x/10
          Son gets Rs. (40/100)(x - 3x/10) = Rs. 7x/25
          Each daughter gets Rs. (1/3)(x - 3x/10 - 7x/25) = Rs. 7x/50
          7x/50 = 224   ∴ x = 1600
          Wife gets Rs. 480

29.   Let the income be Rs. 100   Expenditure = Rs. 80   Savings = Rs. 20
          New Expenditure = (137.5/100)80 = Rs. 110
          New income = Rs. 116 $^2/_3$
          New savings = Rs. 6 $^2/_3$
          Present percentage savings = [(6 2/3)/(116 2/3)]100 = 5 $^5/_7$%

30.   a. Let the original tax be Rs. x
          Let the original consumption be y
          The revenue = Rs. xy
          New tax = (90/100)x = Rs. (9x/10)
          New consumption = (110/100)y = 11y/10
          New revenue = 99xy/100
          Decrease in revenue = xy - 99xy/100 = xy/100
          % decrease in revenue = 1%
          b. Let the consumption be cy
          xy = (90x/100)cy
          c = 10/9 So percentage increase in consumption = 1/9 * 100 = 11 $^1/_9$%

31.   Earlier wages per hour = 120/50 = Rs. 2.40
          New wages per hour = (112.5/100)2.4 = Rs. 2.7
          New number of hours = (92/100)50 = 46
          Earlier weekly wages = Rs. 120
          New weekly wages = Rs. 124.20
          % increase = (4.2/120)100 = 21/6% = 7/2%

32.   A   B   C
          90   100
          110   (100/90)110   100
          Goods worth Rs. 100 are available at Rs. 1100/9 from B
          A person saves (1100/9 -100) = 200/9% = 22 $^2/_9$%

33.   Let the total number of voters be x.
          A defeated B by 640 votes, which was (60 - 40) = 20 % of the legal votes.
          ∴ 20 (L)/ 100 = 640 ⇒ Legal votes = L = 3200
          ∴ Total votes cast = 3200 + 400 = 3600, which is 90% of the total voters.
          ∴ 90 x / 100 = 3600 ⇒ x = 4000.

34.   Let the incomes of A & B be Rs. 5x & 3x respectively. Let their expenses be Rs. 2y & y
          respectively.
          5x - 2y = 3600. 3x - y = 3600

Solving these two equations simultaneously,
A's income = Rs. 18000, B's income = Rs. 10800

35. a = % change in one side = +4
b = % change in other side = -5
% error in area = (a + b) + (ab/100) = (4 - 5) - (20/100) = - 1.2%

Note: If error/change in the length is a% and that in breadth is b% then percentage change in area = a + b + (ab/100) where a & b take appropriate signs i.e. + for increase & - for decrease.

36. Let the income be Rs. x. Savings = (6/100)x = 3x/ 50 Expenses = 47x/50
New income = 115x/100. Savings = 3x/50 expenses = 109x/100
Expenses increase by 15x/100
% increase in expenditure = (15x/100)(100/94x)100 = $15 \frac{45}{47}$%

37. Let the cost of the machine be Rs. x
Freight = 20x/100 = x/5
Total cost = x/5 + x = 1.20x
New price = Rs. (90x/100) = Rs. 9x/10
Freight = (20/100)(9x/10) = 9x/50
total cost = Rs. 9x/10 + 9x/50 = Rs. 1.08x
1.20x - 1.08x = 480
x = 4000
The cost of the machine is Rs. 4000

38. Let D get x marks
C gets 80x/100 = 4x/5 marks
B gets (125/100)(4x/5) = x
A gets 90x/100 = 720
x = 800
D gets 800 out of 1000
D gets 80% marks

39. Let the C.P. be Rs. 100
Price after 33% discount = Rs. 67
Price after 12.5% discount = (87.5/100)67 = Rs. 58.625
Price after 4% discount = (96/100)58.625 = Rs. 56.28
Single equivalent discount = (100 - 56.28) = 43.72%

40. Let the cost be Rs. 100

Bargain 1	Bargain 2
After 20% = Rs. 80	After 25% = Rs. 75
After 15% = Rs. 68	After 12% = Rs. 66
After 10% = Rs. 61.20	After 8% = Rs 60.72

∴ Second Bargain is better

Exercise 3

1. Let the two parts be 7x & 11x their addition is 7x + 11x = 18x
   but 18x = 90   ∴ x = 5. Therefore, the two parts are 35 and 55.

2. Let the three parts be x/2, x/3, x/4 their addition is x/2 + x/3 + x/4 = 13x/12
   but 13x/12 = 1690, giving x = 1560. Therefore, the three parts are 780, 520, 390.

3. Let the two parts be x, 90-x.
   5/8 of first part = (5/8)x and 2/5 of second part = (2/5)(90 - x).
   (5/8)x /(2/5)(90 - x) = 5/4. Therefore, x = 40.
   ∴, the the two parts are 40,50 .

4. Let the original quantity of milk & water be 3x & x respectively.
   3x + x = 20
   ∴   x = 5
   The quantity of milk and water is 15 liters & 5 liters respectively.
   Let y liters of milk be added.
   New quantity of milk and water is (15+y) & 5 liters respectively.
   (15+y)/5 = 4/1   -given
        y = 5 liters.

5. Let the number of girls be x   The number of boys = 480 - x
   0.4(480 - x) + 0.6x = 232
             x = 200

6. The two friends get Rs. 5x & Rs. 11x respectively
   11x - 5x = 1200
       x = 200
   They get Rs. 1000 and Rs. 2200 respectively
   Total amount = Rs. 3200

7. C,B,A get Rs. x, Rs. 1.20x, Rs. 0.96x respectively
   x + 1.20x + 0.96x = 395
   x = 125
   A,B,C get Rs. 120, Rs. 150, Rs. 125 respectively

8. C/D = 4/3   ∴ D = (3/4)C   B/C = 6/5   ∴ B = (6/5)C
   If C = 1, B : C : D = (6/5) : 1 : (3/4)
   Multiplying throughout by 5 x 4, B : C : D = 24 : 20 : 15
   But A : B = 9 : 8 ∴ if B is 24, A is 27

A	B	C	D
9	8		
	6	5	
		4	3
27	24	20	15

   Let A,B,C,D get Rs.27x,Rs.24x,Rs.20x,Rs.15x respectively
   27x + 24x + 20x + 15x = 4300, x = 50
   A,B,C,D get Rs.1350, Rs.1200, Rs.1000, Rs.750 respectively .

9.

Sudha	Kamla	Kamal
3	4	
	3.5	3
42	56	48
210	280	240

10. The new shares of A,B,C are Rs. 2x,Rs. 3x & Rs.7x respectively
The new total amount is Rs. 558 - 4 - 6 - 8 = 540
2x + 3x + 7x = 540
$\qquad$ x = 45
The new shares of A,B,C are Rs.90,Rs.135 & Rs.295 respectively
The old shares are A = Rs. 90 + 4 = Rs. 94
$\qquad$ B = Rs. 135 + 6 = Rs. 141
$\qquad$ C = Rs. 315 + 8 = Rs.323

11. Manish, Sangeeta & Umesh get Rs.x,Rs.2x,Rs.6x respectively
x + 2x + 3x = 1215 $\qquad$ ∴ x = 135
Manish,Sangeeta & Umesh get Rs.135, Rs.270, Rs.810 respectively

12. The shares of A,B,C are Rs.x,Rs.y,Rs.z respectively
5x = 6y = 8z $\qquad$ (I)
$\qquad$ x + y + z = 1180 (II)
∴ x/24 = y/20 = z/15 = $^{1180}/_{59}$ ⇒ x = 480, y = 400 and z = 300.

13. A,B,C get Rs.x,Rs.y,Rs.z respectively
x = (2/3)(y) $\qquad$ --(I)
y = (1/4)(z) $\qquad$ --(II)
x + y + z = 510 $\qquad$ --(III)
From these three equations, $\qquad$ x = Rs. 60 $\qquad$ y = Rs. 90 $\qquad$ z = Rs. 360

14. A,B,C get Rs.x,Rs.y,Rs.z respectively
x = (3/7)(y + z) $\qquad$ --(I)
y = (2/9)(x + z) $\qquad$ --(II)
x + y + z = 12540 $\qquad$ --(III)
∴x + (3/7) x = 3/7 (7x/3 + x) = 3/7(x+y+z) ⇒ x = Rs. 3762.
Similarly $\qquad$ y = Rs.2280, $\qquad$ z = Rs. 6498

15. A,B,C get Rs.x,Rs.y,Rs.z respectively
(2/5)x + 40 = (2/7)y + 20 = (9/17)z + 10
x + y + z = 600
From these equations, $\qquad$ x = Rs. 150 $\qquad$ y = Rs. 280 $\qquad$ z = Rs. 170

16. Each child, woman & man gets Rs.x, Rs.1.5x, Rs.3x respectively
10x + (6)(1.5x) + (8) (3)x = 258
$\qquad$ x = 6
Each man, woman & child gets Rs.18, Rs.9, Rs.6 respectively

17. Combined shares of all men, all women and all children be Rs.12x, Rs. 15x, Rs. 16x respectively
12x + 15x + 16x = 430 ∴ x = 10
Combined shares of all men, all women and all children are Rs.120, Rs. 150, Rs. 160 respectively
Let the number of men, women & children be m,w,c respectively
m + w + c = 45 $\qquad$ (I)
Each man gets Rs. 120/m $\quad$ Each woman gets Rs. 150/w $\quad$ Each child gets Rs. 160/c
120/m : 150/w : 160/c :: 6 : 5 : 4

Taking two ratios at a time, $w = (3/2)m$ $c = (4/3)w = 2m$ (II)
From (I) & (II),   m = 10, w = 15, c = 20
Each man gets Rs. 120/m = Rs. 12
Each woman gets Rs. 150/w = Rs. 10
Each child gets Rs. 160/c = Rs. 8

18.   The number of fifty ps., twenty five ps. & ten ps. coins be 2x, 5x, 10x respectively
$50(2x) + 25(5x) + 10(10x) = 5200$
        x = 16
The number of fifty ps., twenty five ps. & ten ps. coins is 32, 80, 160 respectively

19.   1 Re.   50 ps.   10 ps.
   2   5
          4   9
----
   8   20   45
The number of 1 Re., 50 ps. & 10 ps. coins be 8x,20x,45x respectively
$1(8x) + 0.5(20x) + 0.10(45x) = 1125$ x = 50
The number of 1 Re., 50 ps. & 10 ps. coins is 400,1000,2250 respectively

20.   The values of 1 Re., 50 ps. & 25 ps. coins be 13x, 11x, 7x respectively
No. of 1 Re. coins = 13x/1 = 13x
No. of 50 ps. coins = 11x/0.5 = 22x
No. of 25 ps. coins = 7x/(0.25) = 28x
$13x + 22x + 28x = 378$
 x = 6
 No. of 1 Re. coins = 13x/1 = 13x = 78
No. of 50 ps. coins = 11x/0.5 = 22x = 132
No. of 25 ps. coins = 7x/(0.25) = 28x = 168

21.   Let the initial fares for I, II & III class be Rs. 8x, Rs. 6x, Rs. 3x respectively
The new fares are I class   (8x-8x/6) = (40) x /6 = 20 x /3
            II class   (6x-6x/12) = (66) x /12 = 11 x/2
          III class   3x
Number of passengers in the three classes are in the ratio 9 : 12 : 26
Therefore the collection from the three classes is Rs. 60y, Rs. 66y, Rs. 78y respectively
$60y + 66y + 78y = 1088$
        $204y = 1088$
          $y = {}^{1088}/_{204}$
Therefore Collection from the I class passengers is $\left({}^{1088}/_{204}\right)$ (60) = Rs. 320, from the II class
passengers is Rs.352 , from the III class passengers is Rs. 416

22.   Let the fares be 10y, 8y, 3y .   The ratio of passengers is 3:4:10
The collection from the three classes is Rs. 30x, Rs. 32x, Rs. 30x respectively
$30x + 32x + 30x = 8050$
x = 87.5
The collection from the II class passengers is Rs. 2800

23.   The daily wages of A, B, C be Rs. 4x, 3x, 2x respectively
The earnings of A, B, C are 5(4x), 7(3x), 9(2x) respectively
$20x + 21x + 18x = 118$
x = 2
The daily wages of A, B, C be Rs. 8,6,4 respectively

24.   Each man, woman and child receive Rs. x, Rs. y, Rs. z respectively
$2x = 5z$ $2y = 3z$
$x = (5/2)z$ $y = (3/2)z$ --(I)
Total earnings of all men   = 6x
Total earnings of all women = 12y

Total earnings of all children = 17z
6x + 12y + 17z = 150   -- (II)
From I & II ,
Each man, woman and child receive Rs. 7.50, Rs. 4.50, Rs. 3.00 respectively

25.   No. of men, women & children is 50,100 & 150 respectively
The wages of men, women & children be 6x, 3x, 2x respectively
The total earnings of men, women & children will be 50(6x), 100(3x), 150(2x) respectively
300x + 300x + 300x = 450
   x = 0.5
The total earnings of men, women & children will be 150, 150, 150 respectively
Earnings of each man = 150/50 = Rs. 3 per day
Earnings of each woman = 150/100 = Rs. 1.5 per day
Earnings of each child = 150/150 = Re. 1 per day
Weekly wages of each man = 7(3) = Rs. 21
Weekly wages of each woman = 7(1.5) = Rs. 10.5
Weekly wages of each man = 7(1) = Rs. 7

26.   The incomes of A & B be Rs.4x & Rs.3x respectively
The expenses of A & B be Rs. 3y & Rs. 2y respectively
The savings of A & B are Rs.(4x -3y) & Rs.(3x -2y) respectively
4x - 3y = 600 -(I)   3x - 2y = 600 -(II)
Solving these two equations simultaneously,
x = 600   y = 600
The incomes of A & B are Rs.2400 & Rs.1800 respectively

27.   The incomes of A & B be Rs.2x & Rs.x respectively
The expenses of A & B be Rs.5y & Rs.3y   respectively
The savings of A & B are Rs.(2x -5y) & Rs.(x -3y) respectively
(2x - 5y)/(x - 3y) = 4/1   (I)
(2x - 5y) + (x - 3y) = 500   (II)
Solving these two equations simultaneously,
x = 700   y = 200
The incomes of A & B are Rs.1400 & Rs.700 respectively

28.   The marks in Hindi, English, Physics, Chemistry & Maths be 5x, 3x, 4x, 6x & 7x respectively
Total maximum marks = 100(5) = 500
Aggregate marks secured by the student = 350 (70% of 500)
5x + 3x + 4x + 6x + 7x = 350
   x = 14
The marks in each subject are
Hindi : 70 (I div) , English : 42, Physics : 56, Chemistry : 84 (I div) ,Maths : 98 (I div)

29.   The student gets 3x, 4x, 6x & 7x marks. Total of marks secured = 20x
20x/Maximum = 0.60
Maximum = 100x/3, Maximum marks in each subject = 25x/3
First subject : 3x(3)(100)/25x = 36%
Second subject : 4x(3)(100)/25x = 48%
Third subject : 6x(3)(100)/25x = 72%
Fourth subject : 7x(3)(100)/25x = 84%

30.   Let the incomes of A, B, C be Rs. x, Rs. y, Rs. z reps.
x + y + z = 33600   (I)
A spends 80% i.e. A saves 20% = 20x/100, B saves 12.5% = 12.5y/100, C saves 10% = 10z/100

$$\frac{20x/100}{16} = \frac{12.5y/100}{17} = \frac{10z/100}{12} \Rightarrow \frac{x}{20} = \frac{y}{34} = \frac{z}{30} = \frac{(x+y+z)}{84} = 400$$

$\therefore$ x = Rs. 8000, y = Rs. 13600, z = Rs. 12000

From this result,
Savings of A, B, C are Rs. 1600, Rs. 1700 & Rs. 1200 respectively

31. Let the number of boys under 15 & over 15 be 5x & 4x respectively
    At the end of the term
    $(5x - 20)/(4x + 20) = 7/8$, ∴ x = 25
    The total number of boys = 125 + 100 = 225

32. The correct amount distributed to A,B,C be Rs. 2x, Rs. 3x, Rs. 4x respectively
    2x + 3x + 4x = 19.50
    x = 6.5/3
    A,B,C should receive Rs. 13/3, Rs. 6.50 & Rs. 26/3 respectively
    Let the incorrect amount distributed be Rs. (1/2)y, Rs. (1/3)y, Rs. (1/4)y respectively
    (1/2)y + (1/3)y + (1/4)y = 19.50 ∴ y = 18
    A,B,C actually receive Rs. 9, Rs. 6 & Rs. 4.5 respectively
    A gains Rs.(9 - 13/3) = Rs. 4.67
    B loses Rs. (6.50 - 6) = Rs. 0.50
    C loses Rs. (26/3 - 4.50) = Rs. 4.17

33. The property of the man be worth Rs. x
    Son1 receives (3/8)x, Remainder is (5/8)x
    Son2 receives (30/100)(5/8)x = (3/16)x
    (3/8)x - (3/16)x = 1800
    x = 9600
    The property of the man is worth Rs. 9600
    Son1 receives Rs. 3600
    Son2 receives Rs. 1800
    Total amount given to charity = Rs. 4200
    Let each charity receive Rs. 2y, Rs. 5y, Rs. 7y respectively.
    2y + 5y + 7y = 4200
    y = 300
    Each charity receives Rs. 600, Rs. 1500, Rs. 2100 respectively.

34. Total bill for the three days = (45.75 + 59.60 + 74.65) = Rs. 180
    Let A,B,C pay Rs. 4x, Rs. 5x, Rs. 6x respectively
    4x + 5x + 6x = 180 ∴ x = 12
    A,B,C have to pay Rs. 48, Rs. 60, Rs. 72 respectively
    A actually pays Rs. 2.25 less, B actually pays Rs. 0.40 less, C actually pays Rs. 2.65 more
    To settle the accounts A pays Rs. 2.25 to C B pays Rs. 0.40 to C

35. Earlier number of employees = 9x   Present number of employees = 8x
    Earlier wages = 14y   Present wages = 15y
    Earlier wage bill = 9x(14y) = 126xy
    126xy = 1680 ∴ xy = $^{1680}/_{126}$
    New wage bill = 8x(15y) = 120xy = 120(1680/126) = 1600
    Ratio of earlier wage bill to present wage bill is 1680 : 1600 :: 21 : 20
    The difference in wage bills is Rs. 80

36. Let each child get Rs. x, each woman gets Rs. 1.25x, each man gets Rs.(1.25)(1.25x) = Rs. 1.5625x
    8(1.5625x) + 10(1.25x) + 12x = 2368 ∴ x = 64
    Each woman gets Rs. 1.25(64) = Rs. 80

37. Total amount = Rs. x
    Boy1 gets Rs. x/4 Remainder Rs. 3x/4
    Boy2 gets Rs. 3x/20 Remainder = 3x/5
    Each girl gets Rs. 3x/10 i.e. 3/10 of the entire amount.

38. The man, woman & child get daily Rs.7x, Rs.4x, Rs.3x respectively
Total earnings $2(7x) + 3.5(4x) + 4(3x) = 60$ $\therefore x = 1.5$
The man, woman & child get daily Rs.10.5, Rs.6, Rs.4.5 respectively

39. **$A = Q[1 - (q/Q)]^n$**   A = quantity of wine in the final mixture,

$(A/Q) = [1 - (q/Q)]^n$ Q = original volume,

$36/49 = [1 - (4/Q)]^2$ q = quantity of wine taken out each time,
Q = 28 gallons   n = number of times the operation is carried out

40. Step 1: Let the remaining people be x $\therefore$ the number of people voting last for C & D is x
$\therefore$ previous remainder is 2x
Step 2: If remainder is 2x, the number of people voting for B, C, D is 2x
$\therefore$ the previous remainder is 4x
Step 3 : If remainder is 4x, the number of people voting for A, B, C, D is 4x
$\therefore$ the previous remainder is 8x
None voted before this
$\therefore$ there are 8x people going in for voting
but x = 840 $\therefore$ 8x = 6720
Number of people who vote first = 6720/2 = 3360 such that
A gets 3360 votes, B gets (3/6)3360 = 1680, C gets (2/6)3360 = 1120, D gets (1/6)3360 = 560
Number of people who vote next = 3360/2 = 1680 such that
B gets 1680, C gets (2/3)1680 = 1120, D gets (1/3)1680 = 560
Number of people who vote last = 1680/2 = 840 such that
C gets 840, D gets 84
$\therefore$ A gets 3360 votes
B gets 1680 + 1680 = 3360 votes
C gets 1120 + 1120 + 80 = 3080 votes
D gets 560 + 560 + 840 = 1960 votes

Exercise 4

1. A can do 1/6 of the work in one day   B can do 1/8 of the work in one day
Together they can do 1/6 + 1/8 = 7/24 work in one day
They can complete the work in 24/7 days

2. Proceed as in Problem 1

3. A & B together do 1/10 of the work in one day.
A does 1/15 of work in a day   B does 1/10 - 1/15 = 1/30 of work in one day
B takes 30 days to do the work alone

4. A does 1/45 of the work in a day
In 5 days 5(1/45) = 1/9 work is completed
B completes the remaining 8/9 of work in 20 days
B does the entire work in 20(9/8) = 45/2 days
B does 2/45 work in a day
A & B together do 1/45 + 2/45 = 2/45 = 1/15 work in a day
A & B complete the work in 15 days

5. Let A , B, C take x, y, z days respectively
1/x + 1/y = 1/8
1/y + 1/z = 1/12
1/x = 1/16
Solving these equations simultaneously,
y = 16   z = 48

Pankaj Gandhi's Academy/Arithmetic

$1/x + 1/z = 1/12$

A & C take 12 days to complete the work

6. A, B, C complete the work in x, y, z days respectively
   A, B, C complete 1/x, 1/y, 1/z work in a day
   A & B together $1/x + 1/y = 1/24$
   B & C together $1/y + 1/z = 1/30$
   C & A together $1/z + 1/x = 1/40$
   From the above three equations we get
   $1/x + 1/y + 1/z = 1/20$
   A, B & C together complete the work in 20 days
   Solving the equations simultaneously
   x = 60, y = 40, z = 120
   A, B, C complete the work in 60, 40, 120 days respectively

7. Let A , B, C take x, y, z days respectively
   $1/x + 1/y = 1/30$
   $1/y + 1/z = 1/24$
   $1/z + 1/x = 1/20$
   Solving these equations simultaneously,
   $1/x + 1/y + 1/z = 1/16$
   A, B, C together complete a job in 16 days
   In ten days 10/16 of the job is completed ,6/16 of the job remains
   x = 48 i.e. A completes 1/48 of the job in one day
   A completes 6/16 of job in (6/16)48 = 18 days

8. Ram completes 3/5 of work in 9 days
   ∴He completes the entire work in (5/3)9 = 15 days
   Ram & Shyam together complete 2/5 work in 4 days. Together they complete the entire work
   in (5/2)4 = 10 days
   Shyam alone completes 1/10 - 1/15 = 1/30 work in a day.
   Shyam takes 30 days to complete the work alone .

9. Mohan does 1/12 of work in a day, Sohan does 1/20 of work in a day
   In 4 days Mohan does 4(1/2) = 1/3 of work, 2/3 of work remains
   Mohan & Sohan together complete 1/12 + 1/20 = 4/30 of the work in a day
   They complete 2/3 of the work in 2/3(30/4) = 5 days

10. Let Ram & Shyam take x & y days respectively to finish the job.
    $1/x + 1/y = 1/8$
    In 3 days 3/8 of the work is done, Shyam completes 5/8 of the work in 15 days
    He completes the job in (8/5)15 = 24 days
    y = 24   x = 12
    Ram takes 12 days to finish the job

11. A & B take 45 & 40 days respectively
    Let A leave after x days
    $x/45 + (x + 23)/40 = 1$
       x = 9 days

12. A, B, C take 120, 150, x days to complete the job
    A works for 80 days, B works for 20 days, C works for 48 days
    $80/120 + 20/150 + 48/x = 1$
    x = 240
    C takes 240 days to complete the work

13. Let A take x days to do a piece of work, C takes 2x days to do the same work, Let B take y
    days to do the same work

$1/x + 1/y = 1/12$
$1/2x + 1/y = 1/15$
Solving these equations simultaneously, $y = 20$
B can complete the work in 20 days

14. A, B, C complete the work in x, y, z days respectively
$1/x + 1/y = 1/40$
in 10 days, 1/4 of work is done
A, B, C together complete 3/4 of the work in 20 days
A, B, C together complete the entire work in 80/3 days
$1/x + 1/y + 1/z = 3/80$
$1/z = 3/80 - 1/40$
$= 1/80$
C completes a job in 80 days
B completes the job in $80(3/2) = 120$ days
A completes the job in 60 days

15. Let Alok, Arun & Ashok take 2x, 3x, 6x days to finish the work
Together they complete in 1 day
$1/2x + 1/3x + 1/6x = 1$
$x = 1$
Ashok, Arun & Alok take 6, 3, 2 days to finish the work.

16. A, B, C complete 1/12, 1/15, 1/24 of the work respectively
Let A, B, C get Rs. x/12, Rs. x/15, Rs. x/24 respectively
$x/12 + x/15 + x/24 = 920$
$x = 4800$
A, B, C get Rs. 400, Rs. 320, Rs. 200 respectively

17. No. of days worked = 6
A does $6/16 = 3/8$ of the work
B does $6/24 = 1/4$ of the work
C does $1 - (3/8 + 1/4) = 3/8$ of the work
Let A, B, C get Rs. 3x/8, Rs. x/4, Rs. 3x/8 respectively
$3x/8 + x/4 + 3x/8 = 40$
$x = 40$
A, B, C get Rs. 15, Rs. 10, Rs. 15

18. Let the three taps empty the cistern in 6, 9 and x hours
$1/6 + 1/9 + 1/x = 1/3$
$x = 18$
The third tap empties the cistern in 18 hours

19. A & B take 75/2 & 45 minutes respectively
The first pipe remains open for 30 minutes
Let the second pipe remain open for x minutes
$30(2/75) + x/45 = 1$
$x = 9$ minutes

20. A & B fill the cistern in 12 & 20 minutes respectively
Let C empty the cistern in x minutes
$1/12 + 1/20 - 1/x = 1/120$
$x = 1/8$
C empties 1/8 of the cistern in 1 minute
C empties 15 gallons per minute
1/8 of cistern is 15 gallons
Capacity of the cistern is $(15)8 = 120$ gallons

21. Let the pipes A & B remain open for x minutes
The pipe C remains open for (x - 7) minutes
$x(1/45 + 1/36) - (x - 7)(1/30) = 1$
x = 46 min.
The cistern gets filled in 46 minutes

22. Let the third tap empty the cistern in x minutes
$50(1/60 + 1/75 - 1/x) = 1$
x = 100 minutes

23. Pipes A & B fill the bath in 20 & 30 minutes
A & B together fill $1/20 + 1/30 = 1/12$ of the tank in a minute
A & B take 12 minutes to fill the tank
Let the third pipe empty the tank in x minutes
$12(1/20 + 1/30 - 1/x) + 3(1/20 + 1/30) = 1$
x = 48 min.

24. Let A work for x days   B works for x - 1 days   C works for x - 1/2 days
$x/6 + (x - 1)/8 + (x - 0.5)/10 = 1$
x = 3 days
The work is finished in 3 days

25. 3 Men = 5 Women
6 Men + 5 Women = 6 Men + 3 Men = 9 Men
3 men finish a work in 12 days
Let 6 men & 5 women i.e. 9 men finish the work in x days
$9x = 3(12)$
x = 4 days

26. Let A & B together get 19/23 of the total B & C together get 8/23 of the total
$A + B = (19/23)(529) = 437$
$B + C = (8/23)(529) = 184$
$A + B + C = 529$
Solving these two equations simultaneously,
A = Rs. 345

27. The first man does 3/5 of the work, Second man does 3/8 of the work
The first man should get $(240)(3/5) = Rs. 144$
The second man should get $(240)(3/8) = Rs. 90$
The boy & girl together get Rs. 6
Let the boy get Rs. 2x, the girl get Rs.x
$2x + x = 6$
x = 2
The boy gets Rs. 4, the girl gets Rs.2

28. Suppose, the contractor's capital is C, the weekly wages per man are M and the weekly subsidy is S. Then, we have C = 52 (42 M – S) and C = 13 (60 M – S). Solving these two equations, we get, S = 36 M and C = 312 M. Let N be the number of men that can be maintained for 26 weeks. So, C = 26 (NM – S). From the three equations, we can conclude that N = 48.

29. A fills up 1/6 tank in a minute
B fills up 1/7 tank in a minute

Minute	Quantity	Total	
First	1/6	1/6	
Second	1/7	13/42	
Third	1/6	20/42	
Fourth	1/7		26/42

| Fifth | 1/6 | 33/42 |
| Sixth | 1/7 | 39/42 |

In six minutes 39/42 of the tank gets filled
3/42 remains empty Pipe A fills the remainder up in (3/42)6 = 3/7 minutes
Total time taken = 6 + 3/7 minutes = $6\frac{3}{7}$ minutes

30. Let the cistern be emptied in x minutes
1/2.5 - 1/x = 1/3 $\Rightarrow$ x = 15 hours
The entire cistern gets emptied in 15 hours. $\therefore$ 1/2 cistern gets emptied in 7.5 hours
After that rate gets doubled. i.e. the remaining half gets emptied in 3.75 hours
Total time taken to empty the cistern is
7.5 + 3.75 = 11.25 hours i.e. 11 hrs. 15 min.

31. Let the waste pipe empty the tank in x minutes
1/10 + 1/12 - 1/x = 1/15
x = 60/7 min. = 8 minutes 34 seconds

32. Let the full flow of water begin after x minutes
Initially: pipe 1 fills (5/6)(1/30) = 1/36 of the tank in a minute
pipe 2 fills (9/10)(1/36) = 1/40 of the tank in a minute
After the clogging is removed
pipe 1 fills 1/30 of tank in a minute
pipe 2 fills 1/36 of tank in a minute
x(1/36 + 1/40) + 15.5(1/30 + 1/36) = 1 $\Rightarrow$ x = 1 min.

33. Let the first tap remain open x minutes after the 2nd tap is closed
20(1/60 + 1/80) + x/60 = 1 $\Rightarrow$ x = 25
Total time taken to fill the cistern = 20 + 25 = 45 minutes

34. Let the waste pipe empty the bath in x minutes
Cold water pipe & hot water pipe fill 1/10 + 1/15 = 1/6 of tank in a minute
They fill the tank in 6 minutes
6(1/10 + 1/15 - 1/x) + 4(1/10 + 1/15) = 1 $\Rightarrow$ x = 9 minutes

35. Let the daily ration of water be x gallons
200x + 200(10) = 180x + 180(16) $\Rightarrow$ x = 44
the total quantity of water in the supply is 200(44) + 200(10) = 10800 gallons

36. In order to drip 2080 drops, time required = 2080 x 3/2 seconds. This is the time required to completely fill the given hemispherical vessel once. In 13 hours the given vessel can be filled (13 x 60 x 60) / (2080 x 3/2) = 15 times.

37. In 1 hour first machine will copy 1500/8 = 375/2 workbooks and the second machine will copy 1500/12 = 125 workbooks. Thus, working together two machines will copy (375/2 + 125) = 625/2 workbooks in 1 hour. So in order to copy 1500 workbooks time required will be 1500/(625/2) = 4.8 hours.

38. A is thrice as good a workman as B this means A is 3 times more efficient than B or in other words time taken by B will be 3 times of the time taken by A . If time taken by A is 't' days, time taken by B will be '3t' days . B takes (3t – t) = 2t days less than A but this value is given as 60. So 2t = 60 i.e. t = 30 days & 3t = 90 days. Time taken by A & B working together will be 1/ (1/30 + 1/90) = 90/4 = 22 ½ days.

39. Time taken by pipe A to fill the tank = 4 minutes.
Let us assume time taken by pipe B to completely fill the tank is 'x' minutes.
Then time taken by pipes A & B to together fill the tank = 1/ (1/4 + 1/x) minutes.
But this value is given as 3 minutes, so 1/ (1/4 + 1/x) = 3 i.e. x = 12 minutes.

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40. In 1 hour pipes A & B will together supply (12 + 15) = 27 litres of water.
In 6 hours pipes A & B will together supply (27 x 6) = 162 litres of water.
Now pipe C can empty the tank in 12 hours, this means that water going out through C in 12 hours will be equal to the total volume of the tank 'V' litres. Therefore volume of water going out through pipe C in 6 hours will be V/2.
Now if all the 3 pipes are opened simultaneously the tank is completely filled in 6 hours.
From this we can conclude that
$$V = 162 - V/2 \Rightarrow V = 162 \times 2/3 \Rightarrow V = 108 \text{ litres.}$$

Exercise 5

1. Sum of seven numbers = S = 235
Sum of first three numbers = S_1 = 23(3) = 69
Sum of last three numbers = S_2 = 42(3) = 126
Fourth number = $S - S_1 - S_2$ = 40

2. Cost of twelve horses = 12(250) = Rs. 3000
Cost of remaining eighteen horses = Rs. 9000
Average cost of eighteen horses = 9000/18 = Rs. 500

3. Sum of the total marks obtained by 500 students = 500×45 = 22500
Sum of the total marks obtained by top 150 students = 150×75 = 11250
Sum of the total marks obtained by last 150 students = 150×25 = 3750
∴ Average of the remaining 200 students = (22500 - 11250 - 3750) / 200
= 37.5

4. Let the average be x Sum of weights = 40x
Weight of the new boy = y
New sum = 40x - 100 + y
New average = (40x - 100 + y)/40
(40x - 100 + y)/40 = x + 1/4
y = 110 kg

5. Mon. + Tue. = 2(56) = 112
Mon. + Tue. + Wed + Thu. = 4(62) = 248
Wed/Thu = 15/19
Solving these equations simultaneously, Wed = 60   Thu.. = 76

6. Average of six players = x   Average of seven players = x + 10
Sum of scores of six players = 6x
(6x + 112)/7 = x + 10
x = 42

7. Original number of boys in class = x   Sum of weights of these boys = 43x
New sum = 43x + 42 + 36.5 + 39 + 42.5   = 43x + 160
Average = (43x + 160)/(x + 4)
(43x + 160)/(x + 4) = 42.5
x = 20

8. Average age of men = x   Sum of ages of 8 men = 8x
Sum of ages of women = w
New sum = 8x - 44 + w
New average = (8x - 44 + w)/8 = x + 2
w = 60
Average age of the women = 60/2 = 30

9. Sum of 8 results = 8(20) = 160   sum of first two results = 2(15.5) = 31

Sum of next three results = 3(21 1/3) = 64
The sixth result be x   Seventh result = x + 4   Eighth result = x + 7
31 + 64 + x + x + 4 + x + 7 = 160
        x = 18
last result = 18 + 7 = 25

10.   The number of clerks be x
Salary of officers = 600(12) = Rs. 7200
Salary of clerks = Rs. 84x
Total salary = 7200 + 84x

Average salary = (7200 + 84x)/(x + 12) = 90
x = 1020

11.   Sum of ages of 7 children = 7(12) = 84   Sum of ages of remaining 6 children = 78
Sum of ages after 5 years = 78 + 30 = 108
Average after 5 years = 108/6 = 18 years

12.   Time taken to travel from A to B = 60/20 = 3 hr.
Time taken to travel from B to A = 60/30 = 2 hr.
Total time to cover 120 km = 5 hr.
Average speed = 120/5 = 24 kmph

13.   Time taken to travel :
1st (1/20) km = (1/20)(1/9) = (1/180) hr.
2nd (1/20) km = (1/20)(1/12) = (1/240) hr.
3rd (1/20) km = (1/20)(1/18) = (1/360) hr. 4th (1/20) km = (1/20)(1/36) = (1/720) hr.
Total time taken = (1/72) hr.
Average speed for 1/5 of km = (1/5)/(1/72) = 14.4 kmph
18.75 is a wrong answer as it is being calculated by taking simple average of all the speeds.

14.   In one hour 1st train travels 27 km,   2nd train travels 41 km
The train lags by 14 km
14 km can be covered in 14/41 = 0.342 hr. = 20.488 min.

15.   Total number of words = 180(12)(16)   Let the total number of pages in the book be x
180(12)(16) = x(18)(20)
 x = 96

16.   The persons attending on the 1st, 2nd & 3rd day be 2x, 5x, 13x reps.
Total collection = 30x + 37.5x + 32.5x = 100x
Average charge = (100x)/(2x + 5x + 13x) = Rs. 5

17.   15/4 tonnes enter in 12 minutes
18.75 tonnes enter in one hour
12 tonnes are thrown out in one hour
Effectively (18.75 - 12) = 6.75 tonnes enter in one hour
60 tonnes enter in 8.889 hours
Average rate of sailing = 40/8.889 = 4 $\frac{1}{2}$ kmph = 4.5 kmph

18.   The mixture contains (10/100)70 = 7 litres of water
It contains (70 - 7) = 63 litres of wine
Let x litres of water be added
(7 + x)/(70 + x) = (37/100)
 x = 30 litres

19.   S.P. of new liquid = Rs. 13.75/litre
C.P. of new liquid = Rs. (100/125)13.75 = Rs. 11

Cost of water = Rs. 0/litre

```
                     R₁              R₂
Costs                12 ＼          ／0
Cost of mixture          ＼11／
Proportion           11              1
```
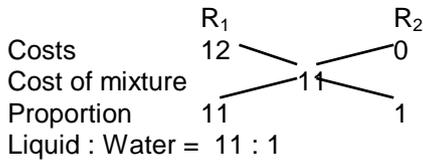
Liquid : Water = 11 : 1

20. Cost per kg of resulting mixture = 146.40/15 = Rs. 9.76

```
                     R₁              R₂
Costs                10.40 ＼      ／8.80
Cost of mixture          ＼9.76／
Proportion           0.96 ／      ＼0.64
```
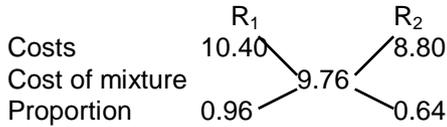
The two varieties should be mixed in the proportion 96 : 64 = 3 : 2

In 15 kg of mixture there should be 9 kg & 6 kg respectively of the two varieties

21.
```
                     Liquid          Water
Weight               1.340 ＼      ／1
Weight of mixture        ／1.270＼
Proportion           0.270           0.070
```

Liquid : Water = 27 : 7

let the quantities of the liquid & water be 27x & 7x in a litre of mixture

$27x + 7x = 1$

$x = 1/34$

Quantity of liquid = 27/34 litres

water = 7/34 litres

22. If the cost prices of each kind of sugar be Rs. 100 then the selling price of one kind is Rs. 107 (7% profit), Selling price of the other kind is Rs. 117 (17% profit)

Selling price of the mixture = Rs. 110

```
                     R₁              R₂
S.P. of each     117 ＼          ／107
Mixture              ＼110／
Proportion           3               7
```
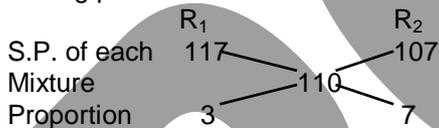
The two kinds of sugar are to be mixed in the proportion 3:7

Let the quantity of each in 100 kg be 3x & 7x

$3x + 7x = 100$

$x = 10$

The quantity of each in 100 kg is 30 kg & 70 kg

23. Let the amounts invested in 5% & 6% stock be Rs. x & Rs. (11000 - x) respectively

Dividend received from each type is

5x/120, 6(11000 - x)/125

$5x/120 + 6(11000 - x)/125 = 490$

$x = 6000$

Amount invested in the 5% stock = Rs. 6000

24. Let the number of workers be x

$85x + 525(15) = 95(x + 15)$

$x = 645$

25. S.P. of mixture = Rs. 14

C.P. of mixture = Rs. 14(100/120) = Rs. 35/3

Let C.P. of the two kinds be Rs. x & Rs. 2x respectively .

```
                     R₁              R₂
Costs                2x ＼        ／x
Cost of mixture          ／35/3＼
Proportion           35/3 - x        2x -35/3
```
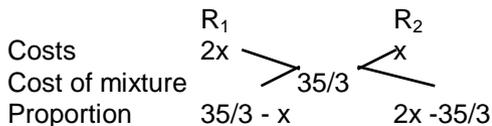
$(35/3 - x)/(2x - 35/3) = 2/3$

x = 8 1/3
The C.P. of the two kinds of tea are
Rs. 8 $\frac{1}{3}$ & Rs. 16 $\frac{2}{3}$

26. The mixture contains 3/4 part milk & 1/4 part water
Let quantity of mixture withdrawn be x
Quantity of milk in the new mixture = 3/4 - 3x/4
Quantity of water in the new mixture = 1/4 - x/4 + x
3/4 - 3x/4 = 1/4 - x/4 + x
 x = 1/3
1/3 part of the mixture should be withdrawn.

27. Mixture 1 contains wine and water in a ratio as
Wine : Water = 3:2
Let the quantity in 3 gallons be 3x & 2x respectively
5x = 3,   x = 3/5
Quantity of wine & water in 3 gallons is 9/5 gallons & 6/5 gallons
Let the quantity of Mixture 2 be p gallons
Wine : Water = 4 : 5
Let the quantity in p gallons be 4y & 5y respectively
4y + 9y = p, y = p/9
Quantity of wine & water in p gallons is 4p/9 & 5p/9
Quantity of wine in resulting mixture = 9/5 + 4p/9
Quantity of water in resulting mixture = 6/5 + 5p/9
9/5 + 4p/9 = 6/5 + 5p/9
p = 5.4
Quantity of second mixture = 5.4 gallons

28.   Alloy 1   Alloy 2
Quantity of tin   10   4
Quantity in mixture   9
Proportion   5   1
The two alloys should be mixed in the ratio 5:1
Amount of copper in the mixture = 5(90/100 + 1(93/100) = 5.43
% of copper in the mixture = [5.43/(5+1)]100 = 90.5%

29.   Silver   Copper
Alloy1 5   1
Alloy2 7   2
Quantity of silver in Alloy1 = 5/6   Quantity of silver in Alloy2 = 7/9
Quantity of silver in mixture = 80/100 = 4/5
                Alloy 1   Alloy 2
Quantity of silver 5/6   7/9
Quantity in mixture   4/5
Proportion   1/45   1/30
The two alloys should be mixed in the ratio (1/45)/(1/30) = 2/3

30. A/Q = [1 - q/Q]^n
A = quantity of wine in final mixture   Q = volume of cask
q = quantity removed   n = number of times the operation is repeated
A/Q = 16/(16 + 9) = 16/25, q = 9, n = 2
16/25 = [1 - 9/Q]^2
Q = 45 litres

31. Let the quantity of the mixture be x
Quantity of spirit = 18x/100   Quantity of water = 82x/100
Quantity of spirit in 8 litres of mixture = 1.44 litres

Quantity of spirit in the new mixture = 18x/100 - 1.44
(18x/100 - 1.44)/x = 15/100
        x = 48 litres

32.   Gold   Copper
                           19   9
        Resultant   15
        Proportion   6   4
        Gold : Copper = 6 : 4 = 3 : 2
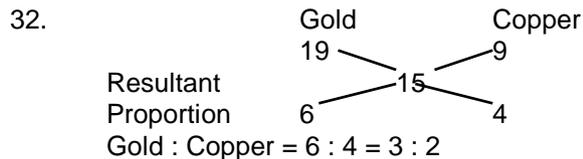
33.   Since he gains $11\frac{1}{9}$ %. His gain is by sells of water in the mixture only.
        $\therefore$ The percentage of water to be added = $11\frac{1}{9}$ % = 100/9 %
        = 1/9 per litre of milk.
        Hence water in 1 litre of mixture = 1/10 litre

34.   S.P. of mixture = Rs. 12.25
        C.P. of mixture = 12.25(100/116 2/3)   = Rs. 10.50

        Let the C.P. of the two liquids be Rs. x & Rs. X + 4 respectively
                           Liquid 1   Liquid 2
        Price   x + 4   x
        Resultant   10.50
        Proportion 10.5   - x   x - 6.5
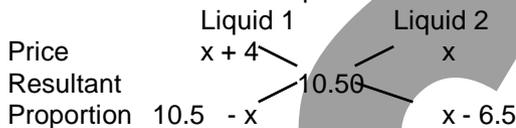
        (10.5 - x)/(x - 6.5) = 5/3
                x = 8
        Prices of the two liquids = Rs. 8 & Rs. 12

35.   Let the mixture contain 4x kg of brandy & (x) kg of water
        New mixture contains 4x kg of brandy & (x + 1) kg of water.
        4x/(x + 1) = 3/1.   x = 3
        Original mixture contained 12 kg of brandy & 3 kg of water

36.   Let the volumes of all the glasses be 10 litres
                           Spirit   Water   Total
                Glass 1   4   6   10
                Glass 2   30/7   40/7   10
                Glass 3   40/9   50/9   10

        Mixture   802/63 1088/63   30
        Spirit : Water = 802 : 1088 = 401 : 544

37.   Total   Petrol Spirit
        Liquid 1   20   8   12
        Liquid 2   21   9   12
        Liquid 3   x   4x/9   5x/9

        Mixture   41+x 17+(4x/9) 24+(5x/9)

        [17 + (4x/9)]/[24 + (5x/9)] = 29/39
                x = 27
        Quantity of third liquid taken = 27 litres

38.   Proceed as in Problem No. 36.

39. Quantity of Spirit   30   12
Resultant   18
Proportion   6   12

The proportion of the two liquids in the resultant is 6 : 12 i.e. 1 : 2
The butler stole 2/3 of the butt as it is the amount of 30% wine replaced by 12% wine

40. Let the weight of the gold be x gm   Weight of the other metal = (18 - x) gm
Let the price of the other metal be Rs. y /gm
$7.20x + y(18 - x) = 74$
$7.20(18 - x) + yx = 60.10$
Solving these two equations simultaneously,   x = 10 , y = 0.25
Quantity of the other metal = 18 - 10 = 8 gm
Price of the other metal = Rs. 0.25/gm

Exercise 6

1. 90 kmph = 90(1000/3600) = 25 m/s

2. $16\,^2/_3$ m/s = 50/3 m/s = (50/3)(3600/1000) = 60 kmph

3. Let the distance covered be d
Speed = distance/time
d = 3(8) = 24 km
Walking at 4 kmph, time taken = 24/4 = 6 hours

4. Let the distance traveled be x
Time taken at 4 kmph = x/4 hr. = 60x/4 min. = 15x min.
Time taken at 5 kmph = x/5 hr. = 60x/5 min. = 12x min.
difference in time = 2.5 - (-5) = 7.5
15x - 12x = 7.5
x = 2.5 km

5. Distance traveled on foot = x km, ∴distance traveled on bicycle =(61 - x) km
time taken to travel on foot = x/4
time taken to travel on bicycle =(61-x)/9
x/4 + (61 - x)/9 = 9
x = 16 km

6. Speed of the tonga be x kmph
i. time taken = 16/4 + 24/x = 4 + 24/x hr.
ii. Time taken =16/x + 24/4 = 6 + 16/x hr.
6 + 16/x - 4 - 24/x = 1
        x = 8 kmph

7. Let the total distance traveled be 6x km
i. 2x traveled at 10 kmph, 2x traveled at 9 kmph, 2x traveled at 8 kmph
time taken = (2x/10) + (2x/9) + (2x/8) = 2x(121/360) = 121x/180
ii. 3x traveled at 10 kmph, 3x traveled at 8 kmph
time taken = (3x/10 + (3x/8) = 3x(18/80) = 27x/40
27x/40 - 121x/180 = (1/2)(1/60)
        x = 3
total distance = 6(3) = 18 km

8. Let the speeds of the two trains be s & (s + 8) kmph respectively
They approach each other with a relative speed of s + s + 8 = 2s + 8
162/(2s + 8) = 6
  s = 9.5 kmph

The speeds of the two trains are 9.5 kmph & 17.5 kmph

9. Let the speed of the train be s kmph, The train approaches the man at the rate of s + 5 kmph
    $(100/1000)[1/(s + 5)] = 7.2/3600$
    $s = 45$ kmph

10. Let the speed of the train be s kmph, the train approaches the man at relative speed of $(s - 4)$ kmph
    $(300/1000)[1/(s - 4)] = 30/3600$
    $s = 40$ kmph
    the distance to the station is $40(15/60) = 10$ km
    The man reaches the station in $10/4 = 2.5$ hr.

11. The mail train approaches the goods train at a relative speed of $80 - 32 = 48$ kmph
    The goods train has covered a distance $= 32(6) = 192$ km in 6 hr.
    To cross completely, the mail train has to cover a distance $= 192+(130/1000) = 192.13$ km
    time taken $= 192.13/48 = 4$ hr. 9 ¾ seconds. And distance $= (192.13/48)*80 = 320.22$ km.

12. In one hour the earlier rider covers a distance $= 10$ km
    The distance between the two $= 80 -10 = 70$ km
    the two riders approach each other with a relative speed of $10 + 25 = 35$ kmph
    the time taken $= 70/35 = 2$ hr.
    they meet each other at $(8 + 2) = 10$ a.m.

13. Length of the train $= 24(60)+60 = 1500$ m.
    Total distance to be covered $= 1.5 + 1.5 = 3$ km
    Time taken $= 3/60 = (3/60)60 = 3$ minutes

14. Let the distance between the two towns be x km.
    i. time taken $= x/60$ hr.
    ii. time taken $= x/40$ hr.
    $x/40 - x/60 = (16 - 10)/60 = 6/60$
    $x = 12$ km

15. Let the length of the first train be l then the length of the second train is l/2
    The two trains approach each other at relative speeds of $(60 + 90) = 150$ kmph
    Time taken to pass each other
    $(3l/2)/150 = (9/2)(1/3600)$
    $l = 0.125$ km $= 125$ m
    let the length of the tunnel be x km
    time taken by the first train to pass through it
    $(x + 0.125)/60 = [4(60) + 37.5]/3600$
    $x = 4.5$ km i.e. 4 km 500 metres

16. The two trains move at a relative speed of $(75 - 45) = 30$ kmph
    The first train passes the passenger in
    $(150/1000)(1/30)3600 = 18$ seconds
    Let the length of the second train be l metres
    $(l + 150)/ [(1000)(30)] = 30/3600$
    $l = 100$ m

17. The length of the faster train be l m.
    The two trains move at a relative speed of $(72 - 54) = 18$ kmph
    $(192 + l)/[(1000)(18)] = 1.5/60$
    $l = 258$ m
    If the trains move in opposite directions they approach each other at a relative speed of $(72 + 54) = 126$ kmph
    The time taken to pass completely $(192 + 258)/[(1000)(126)]$ $3600= 12$ 6/7 sec

18. The two trains move relative to each other at a speed of (50 - 30) = 20 kmph
Let the length of the faster train be l m
Time taken $3600l/[(1000)(20)] = 18$
$\therefore l = 100$ m

19. Let the speed upstream & speed downstream be S_u & S_d respectively
$S_u = (1000/30)(1/60) = 5/9$ m/s
$S_d = (1000/10)(1/60) = 5/3$ m/s
Let the speed of the boat & the current be x m/s & y m/s respectively
$x + y = 5/3$
$x - y = 5/9$
Solving these two equations simultaneously,
$y = 5/9$ m/s $= 5/9 (3600/1000) = 2$ kmph
The speed of the current = 2 kmph

20. Let the speed upstream & speed downstream be S_u & S_d respectively
$21/S_u + 21/S_d = 10$
$7/S_d = 3/S_u$
Solving these two equations simultaneously,
$S_d = 7$ kmph $S_u = 3$ kmph
Let the speed of the boat & the current be x m/s & y m/s respectively
$x + y = 7$
$x - y = 3$
$x = 5$ kmph $y = 2$ kmph

21. Total time taken = 100/100 + 100/200 + 100/300 + 100/400 = 1 + 1/2 + 1/3 + 1/4 = 25/12 hours
Average speed = (400/25)12 = 192 kmph

22. Speed1 = 9 km per day. Speed2 = 15 km per day
In six days both travel 54 km & 90 km respectively
The slower man is 36 km behind
Now the speeds become 18 km per day & 15 km per day
They come closer at a relative speed of (18 - 15) = 3 km per day
They cover 36 km in (36/3) = 12 days
Total time taken = 6 + 12 = 18 days

23. $S1/S2 = \sqrt{t1}/\sqrt{t2}$
$S1/S2 = \sqrt{20}/\sqrt{5} = \sqrt{4}/1 = 2/1$
One train travels twice as fast as the other.

24. Let the speed of the bus be s kmph
The car approaches the bus at a relative speed of (38 - s) kmph
Total distance traveled = 40 + 60 = 100 m = 1/10 km
Time taken = 1/10(38 - s) = 20/3600
s = 20 kmph

25. Length of the train = l m
Speed of the train = s kmph
Relative speeds = s - 2 kmph & s - 4 kmph
$l/1000(s - 2) = 9/3600$
$l/1000(s - 4) = 10/3600$
Solving these two equations simultaneously,
$l = 50$ m $s = 22$ kmph

26.   In 15 seconds the two cars travel
$$36(15/3600) = 0.15 \text{ km}$$
$$48(15/3600) = 0.2 \text{ km}$$
The distance between them is
$$\sqrt{[(0.2)^2 + (0.15)^2]} = 0.25 \text{ km} = 250 \text{ m}$$

27.   Let the speed of the carriage be s kmph $\therefore$ Relative speed = (s - 3) kmph
$$100/1000(s - 3) = 4/60$$
$$S = 4.5 \text{ kmph}$$

28.   In two minutes the monkey climbs (12 - 5) = 7 metres
It will climb 56 metres in 16 minutes
The remaining 7 metres it will climb in 7/12 minutes = (7/12)60 = 35 seconds
Total time taken = 16 minutes 35 seconds

29.   In one hour the train covers 50 km at 50 kmph, 40 km at 40 kmph
The difference in distance covered = 10 km
It would have covered this distance in 10/50 = 0.2 hours = 12 minutes
It stops for 12 minutes in an hour

30.   In one minute the hare moves 12/60 = 1/5 km = 200 m
Total distance between the two = 200 + 100 = 300 m
Relative speed = 16 - 12 = 4 kmph
Time taken = 300/1000(4) = 3/40 hours = (3/40)60 = 4.5 minutes
The distance = 12(4.5/60) = 0.9 km = 900 m
Total distance covered = 900 + 200 = 1100 m

31.   Let S_1 = 5 kmph, S_2 = 20 kmph
$\therefore$ $S_1 / S_2 = 5/20 = 1/4$
Let the corresponding times be t_1 and t_2
Since distance is constant, $S_1 / S_2 = t_2 / t_1$ $\therefore$ $t_1 / t_2 = 4/1$
If the usual time is t , t_1 = (t+15) minutes, t_2 = (t-30) minutes
$\therefore t_1 - t_2 = 45$

        $t_1 : t_2 : (t_1 - t_2)$
        4 : 1 :   3
        ?   45

$\therefore t_1 = 60$ minutes = 1 hour
but $d = S_1 \times t_1$
$\therefore \Box \Box d = 5 \text{ kmph} \times 1 \text{ hour}$
Distance between the two roads is 5 km.

32.   Let the time in which the bars become of same length be t
Let the speed with which wires are drawn = s_1 and s_2
$\therefore$ $s_1 = 14/7 = 2$ cm/minutes, $s_2 = 20/5 = 4$ cm/min.
Let the lengths of bars used up in drawing wires be l_1 and l_2.
$l = S \times t$
$\therefore$ $l_1 = (2 \times t)$ cm , $l_2 = (4 \times t)$ cm
But, $14 - l_1 = 20 - l_2$
$\therefore, 14 - 2t = 20 - 4t$
$\therefore, t = 3$ minutes.

33.   Both men travel the same total distance d
If their times for the above distance are t_1 and t_2, t_1 = 110 mins and t_2 = 132 minutes.
$\therefore s_1 = d / t_1 = d / 110$, $s_2 = d / t_2 = d / 132$
Let the two men meet t minutes from 2.20 p.m.
$\therefore$ the first travels for t minutes and second travels for (t- 20) minutes before they meet
Total distance $d = s_1 t + s_2 (t-20)$

$$\therefore d = \frac{d}{110} \times t + \frac{d}{132}(t-20)$$

$$\therefore 1 = \frac{t}{110} + \frac{(t-20)}{132}$$

$\therefore$ t = 69.09 minutes = 1 hour 9.09 minutes

$\therefore$ They meet at 3.29 p.m.

34. Let speeds by air, rail, and tonga be s_a, s_r and s_t respectively.

If $s_t = x$, $s_r = 4x$, $s_a = 40x$

Total time = 3 1/2 hrs = 7/2 hrs

$$\therefore \frac{7}{2} = \frac{300}{s_t} + \frac{60}{s_r} + \frac{30}{s_a}$$

$$= \frac{300}{40x} + \frac{60}{4x} + \frac{30}{x}$$

$\therefore$ x = 15, 40x = 600

Speed by air is 600 kmph.

35. In 10 kms. of traveling it can save 5 minutes.

$\therefore$ to save 40 minutes it has to travel $40 \times 10 / 5 = 80$ km.

$\therefore$ Total distance traveled = 80 + 25 = 105 km.

36.

Let A be the house and B be the workplace.

The workplace is 8 km away. Had he walked all the way at 4kmph he would have taken 2 hrs to reach his workplace, which would be just in time.

Therefore, total time = 2 hrs

Referring to the above diagram, the total time is made up

    i. walking from A to C for 5 minutes = 5/60hrs = 1/12 hrs.

      now, 4 km in 1hour

     $\therefore$ 1/3 km in 1/12 hrs

     Distance AC = 1/3 km

   ii. If x is the increased speed, time for walking from C to A = AC / x hrs = 1/3x .

   iii. Walking from A to B, time taken = AB/x = 8/ x hrs.

$$2 = \frac{1}{12} + \frac{1}{3x} + \frac{8}{x}$$

$\therefore$ $x = 4\,^8/_{23}$ kmph

37.
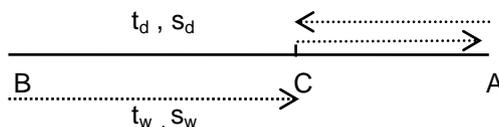
Let the speed driving be s_d and the speed walking be s_w

         $\therefore$ **s_w = 3 kmph**

Referring to the above diagram, if A represents the home and B represents the school, the father starts at his usual time but meets his son on the way at C. So, in going from A to C and back to A he saves the time he would have used commuting from C to B and back to C. Since they reach back 15 minutes earlier than usual, 15 minutes must be that time saved. Therefore time taken to drive from C to B and back is 15 minutes. Therefore time taken to drive the distance BC = t_d = 15/2

$$\therefore \quad t_d = \textbf{7.5 minutes}.$$

Dinky starts 1 hr earlier than usual. Had he moved at driving speed, i.e. speed of the car, he would have reached 1 hr earlier. But he reaches only 15 minutes earlier. Therefore he loses 45 minutes ,i.e. by traveling partly by foot and partly by car, he takes 45 minutes longer than if he would have traveled the whole distance by car.

Therefore time taken to walk the distance BC = t_w = 7.5 + 45 = 52.5 minutes.

$$\therefore \quad t_w = \textbf{52.5 minutes}$$

But for a constant distance BC, $S_d / S_w = t_w / t_d$

$$\therefore \quad \frac{s_d}{3} = \frac{52.5}{7.5}$$

$$\therefore s_d = 21 \text{ kmph}$$

38. Use the shortcut formula for any constant distance

$$\frac{S_1}{S_2} = \frac{\partial S_1}{\partial S_2} \; \frac{\partial t_2}{\partial t_1}$$

where if S is the normal speed and t is the normal time,

S1 and S2 are two different speeds at which the same distance is traveled

∂S_1 and ∂S_2 are the respective deviations from normal speed

∂t_1 and ∂t_2 are the respective deviations from normal time

Therefore, in this case

$$\frac{S_1}{S_2} = \frac{2 \times 30}{4 \times 10}$$

$$= \frac{3}{2}$$

Now, $S_1 = (S + 2)$ kmph and $S_2 = (S - 4)$ kmph

$\therefore S_1 - S_2 = 6$ kmph

$$\begin{array}{ccc} S_1 : & S_2 : & (S_1 - S_2) \\ 3 : & 2 : & 1 \\ ? : & ? : & 6 \end{array}$$

$\therefore S_1 = 18$ kmph and $S_2 = 12$ kmph $[S = S_1 - 2$ or $S = S_2 + 4]$

$\therefore S = 16$ kmph

39. Let the initial speed be s_1 and the later speed be s_2
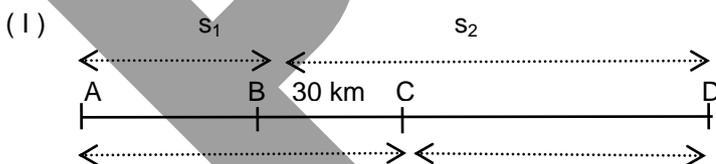
Referring to the above diagram, where both condition (I) and condition (II) have been represented,

When speed decreases by 1/2 the usual speed , then for any constant distance, time would be 2 times the usual.

$\therefore$ it would become once more than the usual time.

So in condition (I), once the usual time taken to travel CB = t_{CB} = 45 minutes

and in condition (II), once the usual time taken to travel DB = t_{DB} = 45 - 15 = 30 minutes

Since in condition (I) and condition (II) the time taken to travel DB is the same,

once the usual time taken to travel CD = t_{CD} = t_{CB} - t_{DB} = 45-30 = 15 minutes.

$\therefore$, the usual time taken to travel CD, t_{CD} = 15 minutes

$\therefore$, the usual speed = CD$/ t_{CD}$ = 30/($^{15}/_{60}$) kmph = 120 kmph

The usual speed is 120 kms.

40. Let the speed of the boat in still water be x kmph.

Speed downstream = $(x + 4)$ kmph

Speed upstream = $(x - 4)$ kmph

Time taken to go 6 km downstream = 6/(x + 4)
Time taken to go 6 km upstream = 6/(x - 4)
6/(x + 4) + 6/(x - 4) = 2 hours
Solving the equation x = 8.

Exercise 7

1. When A runs 1760 metres, B runs 1716 metres.
    When B runs 1320 metres, C runs 1290 metres.
    When B runs 1716 metres, C runs 1290(1716/1320) = 1677 metres.
      A   B   C
    1760   1716   1677
    When A runs 880 metres, C runs 1677(880/1760) = 838.5 metres.
    A beats C by (880 - 838.5) = 41.5 metres.

2. A covers the distance of 2km. in (2 / 20) hour i.e. 360 sec.
    B covers the distance of (2000 - 100) i.e. 1900 m in 360 + 20 i.e. 380 sec.
    B's speed = (1900/380) = 5 m/s = 5 x (3600/1000) = 18 kmph.

3. Let the speeds of A & B be x m/s & y m/s respectively.
    i. A runs 1 km in (1000/x) seconds.
    B runs 960 m in (960/y) seconds.
    So, (960/y) – (1000/x) = 19   (I)
    ii. B runs 1 km in (1000/y) seconds.
     A runs 960 m in (960/x) seconds.
     So, (1000/y) – (960/x) = 30   (II)
    Solving (I) & (II) simultaneously, x = 8 m/s and y = 6.667 m/s.
    From these two results, A runs a km in 125 seconds and B runs a km in 150 seconds.

4. Mohan   Ramesh   Dinesh
    100   80
   100   80
    i.e. When Mohan scores 100, Ramesh scores 80 & when Ramesh scores 100, Dinesh scores 80.
    When Ramesh scores 80, Dinesh scores (80)(80)/100 = 64. Therefore, Mohan scores 100, Dinesh scores 64, i.e. Mohan can give Dinesh 36 points in 100.

5. A & B move at a relative speed of (160 - 120) = 40 m/minute
    A & B come together in 1760/40 = 44 minutes
    A & C move together at a relative speed of (160 -105) = 55 m/ minutes
    A & C come together in 1760/55 = 32 minutes
    A, B & C come together in 352 minutes (LCM of 44 & 32)
    A, B & C come together in 5 hours 52 minutes

6. Whenever A runs 100m, B and C run 96m & 84m respectively. Therefore when B will cover a distance of 100m, C will cover (84/96)100 = 87.5m. Now difference between distance covered by A & B = (100 –87.5) = 12.5m. Time taken by C to cover this 12.5m = 5sec. Therefore speed of C = 2.5m/s and it will take 400 sec. to cover the distance of 1km. We know that time taken by A to cover 100m is same as time taken by C to cover 84m and from this we can conclude that ratio in their speeds is 100 : 84 i.e. 25 : 21. For a constant distance of 1 km. the time taken by A & C will be in the inverse ratio of their speeds i.e. 21 : 25. Hence time taken by A will be (21/25)400 = 336 sec. and C will complete the race 64 sec. after A.

7. Let S be the starting point of the race. A and B run at speeds Sa and Sb. A is the point where A and B meet for the first time and B is the point where A and B meet the second time. When they meet, A and B cover different distances in the same time. So, the ratio of the distances covered is the same as the ratio of their speeds. So, Sa/Sb = 450/(300 + X) = X/750. Solving this equation, we get X =450 m. The length of the track is 450 + 450 + 300 = 1200 m.

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8.	When A runs 1600 m, B runs 1560 m. So, the ratio of the distances covered by A and B is 40:39. When B runs 1200 m, C runs 1140 m. So, the ratio of the distances covered by them is 20:19. In order to compare A and C, we will have to combine the two different scales into a common scale. On the common scale, the ratio of the distances covered by A, B and C is (40 x 20) : (39 x 20) : (39 x 19). Therefore, when A runs 800 m, C will run 741 m. A thus beats C by = (800 – 741) = 59 m in a 800 m race.

9.	A covers a km at the speed of 10 kmph. So, he takes 6 minutes to cover the distance. B will take (6+3) i.e. 9 minutes and C will take 9 + 1 = 10 minutes to cover the same distance. Therefore, C's speed is 1/(10/60) kmph = 6 kmph.

10.	To win the race Arjun will have to cover the distance of 200m while Karna will have to cover the distance of only 195m as Arjun gives Karna 5m. Speed of Arjun is 15kmph so time taken by arjun will be (200 x 18)/(15x5) i.e. 48sec. Arjun beats Karna by 6 sec. so time taken by Karna to cover the distance of 195m will be (48 + 6) i.e. 54 sec. Hence speed of Karna = (195/54)(18/5) = 13kmph.

11.	The ratio of speed of Ram & Shyam is 4:5. Ram runs at a speed of 20/9 m/s and so speed of Shyam will be 25/9 m/s. Suppose, Shyam gives Ram X m in a 1500 m race. Ram, therefore, runs a distance of (1500 - X) m. Now if Shyam continues to run till Ram completes the race, he will cover a distance of [1500 + (25/9)9] = 1525m. Thus in the same time, Ram & Shyam run (1500 –X) and 1525m. respectively.This ratio of distances should be eual to the ratio of speeds. Therefore

	(1500 –X) / 1525 = 4/5 i.e. X = 280m.

12.	When A runs 1000 m, B and C will run 880 m and 960 m respectively. The ratio of distances = the ratio of speeds i.e. 25:22:24. Suppose A, B and C run a distance equal to the LCM of 25, 22 & 24 i.e. 6600m, we can find the time required by them to cover that distance on the ratio scale.

	A	B	C
Speed:	25	22	24
Time:	264	300	275

	The difference in the times for B and C on the ratio scale is 25, while the actual difference is 5 seconds, giving a factor of 1/5. So, the times taken by A B and C to cover a distance of a km each are 52.8 seconds, 60 seconds and 55 seconds respectively. The total time to complete this relay race is167.8 seconds or 2 minutes 47.8 seconds.

13.	A beats B by $7^1/_7$ seconds. A gives B a start of 5 seconds and still beats him by 15 m. Therefore, B takes $7^1/_7$ – 5 = $2^1/_7$ seconds to run 15 m. Therefore, B's speed is 15/(15/7) = 7 m/s. To complete a 400 m race, B takes 400/7 seconds and A takes 400/7 - 50/7 = 50 seconds. Therefore, A's speed is 400/50 = 8 m/s.
	Alternate solution: Let, A and B run at speeds Sa and Sb respectively. In RACE 1, when A runs 400 m, B runs {400 – (5Sb + 15)} m in the same time. Therefore, (400/Sa) = {400 – (5Sb + 15)}/Sb. In RACE 2, (400/Sa) = {(400/Sb) – $7^1/_7$}. Solving the two equations, we get Sa = 8 m/s and Sb = 7 m/s.

14.	Suppose A and B are running at speeds Sa and Sb respectively. If A continues to run till B completes the race, (1000/Sa)+19 = 960/Sb.
	Also, as B gets a start of 40 m, 960/Sa =(1000-30Sb)/Sb
	Solving the two equations, we get Sa = 8 m/s and Sb = 20/3 m/s. Therefore, the ratio of their speeds is 6:5.

15.	Suppose the two bodies need X and (X + 2) seconds to complete one round of the track. The two bodies will meet at a time equal to the LCM of the times they require to complete one round each. It has been given that LCM [X, (X + 2)] = 12 seconds. It can therefore be estimated that X = 4 seconds and (X + 2) = 6 seconds.

16.   When they meet, the faster man has completed 4 ½ rounds, whereas the slower man has completed 3 ½ rounds. Since these distances have been covered in the same time, the ratio of distances = the ratio of speeds = 4.5:3.5 = 9:7.

17.
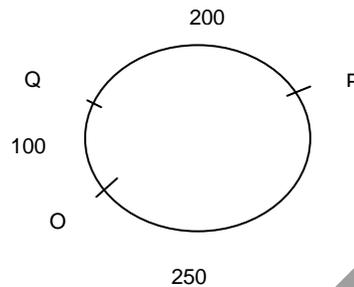
   Suppose Shyam and Ram start from point O. Ram starts when Shyam is at point Q. The distances travelled in the same time give the ratio of speeds. Therefore, Ss:Sr = 200:250 or 4:5. Let their speeds be 4x and 5x respectively. After meeting at point P, Ram has to run 300 m, while Shyam has to run 250 m to come back to the starting point. To do so, Ram will take 300/5x = 60/x units of time and Shyam will take 250/4x = 62.5/x units of time. Therefore, Ram will reach the starting point before Shyam. At that instance in time, Shyam will have run (60/x)4x = 240 m (from the point P). So, Shyam will be 10 m away from the starting point.

18.   When Ranjan scores 100, Rajeev scores 90. When Rajeev scores 75, Rakesh scores 60 and Ritesh scores 50. By scaling of ratios, when Ranjan scores 400, Ritesh scores 240. So, Ranjan gives Ritesh s160 points in a game of 400.

19.   Abhimanyu scores 90 when Chandraprakash scores 100. When Bharat scores 60, Devadatta scores 54. When Bharat scores 50, Abhimanyu scores 45. So, in a game of 200 points, Abhimanyu will score 180, Bharat will score 200, Chandraprakash will score 200 and Devadatta will score 180. Therefore, the teams could be (Abhimanyu & Bharat) v/s (Chandraprakash & Devadatta) OR (Abhimanyu & Chandraprakash) v/s (Bharat & Devadatta).

20.   When Anwer scores 28, Samar scores 20. If Anwer joins Samar in the beginning of the game then Samar's score would have been 100 when Anewr scores 140. But Anwer joins when Samar is at 88. Therefore when Anwer scores 140,Samar will have a score 100+88 = 188.and Parija will be at 200 because it is given that when Parija scores 50, Samar scores 47. Therefore Parija beats Samar by 12 points.

Exercise 8

1.   i. At 4 O' clock the hour hand is at 4 and the minute hand is at 12 i.e. they are 20 minute spaces apart. To be together the minute hand must gain 20 minute spaces over the hour hand.
Now , 55 minute spaces are gained in 60 minutes,
∴ 20 minute spaces will be gained in (60 x 20)/55 = 21 $^9/_{11}$ minutes

ii. To be at right angles the two hands are 15 minute spaces apart. This is possible in two of the following cases :
Case 1: When the minute hand is 15 minute spaces behind the hour hand. to be in this position, the minute hand will have to gain (20 - 15) = 5 minute spaces
Now , 55 minute spaces are gained in 60 minutes,
∴ 5 m.s. will be gained in (60 x 5)/55 = 5 $^5/_{11}$ minutes ∴ they are at right angles at 5 $^5/_{11}$ minutes past 4

Case 2: When the minute hand is 15 minute spaces ahead of the hour hand. To be in this position the minute hand will have to gain (20 + 15) = 35 minute spaces.
Now , 55 minute spaces are gained in 60 minutes,
∴ 35 m.s. will be gained in (60 x 35)/55 = 38 $^2/_{11}$ ∴ they are at right angles again at 38 $^2/_{11}$ min. past 4

iii. To be in the opposite directions they will be 30 minute spaces apart. So the minute hand will have to gain (30 + 20) = 50 minute spaces
Now , 55 m.s are gained in 60 minutes, ∴ 50 m.s will be gained in (60 x 50)/55 = 54 $^6/_{11}$ minutes
∴ they will be in opposite directions at 54 $^6/_{11}$ minutes past 4

iv. Minute hand is 13 m.s behind the hour hand. The minute hand has to gain (20 - 13) = 7 m. s
Now , 55 minute spaces are gained in 60 minutes,
∴ 7 minute spaces will be gained in (60 x 7)/55 = 7 $^7/_{11}$ minutes
∴ the minute hand will be 13 minute spaces behind the hour hand at 7 $^7/_{11}$ minutes past 4

v. Minute hand is 13 minute spaces ahead of the hour hand. in this case the minute hand has to gain (20 + 13) = 33 minute spaces
Now , 55 minute spaces are gained in 60 minutes,
∴ 33 minute spaces will be gained in (60 x 33)/55 = 36 minutes
∴ the minute hand will be 13 minute spaces ahead of the hour hand at 36 minutes past 4

vi. To be equidistant from the figure 5, the minute hand and the hour hand have to be x m.s. away from the figure 5.
Thus the hour hand has to move 5 - x minute spaces and the minute hand has to move 25 + x spaces.
But (5 - x)m.s. of hour hand = 12 (5 - x) m.s. of min. hand
∴ 12 (5 - x) = 25 + x ⇒ x = 35 / 13
The required time is (25 + 35/143) = 27 9/13 min. past 4 o' clock.

2. In a correct clock, the minute hand overlaps the hour hand every 65 5/11 minutes.
But they are together after 66 minutes.
∴ gain in 66 minutes = 65 5/11 - 66 = - 6/11 minutes. Negative gain is a loss.
∴ loss in 1 hour = (6/11) x (60/66) minutes = 60/121 minutes

3. The time from 10 a.m. Friday to 2 p.m. the following Monday is 76 hrs. The clock gains 2 min. per hour.
∴ Gain in 76 hours = 152 min. = 2 Hours 32 minutes
∴ Time shown will be 4 hours 32 minutes.

Other clock gains 76 min. = one hour 16 min.
∴ It will show 3 Hours 16 min.

4. Time form 12 noon Saturday to 12 p.m. following Sunday = 8 days 12 hours. Thus the watch is going to gain (2 + 4 4/5) minutes = 34/5 minutes in 8 days 12 hours i.e. 204 hours
The watch must have been correct when it just gained 2 minutes
34/5 minutes are gained in 204 hours   ∴ 2 minutes are gained in (2 x 204)/(34/5) = 60 hours
60 hours from 12 noon Saturday is 2 days 12 hours ∴ the watch was correct at midnight Monday.

5. Note that at 1:00 the minute hand and hour hand are 5 minutes apart from each other. Then between 1:00 and 2:00, only once, i.e. some time after 1:10, minute hand and hour hand will be 6 minutes apart from each other. Similarly at 11:00 the minute hand and hour hand are 5 minutes apart from each other. Then between 11:00 and 12:00, only once, i.e. some time before 11:50, minute hand and hour hand will be 6 minutes apart from each other. While within every other hours, it can be observed twice that the hour hand and minute hand are 6 minutes apart from each other. Therefore the answer is 44 times.

6. We have - θ = 6 (40 - 11x45/12) = -30/4. At 8:45, the angle between the minute hand and the hour hand of the clock will be 30/4°. This is the same as the required angle.
Therefore $\pm$ 30/4 = 6 (20 - 11m/12). On solving, m = $20^5/_{11}$ minutes or $23^2/_{11}$ minutes past 4 o'clock.

7. At 4:35 p.m. on Monday, the clock has lost 5 minutes or 300 seconds and at 5:45 a.m. the following Monday, it gained 12 minutes or 720 seconds. The clock has, therefore, gained a total of 1020 seconds over a period of 156hours70minutes (9430 minutes). If the clock gains uniformly, it would have shown the correct time (300 x 9430) / 1020 = 2773.529 minutes after 4:35 p.m. on Monday (after 46.225 hours). The clock, thus, showed the correct time at 2:48:30 p.m. on Wednesday

8.
True time	Clock time	(-)	loss in minutes
72	$65^5/_{11}$		$6^6/_{11}$
24 x 60			?

Therefore, the clock looses $(6^6/_{11})$ x (24 x 60) / 72 = $130^{10}/_{11}$ minutes per day.

9. Suppose Arjun goes out at m_1 minutes past 2 o'clock and comes back at m_2 minutes past 4o'clock.
- θ = 6 (10 - 11m_1 /12) and θ = 6 (20 - 11m_2 /12). But m_2 = m_1 - θ/6. On solving the equations, we get, m_1 = $22^{98}/_{121}$. Therefore, Arjuna went out at $22^{98}/_{121}$ minutes past 2 o'clock.

10. From 7:25 p.m. Friday to 1:45 p.m. the following day, there are a total of (35 + 12x60 + 5x60 + 45) = 1100 minutes. In 24hours, i.e.1440 minutes, there is a loss of 12 minutes. So, in 1100 minutes there will be a loss of 12 x 1100/1440 = 55/6 minutes. Therefore, the clock will show 45-(55/6) = 35 minutes and 50 seconds past 1:00 p.m.

11. At 8:30, θ = 6(40 - 11x30/12) = 75.
-75 = 6(35 - 11m/12). On solving m = $51^9/_{11}$ minutes past 7o'clock.

12. a. 180 = 6(45 - 11m/12). Therefore, m = $16^4/_{11}$ minutes past 9 o'clock.
b. $\pm$ 90 = 6(45 - 11m/12). Herefoe, m = $32^8/_{11}$ or m = $65^5/_{11}$. The only possible answer is m = $32^8/_{11}$ minutes past 9 o'clock.
c. 8 minute spaces = 48° So, $\pm$ 48 = 6(45 - 11m/12). Therefore, m = $40^4/_{11}$ or m = $57^9/_{11}$.

13. From 7:25 p.m. on Tuesday to 12:00 midnight on Wednesday is a total of (35 + 28x60) = 1715 minutes. During this time, the clock gains 18 minutes. Therefore, in one day (24 x 60 minutes), the clock will gain =(18/1715)*24*60=$15^{39}/_{343}$ minutes.

14. When the true time is 60 minutes, the clock shows 61 minutes. So, when the clock shows a time of 5 hours, the true time is (5 x 60) / 61 = 4 hours $55^5/_{61}$ minutes. Therefore, the true time is $55^5/_{61}$ minutes past 5 o'clock.

15. When the true time is 12 hours (720 minutes), the clock shows 723 minutes. So, when the clock shows a time of 1442 minutes, the true time must be (1442 x 720) / 723 = $1436^4/_{241}$ minutes. Therefore, the true time is $56^4/_{241}$ minutes past 6:00 a.m.

16. $\theta = \pm 90/3 = \pm 30$. So, $\pm 30 = 6(20 - 11m/12)$ and $m = 16^4/_{11}$ or $m = 27^3/_{11}$. Therefore, the time is $16^4/_{11}$ minutes past 4 o'clock or $27^3/_{11}$ minutes past 4 o'clock.

17. There are 10 intervals between 11 strokes. The first clock has intervals of 3 seconds each, while the second clock has intervals of 2 seconds each. The first clock will strike the fifth stroke after 4 intervals, i.e. 12 seconds. The second clock will strike the seventh stroke after 6 intervals, i.e. 12 seconds. Therefore, there is no time interval between the fifth stroke of the first clock and the seventh stroke of the second clock.

18. First clock loses 2 minutes per hour,the second clock gains 4 minutes. So difference in timings of two clocks is 6 minutes per each hour after 6.10 p.m.Now from 6.10 p.m.Saturday to 12.00 noon next day is a period of 1070/60 hours. So required difference in timings of two clocks is 6x1070/60
i.e. 107 minutes.

19. 15th August 1987 means : (1986 years + 7 months + 15 days)
   1600 years have 0 odd days, 300 years have 1 odd day
   86 years contain 21 leap years & 65 ordinary years and therefore (42 + 65) or 107 or 2 odd days.
   $\therefore$ 1986 years give 0 + 1 + 2 = 3 odd days
   Number of days from 1 Jan 1987 to 15 Aug 1987 = 31 + 28 + 31 + 30 + 31 + 30 + 31 + 15
   = 227 i.e. 3 odd days
   $\therefore$ total number of odd days = 3 + 3 = 6 $\therefore$ the day on August 15, 1987 was Saturday

20. From Friday, 62 days = 8 weeks and 6 odd days. Therefore the day is Thursday.

Exercise 9

1. Profit to be shared in proportion to the capital invested, periods being equal
    A,B,C get Rs. 6300x, Rs. 4200x, Rs. 10500x respectively
    6300x + 4200x + 10500x = 12100 $\Rightarrow$ x = 121/210
    A, B, C get Rs. 3630, Rs. 2420, Rs. 6050 respectively

2. Profit to be shared in proportion to the capital invested, periods being equal.
    A,B,C,D get Rs.(1/3)x, Rs. (1/4)x, Rs.(1/5)x, Rs. (13/60)x respectively.
    (1/3)x + (1/4)x + (1/5)x + (13/60)x = 2460. Therefore, x = 2460.
    A,B,C,D get Rs.820, Rs. 615, Rs.492, Rs. 533 respectively
    A, B, C get Rs. 6000, Rs. 7800, Rs. 9600 respectively.

3. Total capital invested = Rs. 42500 and the total interest paid = (5)(1)(42500)/100 = Rs. 2125.
    The profit that can be distributed among the three partners is 2725 – 2125 = 600. This amount of Rs. 600 is divided among the partners in the ratio 4:5:6, i.e.,Rs. 160, Rs. 200 and Rs. 240 respectively.
    Each person gets the interest due to him and the profit as his share of the total profit.
    Therefore, Naresh gets 500 + 160 = Rs. 660, Suresh gets 625 + 200 = Rs. 825 and Mahesh gets 1000 + 240 = Rs. 1240.

4. B's salary = (25/300)2100 = Rs. 175
    Profit to be distributed = Rs. (2100 - 175) = Rs. 1925
    A & B get Rs. 3600x & Rs. 3400x respectively
    3600x + 3400x = 1925 $\Rightarrow$ x = (11/40)
    A gets Rs. (11/40)3600 = Rs. 990
    B gets Rs. 175 + Rs. (11/40)3400 = Rs. 1110

5. Ram , Tony & Khilona Sing get Rs. 6400(10)x, Rs. 8100(8)x, Rs. 9200(6)x respectively
    64000x + 64800x + 55200x = 35259 $\Rightarrow$ x = 35259/184000
    Ram , Tony & Khilona Sing get Rs. 12264, Rs.12417.30, Rs.10577.70 respectively

6. A's salary = (5/100)7400 = Rs. 370
    Profit to be distributed = Rs. (7400 - 370) = Rs. 7030
    A, B, C get Rs. 6500(6x), Rs. 8400(5x), Rs. 10000(3x) respectively
    6500(6x) + 8400(5x) + 10000(3x) = 7030 $\Rightarrow$ x = (703/11100)
    A gets Rs. 370 + 6500(6)(703/11100) = Rs. 2840
    B gets Rs. 8400(5)(703/11100) = Rs. 2660
    C gets Rs. 1000(3)(703/11100) = Rs. 1900

7. A, B, C get (1/2)(1/3)x, (1/3)(1/4)x, (1/6)(1)x respectively
    (1/2)(1/3)x + (1/3)(1/4)x + (1/6)(1)x = 1000 $\Rightarrow$ x = 2400
    A, B, C get Rs. 400, Rs. 200, Rs. 400 respectively

8. A gets (1/3)15x + (1/6)15x; B gets (1/4)30x C gets (1/5)30x
    (1/3)15x + (1/6)15x + (1/4)30x + (1/5)30x = 4340 $\Rightarrow$ x = (620/3)
    A gets Rs. 1550 B gets Rs. 1550 C gets Rs. 1240

9. | | A | B |
    | --- | --- | --- |
    | Investment | 8400 | x |
    | Period | 12 | 10 |
    8400(12)/(10x) = 3/2 $\Rightarrow$ x = 6720 Therefore, B's capital = Rs. 6720

10. Monthly rent Rs. 30
    For the first five months A,B C pay Rs 50 each
    For the next four months A & B pay Rs. 60 each

For the remaining 15 months A pays Rs. 450
A pays Rs. (50 + 60 + 450) = Rs. 560
B pays Rs. (50 + 60) = Rs. 110
C pays Rs. 50

11.

	A	B	C	D
Cows (C)	12	10	14	9
Period (P)	6	5	10	7
C x P	72	50	140	63

The rent to be shared in the ratio 72 : 50 : 140 : 63
A, B, C, D pay Rs. 72x, 50x, 140x, 63x respectively
$72x = 36$ $x = 0.5$
B pays Rs. 25, C pays Rs. 70, D pays Rs. 31.50
Total rent = Rs. 162.50

12.

	A	B	C1	C2
Sheep (S)	500	750	400	400
Period(P)	6	6	6	6
Factor	2	1	2	1

Rent to be shared in the ratio (1000)6:750(6):[(2400)2 + (2400)1]
i.e. 6000:4500:7200 i.e. 20:15:24
A pays 20/59, B pays 15/59, C pays 24/59

13. A, B, C receive 2/3, 1/6 & 1/6 respectively
i.e. profits are divided in the ratio 4:1:1 ∴ capitals are in the ratio 4:1:1
Let capital invested by A be 4x
A's earlier profit 4x(5/100)   A's new profit 4x(7/100)
4x(7/100) - 4x(5/100) = 200
x = 2500
Capitals of A, B, C are Rs. 10000, Rs. 2500, Rs. 2500 respectively

14. The three persons invest Rs. x, Rs. x + 1000, Rs. x + 2000 respectively
x + x + 1000 + x + 2000 = 9000
x = 2000
The investments are Rs. 2000, Rs. 3000, Rs. 4000 respectively
Profit to be shared in the ratio 2:3:4
They receive Rs. 2y, Rs. 3y, Rs. 4y respectively
2y + 3y + 4y = 5400
y = 600
They receive Rs. 1200, Rs. 1800, Rs. 2400

15. A contributes Rs. (25/100)12000 = Rs. 3000
C contributes Rs. x
B contributes Rs. (80/100)x = (50/100)(x + 3000)
         x = 5000
A, B, C contribute Rs. 3000, Rs. 4000, Rs. 5000 respectively

16. Total Profit = Rs. (4630 + 3730) = Rs. 8360
If B gets Rs. 650 less, the distribution is :A = Rs. 5280, B = Rs. 3080
Thus Capitals are in the ratio 5280 :3080 i.e. 12 : 7
Let the capitals of A & B be Rs. 12x & Rs. 7x respectively
A's salary = (6/100)(12x) = 0.72x
B's salary = (6/100)(7x) = 0.42x
Remainder = 8360 - 1.14x
A gets Rs. 0.72x + (8360 - 1.14x)/2
0.72x + (8360 - 1.14x)/2 = 4360

x = 3000
The capitals are : A = Rs 36000
                      B = Rs. 21000

17.   Total hours of each:
       Man1 = 3 x 8 + 3 x 9 = 51
       Man2 = 3 x 9 + 3 x 10 = 57
       Man3 = 3 x 10 + 3 x 11 = 63
       The wages should be shared in the ratio
       51 : 57 : 63 = 17 : 19 : 21
       Man1 receives (17/57)(45.60) = Rs. 13.60
       Man2 receives (19/57)(45.60) = Rs. 15.20
       Man3 receives (21/57)(45.60) = Rs. 16.80

18.   Profits to be shared in the ratio 3:4:5
       Let the total profit be Rs. x
       A's salary = 0.15x   B's salary = 0.10x   Remainder = 0.75x
       A's share of the remainder = (3/12)(0.75x)   = 0.75x/4
       0.15x + 0.75x/4 = 540
          x = 1600
       B gets = Rs. (160 + 400) = Rs. 560   C gets Rs. 500

19.   During Ist year

	A	B	C
Investment	5000	10000	30000

       Profits to be divided in the ratio 5:10:30 i.e. 1:2:6
       Let A, B, C get Rs. x, 2x,6x respectively
       x + 2x + 6x = 9000
        x = 1000
       A, B, C get Rs. 1000, Rs. 2000, Rs. 6000 respectively
       Now A and B put their profits back in business
       During IInd year

	A	B	C
Investment	6000	12000	30000

       ∴ now profits to be divided in the ratio 1 : 2 : 5
       Let A, B, C get Rs. y, 2y & 5y respectively
       ∴ y + 2y + 5y = 10000
       ∴ y = 1250
       ∴ A, B, & C get Rs. 1250, Rs. 2500, & Rs. 6250 respectively
       ∴ During the third year A has Rs. (6000 + 1250) = Rs. 7250 in business
         B has Rs. (12000 + 2500) = Rs. 14500 in business
          C has Rs. 30000 in business

20.   The total profit be Rs. x
       B's salary = Rs. (120)12 = Rs. 1440
       Profit to be distributed = Rs. (x - 1440)
       Earnings of B = Rs. [(x - 1440)/2] + 1440
       Of these B pays A Rs. (10/100)(45000/2) =   Rs. 2250
       Net earnings of A = (x - 1440)/2 + 2250
       Net earnings of B = (x - 1440)/2+1440-2250
                         = (x - 1440)/2 - 810
       (x - 1440)/2 + 2250 = 2[(x - 1440)/2-810]
            x = 9180
       Total profit = Rs. 9180

Exercise 10

1. C.P. = Rs. x S.P. = Rs. 450
   $(90x/100) = 450$
   $x = 500$
   C.P. = Rs. 500
   For 10% profit,
   S.P. = (110/100)500 = Rs. 550

2. S.P. of each pen = Rs. x
   C.P. of 12 pens = Rs. 10x
   S.P. of 12 pens = Rs. 12x
   gain = Rs. 2x
   gain % = (2x/10x)100 = 20%

3. Let the C.P. be Rs.x   loss = Rs. (x - 9)
   Percentage loss = $[(x - 9)/x]100 = x \Rightarrow x^2 - 100 x + 900 = 0$.
   x = 10 or x = 90
   The C.P. is either Rs.90 or Rs. 10

4. C.P. of 50 doz. eggs = 50(4) = Rs. 200
   Expected S.P. = (105/100)200 = Rs. 210
   Eggs remaining = (50)12 - 40 = 560
    = 560/12 = 140/3 doz.
   S.P. of 1 doz. eggs = 210(3/140) = Rs.4.50

5. C.P. of 1st radio = (120/75)100 = Rs. 160
   C.P. of 2nd radio = (120/125)100 = Rs. 96
   Total C.P. = Rs. 256 Total S.P. = Rs. 240
   Net loss = Rs. 16

6. C.P. of each clock = Rs. x
   Case 1:
   S.P. = (110/100)40x + (120/100)50x = 104x
   Case 2:
   S.P. = (115/100)90x = 103.5x
   104x - 103.5x = 40
            x = 80
   C.P. of each clock = Rs. 80

7. Case 1 : C.P. = Rs. x
   S.P. = Rs. (95/100)x
   Case 2: C.P. = Rs. (90/100)x
   S.P. = Rs. (95/100)x + 42
   (95/100)x+42 - (90/100)x = (1/4)(90/100)x
          x = 240
   C.P. = Rs. 240

8. Total number of oranges bought = 2x
   C.P. = (4/3)x + (6/5)x = (38/15)x
   S.P. = (10/3)x
   gain = (10/3)x - (38/15)x = (36/45)x = (4/5)x
   gain % = (4x/5)(15/38x)100 = 31 $^{11}/_{19}$ %

9. The total stock be 100
   20% sold at 10 % profit = (10/100)20, remainder 80%
   80/2 sold at 10% loss = -(10/100)40

Remaining sold at x% profit

$(10/100)20 - (10/100)40 + (x/100)40 = (5/100)100$

x = 17.5 %

10. C.P. of each bicycle = Rs. x
Total C.P. = Rs. 40x
Marked price = Rs. (125/100)x = Rs. 1.25x
S.P.(cash sales)= (90/100)1.25x = Rs.1.125x
S.P.(credit sales)= (95/100)1.25x = Rs.1.1875x
30 bicycles sold for cash, 10 sold for credit
total selling price = 33.75x + 11.875x = 45.625x
gain = 45.625x - 40x = 2025
    x = 360
C.P. of each bicycle = Rs. 360

11. List price = Rs.50
Selling price = (90/100)50 = Rs. 45
C.P. = Rs. x gain % = 25, S.P. = (125/100)x
(125/100)x = 45
 x = 36
C.P. = Rs. 36

12. Marked price = Rs. x
S.P. = Rs.(80/100)x = Rs. (4/5)x
C.P. = Rs. (4x/5)(100/120) = Rs. (2/3)x
gain = (4/5)x - (2/3)x = 40
        x = Rs. 300

13. New price = (87.5/100)126 = Rs.110.25
Difference in old & new prices = Rs. 15.75
One can buy 9 kg more
New price of sugar = Rs. 15.75 for 9 kg
i.e. Rs. 1.75 per kg
Old price of sugar = 1.75(100/87.5) = Rs. 2 per kg
Rise of 12.5% makes the price 2(112.5/100) = Rs. 2.25 per kg
One can buy 126/2.25 = 56 kg of sugar

14. Salesman buys 1100 gm when he wants 1000 gm and actually sells 900 gm when he wants to sell 1000 gm for the same amount
total gain = 200 gm   gain % = (200/900)100 = $22 \frac{2}{9}$%

15. C.P. of 1000 gm = Rs.100
C.P. of 900 gm = Rs. 90
The tradesman sells 900 gm for Rs.100
Gain = Rs. (100 - 90) = Rs. 10
Gain % = (10/90)100 = $11 \frac{1}{9}$ %

16. Price of each quintal of wheat = Rs. x
Total C.P. = (90 + 50)x = Rs. 140x
S.P. of 90 quintals = (108/100)90x = 97.2x
S.P. of 50 quintals = (110/100)50x = 55x
Total S.P. = Rs. 152.2x
S.P. at 9% profit = (109/100)140x = Rs. 152.6x
152.6x - 152.2x = 60
   x = Rs. 150
Price of each quintal of wheat = Rs. 150

17. C.P.: Sugar1 = Rs. 3(100/80) = Rs. 3.75
Sugar2 = Rs. 5(100/125) = Rs. 4.00

Let 1 kg of each kind be mixed
C.P. of mixture = Rs. (3.75 + 4.00) = Rs. 7.75
S.P. of the mixture = Rs. (6 + 6) = Rs. 12.00
Gain = Rs. (12.00 - 7.75) = Rs. 4.25
Gain % = (4.25/7.75)100 = $54 \frac{26}{31}$%

18.   S.P. of mixture at 20% profit = Rs. 7.50
C.P. of mixture = (100/120)7.50 = Rs. 6.25

C.P. of contents: 5   7

C.P. of mixture: 6.25

Proportion: 0.75   1.25

The two kinds should be mixed in the ratio 0.75 : 1.25 i.e. 3 : 5

19.   C.P. = Rs. 2400
Copies for sale (2000 - 500) = 1500
In every set of 25 copies, 1 copy is given free.
No. of copies given free = (1500/25) = 60
Copies actually sold = 1500 - 60 = 1440
Selling price of each = 3.25 (75/100) = Rs. 2.4375
Selling price = Rs. 2.4375(1440) = Rs. 3510
gain = Rs. (3510 - 2400) = Rs.1110
gain % = (1110/2400)100 = $46 \frac{1}{4}$%

20.   The cost of 1 horse & 1 cow be Rs. x & Rs. y respectively
5x + 10y = 10000   --I
gain on five horses = (15/100)5x = 0.75x
loss on 10 cows = (10/100)10y = y
0.75x - y = 375   --II
Solving I & II simultaneously,
cost of one horse = Rs. 1100
cost of one cow = Rs. 450

21.   Price of the horse = Rs. x
Price of the carriage = Rs. (500 - x)
Net gain = (2/100)500 = Rs. 10
Gain on horse = (20/100)x = x/5
Loss on carriage = (10/100)(500 - x) = (500 - x)/10
x/5 - (500 - x)/10 = 10
    x = 200
Price of the horse = Rs. 200

22.   Price of the horse = Rs. x
Price of the carriage = Rs. (1800 - x)
Gain on horse = (20/100)x = x/5
gain on carriage = (30/100)(1800 - x) = (1800 - x)(3/10)
net gain = (25 5/6)(1800/100) = Rs. 465
x/5 +(1800 - x)(3/10) = 465
      x = 750
Cost of horse = Rs. 750

23.   The watch costs Rs. x to A

A sells it to B for Rs. (120/100)x = Rs. 1.2x
B sells it to C for Rs (90/100)1.2x = Rs. 1.08x
1.08x = 216
x = 200
A buys the watch for Rs. 200

24.   The production cost be Rs. x
        Wholesale price = x + (22/100)x + (18/100)x = (140/100)x = (7/5)x
        Retailer buys at (7x/5)(115/100) = Rs. (161/100)x
        Marked price for retailer = Rs. (161x/100)(125/100) = Rs. (161/80)x
        Selling price of retailer = (161x/80)(96/100)
        (161x/80)(96/100) = 483
           x = 250
        The production cost is Rs. 250

25.   Manufacturer's cost = Rs. x
        Selling price = Rs. (118/100)x = Rs. 1.18x
        Wholesaler's selling price = (120/100)1.18x = Rs. 1.416x
        Retailer's selling price = (125/100)1.416x = 1.77x
        1.77x = 30.09
         x = 17
        Manufacturer's cost = Rs. 17

26.   Bookseller buys the book for Rs. (75/100)2550 = Rs. 1912.50
        Total expenses = Rs (1912.50 + 50) = Rs. 1962.50
        Profit = Rs. (2550 - 1962.50) = Rs. 587.50

27.   Price after 1st discount = (9/10) 1000 = Rs. 900
        Price after 2nd discount = (4/5) 900 = Rs. 720
        Additional expenses = (1/10) 720 = Rs. 72
        Total cost price = Rs. 792
        In order to gain 15 % profit, S.P. should be (115/100) 792 = Rs. 910.80

28.   The cost is Rs. 700
                    Bargain 1   Bargain 2
        After 20% = Rs. 560   After 25% = Rs. 525
        After 15% = Rs. 476   After 12% = Rs. 462
        After 10% = Rs. 428.40   After 8% = Rs 425.04
                    Second Bargain is better

29.   List Price = Rs. 160
        Price after 10% discount = (90/100)160   = Rs. 144
        2nd discount rate = x%
        Price after 2nd discount = (100 - x)(144/100)
        (100 - x)(144/100) = 122.40
              x = 15
        2nd discount rate = 15 %

30.   Price after 5% discount = (95/100)2600 = Rs. 2470
        Price after 10% discount = (90/100)2470 = Rs. 2223
        The amount spent on repairs be Rs. x
        The cost = Rs. (2223 + x)
        S.P. = (110/100)(2223 + x)
        (110/100)(2223 + x) = 2750
                x = 277

The amount spent on repairs is Rs. 277

31.	The original room rent = Rs. x
	rent after 30% discount = (70/100)x = Rs. 7x/10
	Rent after 10% discount = (90/100)(7x/10) = 63x/100
	63x/100 = 189
	x = Rs. 300

32.	Let the number of articles be 100
	C.P. of each article be Rs. 100
	Total cost = Rs. 10000
	Marked price = Rs. 130 (30% above cost)
	50 articles sold at Rs. 130
	S.P. = 130(50) = Rs. 6500
	25 articles sold at (85/100)130 = Rs.110.5
	S.P. = 110.50(25)= Rs. 2762.50
	25 articles sold at (70/100)130 = Rs. 91
	S.P. = 91(25) = Rs. 2275
	Total S.P. = Rs. 11537.50
	Profit = 11537.50 - 10000 = Rs. 1537.50
	Profit % = (1537.50/10000)100 = 15.375 % = $15 \frac{3}{8}$%

33.	Let the C.P. be Rs. 100		The dealer demands Rs. 150
	The buyer should pay {(150 -50)/150}100 = 66 2/3 % only, i.e., 33 1/3 % less

34.	Let the list price of the article be Rs.100.
	Amount received by first dealer on cash payment = 100x(75/100)x(85/100) = Rs. 63.75
	Amount received by second dealer on cash payment = 100x(90/100)x(70/100) = Rs. 63.
	So, for the buyer, the second offer is a better one.

35.	Let the selling price excluding sales tax be Rs. x
	x + (4/100)x = 260		x = 250
	discount for cash sales = (6/100)250 = Rs. 15
	For credit sales a customer pays Rs. 15 extra for two months. Therefore for a year he pays Rs. 90 extra

36.	Let the initial sales be 3x. New sales are 5x
	Let C.P. of x articles be Rs. 100 ,			∴S.P. = Rs. 120
	New S.P. = Rs. 114			Initial revenue = Rs. 360x
	New revenue = Rs. 570x			Initial profit = Rs. (360x -300x) = Rs.60x
	New profit = Rs. (570x - 500x) = Rs. 70x		Profits improve in the ratio 6:7

37.	Price after 10% discount = Rs. 90
	Price after 20% discount = Rs.(80/100)90 = Rs. 72
	Money spent = (10/100)72 = Rs. 7.2
	Total outlay = Rs. (72 + 7.2) = Rs. 79.2
	S.P. for 15% profit = (115/100)79.2 = Rs. 91.08

38.	Let the earlier S.P. be Rs. 100 per article
	New S.P. = Rs. 90 per article		Let the C. P. be = Rs. y
	Earlier Sale = x New Sale = 2x		Profits are equal
	100x - xy = 180x - 2xy		y = 80
	C.P. = Rs. 80		Earlier Profit = Rs. 20x
	New S.P. = Rs. 110		New profit = 110(x/2) - 80(x/2) = 15x
	Profit becomes 3/4 of the earlier i.e. it reduces by 1/4 I.e. 25%

39.	Let the charge for the dinner be Rs. x per player
	The contractor should have received Rs. 24x. He actually received Rs. 21x

His C.P. = (100/112.5)24x = (21 1/3)x
His loss = (21 1/3)x - 21x = 1
    x = 3
The charge for the dinner is Rs. 3

40. Let profit in first condition be x.
 In the second condition Sales increase by 40 %. of Rs. 37500 = Rs. 15000.
 $\therefore$ profit will increase by Rs. 15000.
 Yet it will decrease by Rs.9400 (advertising expenses) and further by increase in managers
 incentive which is 1% of increase in sales of Rs. 15000 = Rs 150.
 $\therefore$ increase in profit = Rs. 15000 - Rs.9400 - Rs.150 = Rs. 5450
 Cost of material is 35% of gross sales therefore it will go up by 35% of 15000=5250
 Profit will be reduced to that extent i.e. net increase in profit = 5450-5250=200.

Exercise 11

1. Number of days for which the money is deposited = 146 days = 146/365 i.e. 0.4 year
Interest = 2500(15/2)(0.4)(1/100) = Rs. 75
Amount = Rs. (2500 + 75) = Rs. 2575

2. Interest paid by B to A =(3500)(10)(3/100) = Rs. 1050
Interest paid by C to B=(3500)(11.5)(3/100) = Rs. 1207.50
B's gain = Rs. (1207.50 - 1050) = Rs. 157.50

3. Let Rs. x be deposited in each
Interest(PO) = x(10)(4/100) = (2/5)x
Interest(B) = x(11)(11/200) = (121/200)x
(121/200)x - (2/5)x = 563.75
         x = Rs. 2750

4. Let the rate of interest be r% per annum
Let the amount borrowed be Rs. P
Interest = (P)(r)(6/100)
Amount = P + (P)(r)(6/100)
P + (P)(r)(6/100) = (7/4)P
         r = 12.5 % = 12 $\frac{1}{2}$%

5. Let the period be n years
Interest1 = (900)(4n/100) = 36n
Interest2 = (1100)(5n/100)= 55n
36n + 55n = 364
n = 4 years

6. Let the rate of interest be r% per annum
Interest on Rs. 450 = 450(r)(3/100) = (27/2)r
Amount = 450 + (27/2)r = 504
         r = 4%
Interest on Rs. 615 in 2$\frac{1}{2}$ years   = 615(4)(5/200) = Rs. 61.50
Amount = Rs. (615 + 61.50) = Rs. 676.50

7. Let the sum of money be Rs. x & the rate of interest be r %
x + x (r)(10/100) = 2x
      r = 10 %
Let the period be n years
x + x (10)(n/100) = 3x
n = 20 years

8. Let the sum be Rs. x & the latest period be n years
x + x (6)(n - 2)/100 = 2400   (I)
x + x (5)(n/100) = 2400   (II)
Solving these two equations simultaneously,
n = 12, x = 1500
The periods are 10 years & 12 years, Sum is Rs. 1500

9. Proceed as in Problem 8.

10. Let the sums invested be Rs x at 4%, Rs. (8000 - x) at 5%
Interest 1 = x(4)(1/100) = x/25
Interest 2 = (8000 - x)(5)(1/100)
= (8000 - x)/20
x/25 + (8000 - x)/20 = 350

x = Rs. 5000
Rs. 5000 invested at 4%, Rs. 3000 invested at 5%

11. Let the two parts be P_1 and P_2
∴$P_1 \times 2 \times 6/100 = P_2 \times 3 \times 8 /100$
$\Rightarrow P_1/P_2 = 2/1$ ∴$P_1/2 = P_2/1 = 6000/3$
∴P_1 = Rs. 4000 & P_2 = Rs. 2000.

12. Let the payments be Rs. x , Rs. 2x, Rs. 3x
Total = Rs. 6x
Interest on payment = x(10)(2/100) + 2x(10)(1/100) = 2x/5
If the payment was done at the end of three years the interest would come to
12800x10x3/100 = 3840.
6x + 2x/5 = 12800 + 3840
∴x = Rs. 2600
The three installments are Rs. 2600, Rs. 5200, Rs. 7800

13. Let the rate of interest per month be r
Total amount repaid = Rs. 10   Interest = Re. 1
(r/100)(9+8+7+6+5+4+3+2+1) = 1
r = 100/45
Rate of interest per annum = (100/45)12 = 26 2/3%

14. Amount after 5 years =
1500 + 1500(14)(5/100) = Rs. 2550
Cost of watch = Rs. (2550 - 2100)
            = Rs. 450

15. Let the sum invested be Rs. x
Firm1 :
x + x(8)(r/100) = 2x
r = 12.5%

Firm2 :
x + x(17)(r/100) = 3x
r = 11.7%
The first firm pays better interest

16. Let the rates be r % & (r +1) %
Interest1 = 2500(r)(3/100) = 75r   &   Interest2 = 1500(r+1)(3/100) = 45(r+1)
75r + 45(r+1) = 525; r = 4%
The two rates are 4% & 5%

17. Let A, B, C borrow Rs. x, Rs. y, Rs. z respectively
x + y + z = 29200   (I)
A pays back Rs. x + x(4)(5/100) = Rs. 6x/5
B pays back Rs. y + y(4)(10/100) = Rs.7y/5
C pays back Rs. z + z(4)(15/100) = Rs.8z/5
6x/5 = 7y/5 = 8z/5
i.e. 6x = 7y = 8z (II)
Solving I & II simultaneously,
x = 11200, y = 9600, z = 8400
A, B, C borrow Rs. 11200, Rs.9600,
Rs. 8400 respectively

18. Let the rates of interest be r & 2r. Proceed as in Problem 16

19. Let the amounts be Rs. x & Rs. y

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$x + y = $ Rs. 18750   (I)

$\therefore x(1 + 6 \times 5/100) = y(1 + 4 \times 5/100)$

$\therefore x/12 = y/13 = 18750/25$

$\therefore x = $ Rs. 9000, $y = $ Rs. 9750

20.   Expenses = Rs. 500

Repairs = 1.5(23000)/100 = Rs. 345

Total Expenses = Rs. 845

Returns expected = 23000(8/100) = Rs. 1840

Total rent per year = Rs (1840 + 845)

= Rs. 2685

Rent per month = 2685/12 = Rs. 223.75

21.   Amount = $8000(1 + 5/100)^3$ = Rs. 9261

Interest = Rs. (9261 - 8000) = Rs. 1261

22.   Half yearly interest = 2%

Total period = 2(1.5) = 3

Amount = $5000(1 + 2/100)^3$ = Rs. 5306.04

Interest = Rs. (5306.04 - 5000) = Rs.306.04

23.   Let the rate be r% p.a.

$800(1 + r/100)^2 - 800 = 82$

$r = 5\%$

24.   Let the sum be Rs. x

C.I. = $x(1 + 19/300)^3 - x$ = 0.2023x   S.I. = x(19/3)(3/100) = 0.19x

0.2023x - 0.19x = 184

   x = Rs. 14959

25.   Let the sum be Rs.x Rate of interest be r%

$x(1 + r/100)^2 = 605$ ..   ..   **A**

$x(1 + r/100)^3 = 665. 50$ ..   ..   **B**

Dividing B by A,

r = 10%, x = Rs. 500

26.   Let the sum be P. $\therefore$ Interest on S.I. of first year = Rs. 2

$\therefore 2 = (S.I.) \times 1 \times 5/100 \Rightarrow$ S.I. = Rs. 40

$\therefore 40 = P \times 1 \times 5 / 100 \Rightarrow P = $ Rs. 800.

27.   Let the sum be Rs.P, rate of interest be r%

$\therefore$ Annual S.I. = Rs. 40/2 = Rs. 20

$\therefore 20 \times 1 \times r /100 = 1 \Rightarrow r = 5\%$

$\therefore P \times 1 \times 5 /100 = 20 \Rightarrow P = $ Rs. 400

28.   Let the sum be Rs. x

$x(1 + r/100)^4 = 2x$

$\therefore (1 + r/100)^4 = 2$

Let the sum become 8 times in n years

$x(1 + r/100)^n = 8x \therefore (1 + r/100)^n = 8 = 2^3 = [(1 + r/100)^4]^3$

$= (1 + r/100)^{12} \therefore n = 12$ years

29.   Let the sum be Rs. x

He pays Rs. $x(1 + 4/100) = $ Rs. (1.04)x

He receives Rs. $x(1 + 3/100)^2 = $ Rs. 1.0609x

1.0609x - 1.04x = 104.50
x = Rs. 5000

30. Let the sum be Rs. x

Amount paid if interest is payable yearly $= x(1 + 20/100)^2 = x(120/100)^2$

Amount paid if interest payable half yearly $= x(1 + 10/100)^4 = x(110/100)^4$

$x(110/100)^4 - x(120/100)^2 = 48.20 \Rightarrow x = Rs. 2000$

31. Let the money borrowed be Rs. x

$x(1 + 10/100)^3 + x(1 + 10/100)^2 + x(1 + 10/100) = 2641$

x = Rs. 725.35

32. Let the installments be Rs. x each

Amount to be refunded $= 1230(1 + 5/100)^2 = 1356.075$

$x(1 + 5/100) + x = 1356.075$

x = Rs. 661.50

33. Let A & B get Rs. x & Rs. y respectively

$x + y = 3903$ (I)

$x(1 + 4/100)^7 = y(1 + 4/100)^9$

$\Rightarrow x/(26)^2 = y/(25)^2 = (x+y)/(676 + 625) = 3$

$\therefore x = Rs. 2028$ and y = Rs. 1875

34. Amount to be paid after three years $2000(1 + 25/200)^3 = Rs. 2847.66$

Let the annual installment be Rs.x

$x(1 + 25/200)^2 + x(1 + 25/200) + x = 2847.66$ x = Rs. 840

35. r = - 6 1/4% (decrease)

Let the value at the beginning be Rs. x

$x(1 - 25/400)^3 = 21093.95$

x = Rs. 25600.24

36. Let the number of years be n

Let the sum be Rs. x

$x(1 + 25/100)^n = 2x$

n = 3.10 years

The sum will double in 3.10 years

It will be more then double in 4 years

37. Amount at the end of first year = 5000(1.04) = 5200
Amount at the end of second year = 5200(1.03) = 5356
Amount at the end of third year = 5356(1.02) = 5463.12
Compound Interest = 5463.12 - 5000 = 463.12

38. Let the original amount be Rs. 100

Year	Principal	Interest in that year	Total
1	100	5	105
2	105	5.25	110.25
3	110.25	5.5125	115.7625

The difference between the interest of third year and Second Year = 5.5512 - 5.25 = 0.2625.

$\therefore$ When the difference between the interests is 0.2625, the Principal is Rs. 100

$\therefore$ When the difference is 262.50, the Principal will be Rs. 1,00,000.

$\therefore$ Total interest $= 10^5 [(1 + 1/20)^4 - 1] = Rs. 21550.625$

39. Let B's capital be Rs. x

A's capital = Rs. 441x/400

B receives Rs. $x(1 + 5/100)^3 = Rs. x(1.05)^3$

Let A invest at a rate of r%

A receives $441x/400 + 441xr(3/100)(1/400) = x(1.05)^3$

r = 5/3%

40. Let the original count be x. After the 1st hour, the count is x (1+5/100) = 1.05x. After the 2nd hour, the count is 1.05x (1 - 5/100) = 1.05(0.95)x. After the 3rd hour, the count is [1.05(0.95)x](1+5/100) = (1.05)(0.95)(1.05)x. Therefore, x = 8 x 10^8.

Exercise 12

1. To compare the two stock options, we will have to compare the returns on Rs. 100.
   Stock 1 : 100(3)/86 = Rs. 3.50.   Stock 2 : 100(4)/110 = Rs. 3.60
   Therefore, stock 2 is the better option.

2. Let the investment be Rs. X. The income from each is $3x/80 + 5x/120 = 380$. Therefore, x = Rs. 4800.

3. Let the market value of the other stock be Rs. X.
   $7600(5)/100 - 7600(105/100x)*4.5 = 2$. Therefore, x = Rs. 95.

4. (i) Income from Company1 = 10(8)(500)/100 = Rs. 400.
   Shares sold at Rs. 16 each (60% premium).
   Amount obtained = Rs. (16)500 = Rs. 8000.
   Shares bought at Rs. 4 (20% discount).
   No. of shares bought = 2000.
   (ii) Income form Company2 = 5(5)(2000)/100 = Rs. 500.
   Difference in incomes = Rs. 100.

5. Let the 2 parts be Rs. x & Rs. (5300 - x).
   Income from the first part = x(3.5/98) = 7x/196.
   Income from the second part = (5300 - x)4/100.
   $7x/196 = (5300 - x)4/100$. Therefore, x = Rs. 2800. So, the two parts are Rs. 2800 & Rs. 2500

6. Let the market price of the 6% stock be x.
   Total income = 5% of 2490 = (5/100)2490 = 124.50.
   $N_1i_1 + N_2i_2 + N_3i_3 = 124.50$, where, Where N_1 = number of $Stock_1$, I_1 = income from $Stock_1$, etc.
   $(900/75) 3.5 + (850/68)3 + (740/x)6 = 124.5$. Therefore, $x = 98\,^2/_3$.

7. The semi-annual dividend is 7% of 15600 = Rs. 1092. Therefore, Ganesh's annual income from the stock is Rs. 2184.

8. The investment required to earn an income of 14765 is 4788782. $\therefore$, the investment required to earn Rs. 7.50 is (7.50 x 4788782) / 14765 = Rs. 2432.50. This is the cost price (including brokerage of 2.50). $\therefore$, the market value is 2432.50 – 2.50 = Rs. 2430.

9. Since 6% on Rs. 100 amounts to a return of 8%, the stock has been bought at a discount. The market value of the stock is (100 x 6) / 8 = 75. $\therefore$, the stock has been bought at a discount of 25.

10. Shantanu's income from the 7% stock is 175 x 7 = 1225. If he wants to invest in the 9% stock, his cost price will be 108.50. An investment of Rs. 108.50 gives Shantanu an income of 9. $\therefore$, in order to earn an income of Rs. 1225, Shantanu will have to invest a sum of (108.50 x 1225) / 9 = Rs. 14768.

11. Arvind's income on selling the shares is (8/9) (300 x 475) = 126667. His initial investment was 300 x 425 = Rs. 127500. $\therefore$, Arvind's net loss is Rs. 833.

12. If a person holds one unit of stock, he will get Rs. 3.75 every 3 months. The annual income from this unit will be Rs. 15. $\therefore$, the annual income from 180 units will be 180 x 15 = 2700.

13. An investment of Rs. 94 earns Rs. 8.5. Therefore, an investment of Rs. 100 would earn (8.5 x 100) / 94 = Rs. 9.04. Therefore, the return on investment is 9.04%.

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14. Anuj's initial income was 1628 x 9 = Rs. 14652. He sells his holding at 155 and invests the amount in oil stock at a premium of 48. He will, therefore, hold (1628 x 155) / 148 = 1705 units of this stock. Since his income from this stock is the same as his initial income, he must earn 14652 / 1705 = Rs. 8.59 per unit of stock. Therefore, Anuj must invest in 8.59% stock at 148.

15. In order to decide the best investment, the four options will have to be compared. The option giving the maximum returns per Rupee of investment is the best option. The returns per Rupee of investment from these options are 7.619, 7.407, 7.826 and 11.20 respectively. The best investment option, therefore, is the 14% stock at 125.

16. Since the brokerage is ½, the effective cost price per unit of stock is 105.50. Therefore, the amount of money required to purchase Rs. 2000 stock is 20 x 105.50 = Rs. 2110.

17. A unit of stock of face value is sold for Rs. 101 ¾. In order to realise Rs. 1221 from the sale of stock, the stock must be worth (100 x 1221) / (101 ¾) = Rs. 1200.

18. Suppose, Jaydeep invests Rs. 100 in 6% stock at 75, Rs. 200 in 7% stock at 80 and Rs. 300 in 10% stock at 90. The yield from these three investments is, respectively, Rs. 8, Rs. 17.5 and Rs.33.33. Jaydeep's total investment of Rs. 600 earns him a total of Rs. 58.83. Therefore the average yield on his investment is 9.8%.

19. Suppose, the person invests Rs. X in 2.75% stock at 95. He has a total of X/95 units of stock and his income from this investment is (2.75X/95). As he sells the stock at par, his income from this sale is 100X/95. He now invests this amount in 3.5% stock. His investment now is {(100X/95) - 900}. His income from this stock is (3.5/100) {(100X/95) - 900}. Since his income in both cases is the same, we get, (2.75X/95) = (3.5/100) {(100X/95) - 900}. Therefore, X = Rs. 3990.

20. The total returns amount to {(3.5 x 1980)/99} + {(4.5 x 3220)/105} = Rs. 208. This amount is returns on a total investment of 1980 + 3220 = Rs. 5200. Therefore, the average rate of return is (208/5200) = 0.04 = 4%.